The Application of Symmetry Concepts to Regular Repeating Pattern Design

Pundita Tantiwong

A thesis submitted in accordance with the requirements for
the degree of Doctor of Philosophy

De Montfort University
Faculty of Art and Design

August 2000

The candidate confirms that the work submitted is her own and that appropriate credit has been given where reference has been made to the work of others.

Abstract

This thesis is concerned with the application of geometric symmetry concepts to regular repeating pattern design, which includes aspects of pattern analysis (i.e. the identification of a pattern's geometrical structure) and pattern synthesis (i.e. the construction of patterns). The aim is to develop tools and an understanding which could provide an awareness among designers of the potential for the application of geometric symmetry. Comprehensive explanation of the theoretical principles underlying the geometry of patterns as well as the systematic classification of three design categories, i.e., finite designs, band patterns and all-over patterns has been given in association with widely accepted notations. The potential of symmetry classification as a worthwhile analytical tool is explored through its application to six categories of traditional Thai textile patterns: three categories of village or domestically-produced textiles (*tung, muon khit* and *pha zin*) and three categories of court textiles (*pha lai-yang, pha phuum* and *pha yok*). Data are tested to establish if the patterns from these different categories share particular symmetry preferences, and if certain symmetry preferences may be associated, firstly, with certain patterning techniques and secondly with aspects of Thai culture. The two-fold approach of space sub-division and space filling with respect to geometrical principles of the plane is examined through the use of two fundamental lattices, i.e. a square lattice and an isometric lattice. Certain construction techniques are presented in the context of unit construction, linear construction and a hybrid approach. The evolution of regular repeating pattern concepts is reviewed through groups of designs ranging from traditional Islamic patterns to computer-generated images. Design variations generated from seventeen symmetry groups of all-over patterns are explored in the context of unit content and unit translation. A series of experimental designs are created and presented in illustrative forms accompanied by texts.

Acknowledgements

This PhD submission was made possible by a five-year scholarship awarded by Chulalongkorn University, Bangkok. The scholarship included a Master of Art in Fashion and Textile Studies (1995-96) and a PhD since September 1996 at De Montfort University, Leicester.

I wish to express my gratitude to Professor Tom Cassidy, the first supervisor, and Dr. Michael Andrew Hann, the second supervisor (School of Textile Industries, The University of Leeds), for their invaluable guidance, advice and support.

During 1992 to 1994 I had pursued fieldwork on the Conservation Project of Supplementary-weft Patterns in Yasothorn Province, which was led by Associate Professor Aurapin Panthong and Associate Professor Nuan-noy Boonwong. This provided an extensive volume of photographic materials, which were used for the pattern analysis in this research. I am particularly grateful to Khun Payao Aun-siri who generously provided a photographic collection of court textiles held at the National Museum, Bangkok, and Rosemary Crill who supported the access to an un-displayed collection of Southeast Asian textiles held at the Victoria and Albert Museum, London.

To the following individuals who assisted in various ways, I wish to record my appreciation of the support and critiques of Dr. Adam Hardy, Dr. Henry Ginsburg, Dr. Gary Fozzard, Carolyn Hardaker, Tony Walsh and Kim Mitcham at Stoll, UK, Keith Cass, Noel Healy, Rob Snow, Mei-Chih Huang and Catherine Docherty. Finally, my thanks must go to my parents, Somchairuk and Peter Nuttall, Takerng Pattanopas, Apinya Boonprakob, Worawut Suteeweerakajorn and Tony Rothwell, for their advice and encouragement from the beginning to the final full stop.

Contents

Figure Index

Page

Table Index

"See how various the forms, and how unvarying the principles"

[Jones, cited by Gombrich, 1979, p.55]

Chapter 1 Introduction

> *" The notion of pattern derived from the recognition of a periodic recurrence; that is to say, two elements are required: a repeat motif conjoined with a structured or rhythmic base."*
>
> [Wade, 1982, p.1]

To generate a regular repeating pattern, a motif undergoes repetition on an underlying structure which is governed by the geometrical logic known as geometric symmetry. As a definition cited by El-Said and Parman, symmetry or *symmetria* in classical terminology meant the proportionality between the constituent elements of the whole [El-Said and parman, 1976, p.6]. Therefore, it incorporates the properties of balance, harmony, regularity and order. Geometric symmetry is a symmetry with respect to geometric transformations which involve any transformations that affect only the geometric properties of a system [Rosen, 1975, p.28].

The concept of geometric symmetry was discovered and developed within the sphere of mathematics, however, it has been applied to a diversity of disciplines. As Escher pointed out:

> *" Crystallographers have put forward a definition of the idea they have ascertained which and how many systems or ways there are of dividing a plane in a regular manner. In doing so, they have opened the gate leading to an extensive domain, but they have not entered this domain themselves. By their very nature they are more interested in the way in which the gate is opened than in the garden lying behind it."*
>
> [cited by Bool, 1986, p.10]

Crystallographers have developed concepts related to geometric symmetry for the understanding of the structure of three-dimensional crystals. The application of the concepts to the understanding of two-dimensional phenomena helps in the understanding of the geometry of designs and their construction. In the context of pattern design, geometric symmetry concepts are of use in two respects: in pattern analysis and in pattern synthesis [Hann and Thomson, 1992, p.3].

A wide range of publications has contributed to the study of pattern design; these are from a range of disciplines. *The Grammar of Ornament* by Owen Jones [1856] appears to be one of the first publications in this field, where attention was focused on the cultural and historical aspects of patterns rather than on their geometrical structure. In *Traditional Methods of Pattern Designing* by Christie [1910] (reprinted as *Pattern Design* [1969]), classification of designs is based on design features or sources of ornaments (e.g. plant and animal motifs and isolated, waved, cross-band, interlocking and colour counterchange patterns).

Meanwhile, in *The Sense of Order* by Gombrich [1979], the emphasis is on the manifestation of history, theory and psychology inherent in decorative art and crafts from a great variety of cultures.

From the geometrical viewpoint, publications including *Pattern Design* by Day [1903, reprinted 1979] and *Geometric Patterns & Borders* [1982] by Wade; these suggest the analysis of pattern construction through the use of geometrical structures. The study of patterns from a geometrical viewpoint has been carried out in *Order in Space* by Critchlow [1969], *Connections: The Geometric Bridge between Art and Science* by Kappraff [1991], and publications related to tilings (e.g. *Tiliings and Patterns* by Grünbaum and Shepard [1987]) and Islamic patterns (e.g. *Arabic Geometrical Pattern & Design* by Bourgoin [1973], *Islamic Pattern: An Analytical and Cosmological Approach* by Critchlow [1976], *Geometric Concepts in Islamic Art* by El-Said and Parman [1976], and *Pattern in Islamic Art* by Wade [1976]).

An attempt to broaden the concept of geometric symmetry into the sphere of design, textile design in particular, was carried out by Woods in the mid-1930s. A series of papers published in Transactions of the Journal of the Textile Institute aimed to de-mystify the mathematical rules underlying the geometrical structures of regular repeating patterns [Woods, 1935/6]. The objective was to encourage an awareness among textile designers of the potential for the application of the principles of geometric symmetry to an understanding of regular repeating pattern design. Subsequent works which have adopted theoretical principles of geometric symmetry and terminology developed by crystallographers for the non-mathematician public were produced by Padwick and Walker [1977], Schattschneider [1978], Stevens [1981], Washburn and Crowe [1988] and Hann and Thomson [1992]. All the patterns in these publications are thus grouped according to symmetry groups and include related notation. Publications related to the development of the geometric symmetry concepts applied to a wide scope of pattern analysis and synthesis were provided in series of papers edited by Hargittai [1986/89] and Emmer [1993].

As Washburn and Crowe [1988, p.24] pointed out symmetry seemed to be a diagnostic feature in the perception of design. A regular repeating pattern may be constructed using a combination of one or more of the four symmetry operations (i.e. translation, rotation, reflection and glide-reflection). It can thus be grouped into either one of the finite designs, of classes cn or dn, one of seven classes of band patterns, or one of seventeen classes of all-over patterns depending upon the symmetry groups it admits.

Certain anthropologists, archaeologists and art/design historians have applied symmetry classification to the analysis of patterns of decorated items from archaeological, historic or cultural settings in addition to the customary categorisation by reference to media, styles, cultures and periods, which Hann and Thomson [1992, p.1] believed constituted subjective commentary and superficial analysis. As Hann stated:

> *" A means by which textile and other surface patterns can be classified by reference to the symmetry characteristics of their underlying structures is developed and shown to be an objective, systematic and reproducible means providing meaningful and standardised descriptions of regular geometric patterns."*
>
> [Hann, 1991, p.ii]

A great quantity of relevant literature cited by Washburn and Crowe [1988, pp.24-34] and Hann and Thomson [1992, pp.7-9] indicates that patterns from different cultural settings exhibit their own unique symmetry preferences evidenced by non-random distributions of symmetry classes employed. As noted elsewhere this non-randomness is of importance, for it indicates that symmetry classification is in some way culturally sensitive [Hann, 1992, p.581]. Certain symmetry characteristics shared by patterns from different categories (e.g. cultural settings, archaeological sites, end uses and patterning techniques) suggest cultural association either within one cultural setting or between different cultural settings, which leads to the study of cross-cultural comparison.

Hann explored the potential of symmetry classification applied to two groups of textile patterns: i) case studies from four distinct cultural settings: traditional Javanese batiks, traditional Sindhi ajraks, jacquard-woven French silks (Autumn, 1893) and Japanese textiles produced during the Edo period (1604-1867) [Hann, 1992] and ii) case studies of Japanese textiles which was produced during consecutive times of the Edo period (1604-1867) and employed one of two different patterning techniques [Hann, 1993]. The first research was aimed at testing that patterns from each cultural setting show a non-random distribution of symmetry preference. The latter research was aimed at testing two hypotheses: that the symmetry preferences of a given culture are maintained over time in the absence of external forces of change, and that patterns produced by different techniques will exhibit broadly similar symmetry characteristics.

The link between symmetry and culture was explored further, in the context of Asian cultures by Van Esterik who examined the symmetry features used in Ban Chiang pottery, Thailand, to further understand symbolism and the process of creating symbols by pre-historical communities [Van Esterik, 1979]. Haake recognised an intense link between the particular symmetry characteristics of Javanese batik patterns and the ancient Asian philosophy models of *mancapat* (a model of the cosmos) and dualism (the co-existence of opposites) [Hakke, 1996]. Summerfield explored how motifs, structures and folds in Minangkabau ceremonial garments related to social standards and the rules of *adat* [Summerfield, 1996]. Yu revealed the concept of positive and negative on two co-existing features under two-fold rotation and bilateral reflection in certain Chinese arts and crafts [Yu, 1989].

In the context of this thesis, attention is focused on the application of geometric symmetry concepts to the understanding of regular repeating pattern design. The aim is to establish a theoretical awareness of symmetry concepts as they may be applied to pattern analysis in which the application of symmetry classification to groups of patterns from particular cultural settings is explored, and aspects of pattern synthesis in which the principles are applied to the creation of patterns. To generate background knowledge of geometric symmetry, chapter 2 provides a reasonably comprehensive explanation of the theoretical principles of geometric symmetry and the symmetry classification of finite designs, band patterns and all-over patterns. Associated terminology, notation and schematic illustrations are presented where appropriate.

Chapter 3 examines symmetry characteristics in representative samples of traditional Thai textiles. Six categories of textiles are considered: three categories of village or domestically-produced textiles (*tung, muon khit* and *pha zin*) and three categories of court textiles (*pha lai-yang, pha phuum* and *pha yok*). Each group has an individual end use and was produced by one of three patterning techniques (i.e. supplementary-weft, weft-ikat and printing/ painting). The study was aimed at testing two hypothesis: that the patterns from these different categories share particular symmetry preferences, and that certain symmetry characteristics may be associated firstly, with certain patterning techniques and secondly, with aspects of Thai culture.

Rather than the analysis of existing patterns, geometric symmetry concepts can also be applied to the context of pattern creation. Although the symmetry concepts are not new in the sphere of mathematics its application may be unfamiliar to designers. This, as Horne pointed out, may be because of the impenetrable theories and incomprehensible language and terminology used in mathematical domain, and the restriction of rigid geometric features that may hinder designers' creativity [Horne, 1997, p.412].

Since symmetry classification has been extensively employed by anthropologists, archaeologists and art/design historians for the analysis of patterns from different cultural sources, the clarification of the theoretical concepts has then been developed practically for those from the non-mathematical domain.

Rigid geometric structures are deformable, and offer infinite possibilities for the designer. Snowflakes, for example, exhibit natural symmetry in which unpredictable and infinite varieties of six-pointed designs are bounded in invariant hexagonal shapes; no two are identical [Horne, 1997, p.1].

The concept could interface pragmatically with designers' customary pattern construction means. Watson [1954, 6th ed. 1996, p.264] identified three possible ways to create figurative jacquard designs: i) by geometric arrangement, ii) by the conventional treatment of natural and artificial forms, and iii) by the adaptation or reproduction of earlier designs.

Designers may use a geometric structure as a guideline not only for the packing of square-/rectangle-shaped repeating units on the infinite plane, but also for the distribution of decorative elements within each square-/rectangle-shaped repeating unit. A rigid geometric structure can be modified by applying conventional treatment of natural and artificial ornaments, and can also be re-used to generate new combinations.

Over centuries a conceptual bridge between mathematical geometric principles and design application has been built by certain artists, artisans and designers. The Muslims have developed geometrical concepts through the abstraction of polygonal patterns, in which figures referred to as *"any likeness of any thing that is in the heaven above, or that is in the earth beneath, or that is in the water under the earth"* are prohibited according to Islamic proscription [Bool, 1986, p.23]. However, for Escher, figurative designs seem to be preferable. Escher's attempt to create chaotic figurative designs in order has proved to be an influential achievement that has inspired and fascinated both scientists and designers. (Related publications include MacGillavry [1976], Locher [1982], Coxeter [ed.al., 1986], Schattschneider [1990/93] and Ernst [1994].)

Various chaotic figures were packed on the underlying geometric structures derived from surface decoration at the Alhambra, Spain. As remarked by Escher (in Locher):

> *"I find the emphasis upon what is 'in the water under the earth' particularly striking, because fish make such suitable motifs for filling my planes... Try as I will, I cannot accept that something as obvious as making adjacent figures recognisable... My experience has taught me that the silhouettes of birds and fish are the most gratifying shapes to all for use in the game of dividing the plane. The silhouette of a flying bird has just the necessary angularity, while the bulges and indentations in the outline are neither too pronounced nor too subtle."*
>
> [cited by Coxeter, 1986, p.16]

The use of just bird and fish motifs led Escher to create a variety of patterns. The birds and fish reveal the design theme of duality where the birds are "water" for the fish and the fish are "air" for the birds.

The interplay of repeating motifs and underlying structures seems to be the most significant issue for pattern design. Different combinations cause different design outcomes. In some patterns, motifs are obviously seen more explicitly than the pattern structures. Meanwhile in other patterns the reverse is true.

An application of symmetry concepts to the creation of extensive varieties of repeating patterns has been explored in chapter 4 and 5. Chapters 4 discusses the principles of pattern construction by which the relationship between repeating units and underlying structures is examined through the two-fold approaches of space sub-dividing and space filling. As Escher explained:

> *"A plane, which should be considered limitless on all sides, can be filled with or divided into similar geometrical figures that border each other on all sides without leaving any empty spaces. This can be carried on to infinity according to a limited number of systems."*
>
> [cited by Bool, 1986, p.11]

Space-filling patterns, examples of which include mosaics, tilings, lattices, networks or tessellations, can be viewed as vertices or point conditions, lines or reticulations of the surface, or as the fitting together of regular shapes to fill a surface [Critchlow, 1969, p.60]. The organisation of an array of points, a series of lines and a construction of planes is governed by the geometric principles of the plane, which basically involve three fundamental shapes (i.e. a square, an equilateral triangle and a hexagon), two associated lattices (i.e. a square lattice and an isometric lattice), and the four symmetry operations.

An investigation is made of five examples, i.e., Islamic pattern construction, seventeen geometric symmetry structures, linear construction, textile repeating formats and weave structure formats. Islamic pattern construction and seventeen geometric symmetry structures reveal the basis of space filling referred to as additive construction, that is the means by which a repeating unit is modified prior to being constructed on the associated structure. Whereas linear construction exhibits the basis of space sub-dividing, subtractive construction is the means by which an underlying structure is primarily generated by the construction of series of lines.

In cases of Islamic patterns, three proportional polygons (i.e. the square and the root two system of proportion, the hexagon and the root three system of proportion, and the pentagon and the golden ratio) underlie systematic space sub-division within a unit and the construction of repeating units on either square, hexagonal, rectangular or rhombic lattices. The seventeen symmetry groups determine numbers and orientations of fundamental regions to be constructed on their individual structures. The connection of lines and space along the unit edges or the intersection of series of lines produces a continuous polygonal network relating every constituent part to an all-over structure. Decorative elements can then be filled in the intervals. Straight lines bounded around each polygonal constituent unit may be replaced by any sections or curves shared by adjacent units.

In the context of textile design, perpendicular directions of fabric width and length, and mechanical means of manufacture determine the shape and size of the repeating unit which is usually bounded in either a square or a rectangle. A variety of repeating formats and weave structure formats reflect the hybrid approach by which designers can apply the basis of space filling and space sub-dividing to any stages of the distribution of motifs within a unit and the construction of repeating units across fabric width and along fabric length.

Case studies from four design categories (i.e. two-dimensional graphics, three-dimensional objects, computer-generated images and contemporary household products) reveal the development of the repeating pattern concepts, in which rigid geometric structures have been transformed into varieties of designs.

Chapter 5 is built up on the development of the concepts of pattern constructions discussed in chapter 4. Patterns having the same symmetry class may look different due to various features. For example, when two colours are symmetrically applied, there are the total of twenty-three possibilities of counterchange designs generated from seven symmetry groups of band patterns and forty-six possibilities of counterchange designs generated from seventeen symmetry groups of all-over patterns. The principles of counterchange designs in which the symmetry operations involve the systematic interchanging of colours has not been mentioned here due to the huge scope and variations possible. However, designers may take it into account as a practical way to obtain a variety of designs on one pattern structure. A wide-ranging collection of publications has contributed to the development of the further understanding of the geometrical principles of counterchange designs. Important work has been done by Woods [1935/6, Part I, II, III and IV], Shubnikov and Belov [1964], Loeb [1971], Schattschneider [1986], Grünbaum and Shephard [1987, Chapter 8], Washburn and Crowe [1988], Lin [1995, Chapter 6] and Horne [1997, Chapter 6].

Varieties of the unit contents (e.g. types of motifs, numbers of motifs and motif orientations with reference to the symmetry groups and the unit boundaries) are also the critical features that may produce varieties of designs within each symmetry group. According to the fact that a fundamental region is not necessarily an asymmetrical motif in every case. It may contain a group of motifs admitting a collection of symmetry operations, known as a symmetry-obtained unit. To present a variety of designs that may be generated within each symmetry class, Horne [1997, Chapter 3] applied a wide-range of finite designs of classes cn and dn as the fundamental regions to the construction of configuration designs of finite designs of classes cn and dn, seven classes of band patterns and seventeen classes of all-over patterns.

Chapter 5 of this thesis is focused on an investigation of varieties of seventeen symmetry groups of all-over patterns which may be generated from the combinations of two construction elements, i.e., different features of unit cells and different repeating formats.

NOW NO SWIMS ON MON

Two-fold rotation of the same meaning, cited by Escher in Schattschneider, 1990, p.20

Chapter 2 Principles of Symmetry and the Classification of Regular Repeating Patterns

2.1 Introduction

As pointed out by Hann and Thomson [1992, p.1], a regular repeating pattern is a design in the plane which exhibits a repetition of a motif or motifs at regular interval. A tiling (sometimes referred to as a tessellation) may be the special case where the collection of closed shapes covers the plane without gaps or overlaps. Geometric symmetry principles have been applied to the description and understanding of how any such motif undergoes repetition to generate the entire design.

This chapter proposes to provide a basic understanding of geometric symmetry and in particular the principles as they may be applied by designers to the understanding of various aspects of repeating pattern design. The focus is on the creation of designs as well as their classification.

An array of literature from the sphere of design (e.g. Stevens [1981], Washburn and Crowe [1988], Hann and Thomson [1992] and Horn [1997]) presents general descriptions in terms of terminology, definition, classification and notation. The comprehensive discussion of geometrical proof and explanation is available from mathematical and crystallographic publications (e.g. Woods [1935/36], Grünbaum and Shepard [1987] and Kappraff [1991]).

There is a range of different terminology used in publications from various fields. Therefore, the terms used in this chapter will be typed in italicised form when they are firstly mentioned. Also, there are certain terms that may cause a degree of ambiguity. The terms *motif* and *figure* are used to mean a recurring part of a decorative area or graphic in the plane. The term *design* is applied to any motifs or a set of motifs, which admit at least one symmetry operation. While the term *pattern* refers to the design in which a repetition undergoes by translation.

2.2 The Principles of Geometric Symmetry

As noted by Schattschneider, symmetry is the action or transformation that produces the symmetric properties of the design [Schattschneider, 1986, p. 678]. A motif is said to be asymmetrical when it cannot be divided into two or more smaller identical parts by lines which meet at a centre point [Schattschneider, 1990, p.32]. On the other hand, it is said to be symmetrical when it consists of two or more parts of identical size, shape and content [Hann and Thomson, 1992, p.1].

The smallest identical part including a motif and its enclosing area, which can be repeated to complete the design without gaps or overlaps, is called as a *fundamental region*. A repetition of fundamental regions is basically governed by the relevant *symmetry operations*, or in the terminology of mathematicians as *isometries* (iso = equal, metron = measure) [Schattschneider, 1986, p.673 and Grünbaum and Shepard, 1987, p.26].

Woods [1935, T.341] defined a symmetry operation as a transformation which shifts a motif from one such equivalent position to another, whereas Grünbaum and Shepard [1987, p.26] provided more specific definition as a congruence transformation or any mapping of the Euclidean plane onto itself which preserves all distances. A design is symmetrical when it admits one or more symmetry operations in addition to the *identity symmetry*, in which the motif seems to have no movement at all.

There are merely four distinct types of symmetry operations in which a motif can be related to a congruent copy of itself. *Translation* and *rotation* are denoted as direct isometries, while *reflection* and *glide-reflection* are denoted as indirect isometries, by which a motif is reversed, e.g. from left-hand motif to right-hand motif.

- **Translation** by which a motif undergoes repetition in any direction at regular intervals, whilst retaining the same orientation. In Figure 2.1, a letter "p" is repeated horizontally by translation *T*, which shifts "p" from the initial position on the left-hand side to its congruent copy on the right-hand side. A translation *–T* is an *inverse symmetry*, which occurs in the opposite direction of translation *T* to shift the congruent copy back to the initial position.

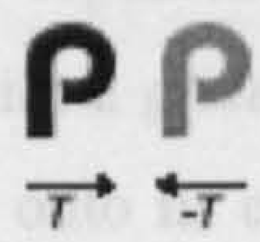


Figure 2.1 Translation

- **Rotation** is the means by which a motif undergoes repetition about a fixed point through a certain fractional angle of 360°, that leaves the motif exactly coinciding with its original position. The number of equivalent positions where motifs locate about a fixed point indicates the order of rotation, referred to as *n-fold rotation*, and hence the centre of rotation is called as an *n-fold rotational centre*, where n is an integer. Examples of n-fold rotations, where n is 1, 2, 3, 4, 5 or 6 are shown in Figure 2.2a-f

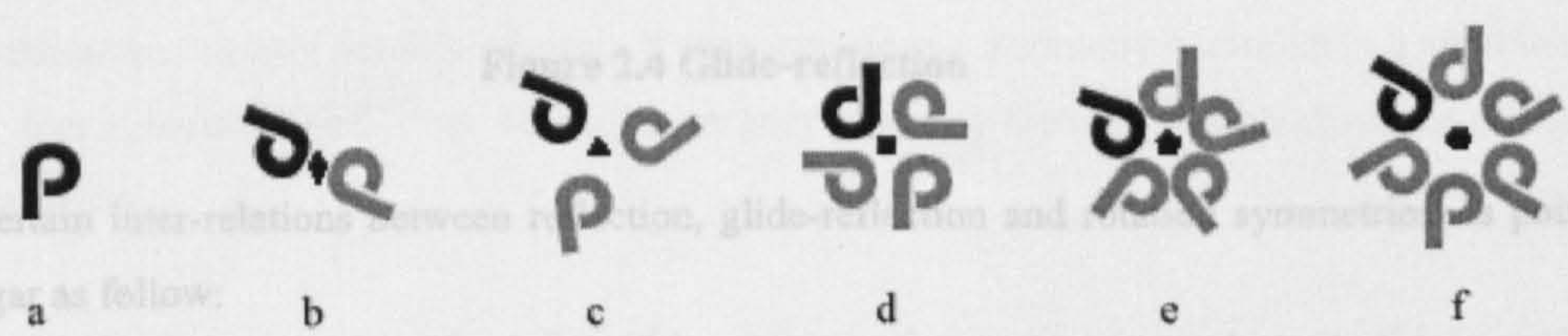


Figure 2.2a-f N-fold rotations, where n = 1-6

In Figure 2.2a, a letter "p" presents a rotation of 360° (n = 1), by which a letter "p" is shifted onto its original position. This symmetry produces the same effect of having no movement as the identity symmetry, and the motif is thus identified as asymmetrical. In Figure 2.2b, a rotation of 180° (n = 2) is evident, or; this is also known as a *half-turn*, about a two-fold rotational centre symbolised by a diamond. In Figure 2.2c, a rotation of 120° (n = 3) is evident about a three-fold rotational centre symbolised by an equilateral triangle. In Figure 2.2d, a rotation of 90° (n = 4) is evident about a four-fold rotational centre symbolised by a square. In Figure 2.2e, a rotation of 72° (n =5) is evident about a five-fold rotational centre symbolised by a pentagon. In Figure 2.2f, a rotation of 60° (n = 6) is evident about a six-fold rotational centre symbolised by a hexagon.

- **Reflection** is an axial symmetry by which a motif undergoes repetition by producing its mirrored image across an imaginary straight line, known as a *reflection axis*. In Figure 2.3, a letter "p" is shifted horizontally onto its mirrored image on the right-hand side with respect to a vertical reflection axis, which is represented by a double line.

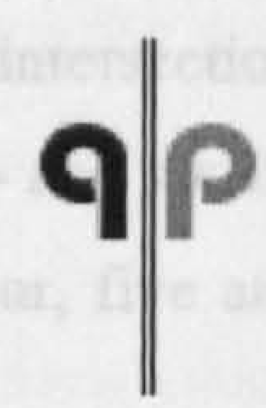


Figure 2.3 Reflection

- **Glide-reflection** by which a motif is repeated in one action through a combination of translation and reflection in association with a *glide-reflection axis* [Hann and Thomson, 1992, p.4]. In Figure 2.4, a letter "p" is shifted onto its congruent copy (above right) by a reflection followed by a translation which is parallel to the reflection axis, or by a translation followed by a reflection in a reflection axis parallel to the translation vector. A glide-reflection axis is represented by a dashed line which denotes both reflection axis and translation vector.

Figure 2.4 Glide-reflection

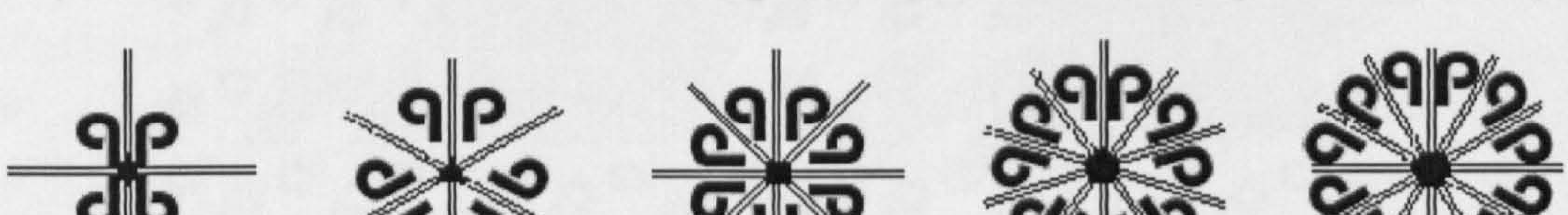
There are certain inter-relations between reflection, glide-reflection and rotation symmetries, as pointed out by Hoggar as follow:

- The presence of both rotation and reflection implies at least two reflection directions. Similar for glide-reflections.
- The presence of non-parallel reflections, glide-reflections or a combination implies rotations.
- The least angle between the lines of two reflections, glide-reflections or a combination is π/n, where n is the number of reflection or glide-reflection axes
- The crossing at right angles of a reflection or a glide-reflection axis with another axis implies the presence of a two-fold rotational centre.

[Hoggar, 1992, p.63]

Since there are no limited numbers of reflection axes intersecting at one point, examples in Figure 2.5 exhibit four cases where two, three, four, five and six reflection axes intersecting each other at 90°, 60°, 45°, 36° and 30°. In Figure 2.5a, an intersection of two reflection axes at the right angle produces two-fold rotation about the intersecting point. This may be identified either as reflections in perpendicular directions in which a letter "p" is shifted horizontally onto its mirrored image on the right-hand side across a vertical reflection axis, then a combination of two letters is shifted vertically onto their mirrored image in the lower part across a horizontal reflection axis, or as a two-fold rotation of a pair of motifs admitting vertical or horizontal reflection. The intersections of three, four, five and six reflection axes (Figure 2.5b-e), which produce three-, four-, five- and six-fold rotations about the intersecting points may be identified either as the reflections in three, four, five and six directions, or as three-, four-, five- and six-fold rotations of pairs of mirrored motifs.

Figure 2.5a-e Illustrations show the intersections of two, three, four, five and six reflection axes, by which produce two-, three-, four-, five- and six-fold rotational centres at the intersecting points.



a b c d e

A *symmetry group* is a collection of all symmetry operations which underlies the transformation of a motif which is superimposed exactly onto its original. It may include one symmetry operation or a combination of any of the four symmetry operations. Moreover, as pointed out by Stevens, it has to have the following three characteristics:

i) Each operation can be followed by the second operation to produce the third operation that itself is a member of the group.

ii) Each operation can be undone by another operation, that is to say, for each operation there exists an inverse operation.

iii) The position of the pattern after an operation can be the same as before the operation, that is, there exists an identical operation which leaves the figure unchanged.

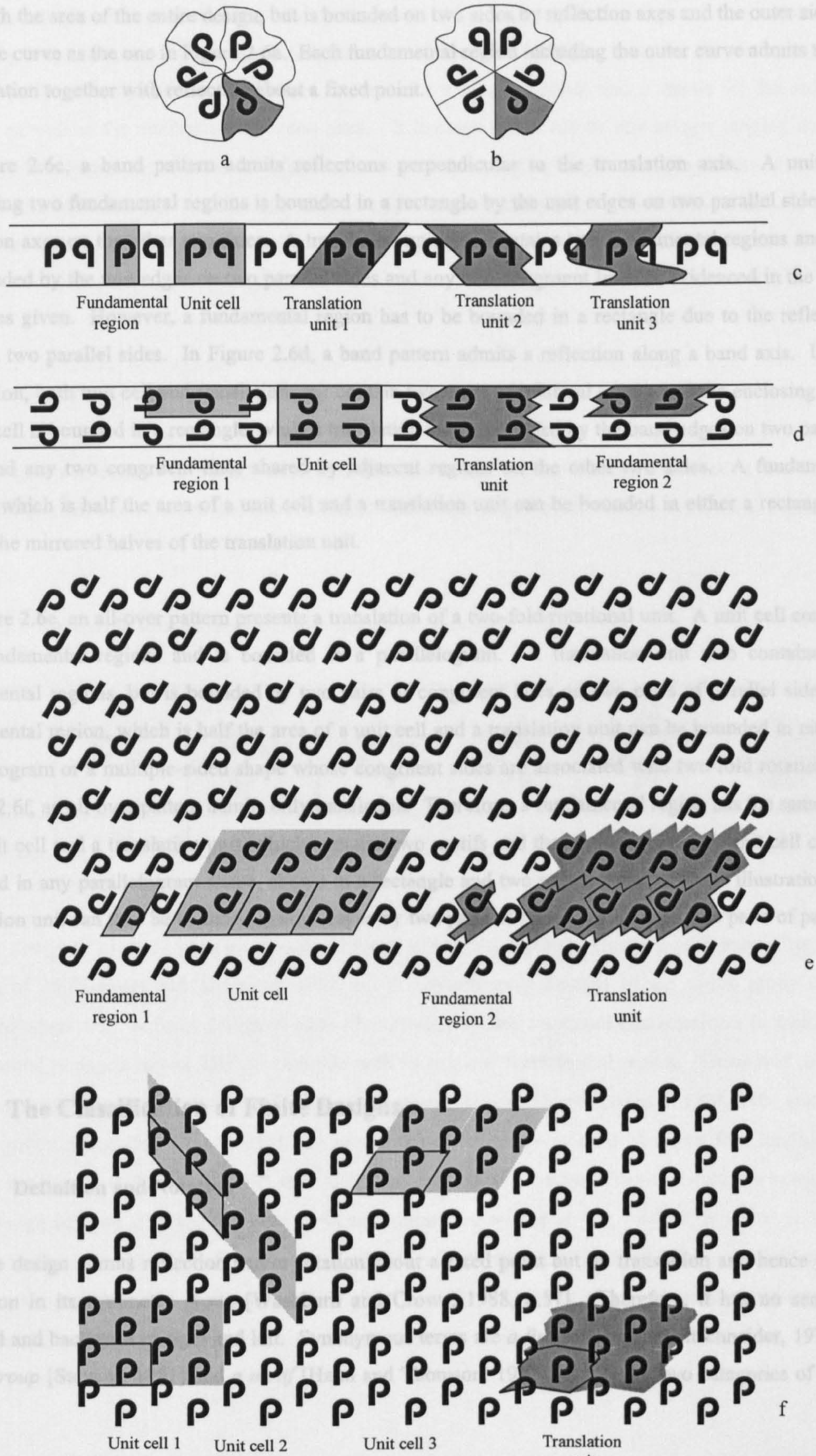
[Stevens, 1981, p.11]

There are three categories of designs distinguished by their symmetry group arrangements. A design that exhibits symmetry about a fixed point, with no translation, is regarded as a *finite design*. A design which undergoes repetition by successive translation in one direction is regarded as a *band pattern*. A design which is translated successively in two non-parallel directions is regarded as an *all-over pattern*.

In cases of band and all-over patterns, where the repeating unit and its relevant area admits successive translations on a framework of corresponding points, is called a *lattice* or *net*. Hammond defined a lattice as an array of points in space in which the environment (i.e. the spatial distribution and orientation of the surrounding motifs) of each point is identical [Hammond, 1997, p.34]. By connecting the lattice points with sets of parallel lines, the plane is divided into parallelograms. Any such parallelogram is called as a *lattice unit* or a *unit cell*. Each unit cell must have the same shape and content and can be successively translated to produce an infinitely repeating design.

The smallest area of the plane under successive translation is generally referred to as a *translation unit*. Although it contains the same area as a unit cell, it is not necessarily bounded in a parallelogram shape. As noted by Horne [1997, p.96], the region of a fundamental region and a translation unit can be represented in any shape if they are not bounded entirely by reflection axes and/or the exterior boundaries of the whole design. Each pair of straight lines of the parallel sides can be replaced by a pair of any congruent curves or lines. Illustrations in Figure 2.6a-f show examples of boundaries of fundamental regions, unit cells and translation units on finite designs, band patterns and all-over patterns respectively.

Figure 2.6 a-f Examples show possible boundaries of fundamental regions, unit cells and translation units of finite designs (a-b), band patterns (c-d) and all-over patterns (e-f).



In Figure 2.6a, a fundamental region is one-sixth the area of the entire design and is bounded on two sides by a pair of any congruent curves shared by adjacent regions and another curve at the other side. Both curves are repeated about a fixed point by six-fold rotation. In Figure 2.6b, a fundamental region is also one-sixth the area of the entire design, but is bounded on two sides by reflection axes and the outer side by the same curve as the one in Figure 2.6a. Each fundamental region including the outer curve admits three-fold rotation together with reflection about a fixed point.

In Figure 2.6c, a band pattern admits reflections perpendicular to the translation axis. A unit cell containing two fundamental regions is bounded in a rectangle by the unit edges on two parallel sides and reflection axes on the other two sides. A translation unit also contains two fundamental regions and can be bounded by the unit edges on two parallel sides and any two congruent lines as evidenced in the three examples given. However, a fundamental region has to be bounded in a rectangle due to the reflection axes on two parallel sides. In Figure 2.6d, a band pattern admits a reflection along a band axis. Under translation, both unit cell and translation unit contain two pairs of bilateral motifs and the enclosing area. A unit cell is bounded in a rectangle, while a translation unit is bounded by the band-edges on two parallel sides and any two congruent lines shared by adjacent regions on the other two sides. A fundamental region, which is half the area of a unit cell and a translation unit can be bounded in either a rectangle or one of the mirrored halves of the translation unit.

In Figure 2.6e, an all-over pattern presents a translation of a two-fold rotational unit. A unit cell contains two fundamental regions and is bounded in a parallelogram. A translation unit also contains two fundamental regions, but is bounded by two pairs of congruent lines on two pairs of parallel sides. A fundamental region, which is half the area of a unit cell and a translation unit can be bounded in either a parallelogram or a multiple-sided shape whose congruent sides are associated with two-fold rotation. In Figure 2.6f, an all-over pattern admits only translation. Therefore, a fundamental region has the same area as a unit cell and a translation unit, which contains two motifs and the enclosing area. A unit cell can be bounded in any parallelogram shape, shown in a rectangle and two parallelograms in the illustration. A translation unit can thus be bounded in any shapes by two pairs of congruent lines on two pairs of parallel sides.

2.3 The Classification of Finite Designs

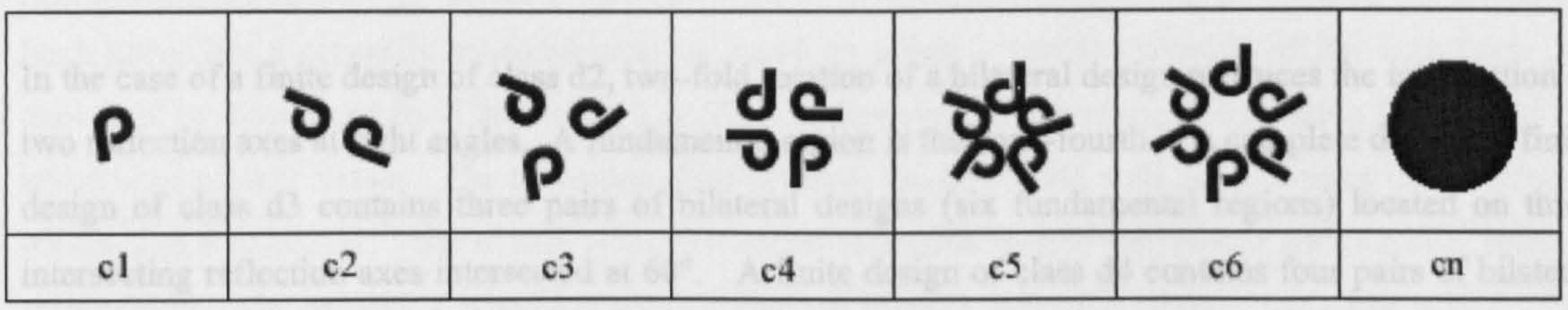
2.3.1 Definition and Notation

A finite design admits reflection and/or rotation about a fixed point but no translation and hence glide-reflection in its symmetry group [Washburn and Crowe, 1988, p.57]. Therefore, it has no sense of forward and backward or right and left. Synonymous terms are *a finite group* [Schattschneider, 1978], *a point group* [Stevens, 1981] and *a motif* [Hann and Thomson, 1992]. There are two categories of finite designs, i.e., a *cyclic group* and a *dihedral group*. A cyclic group exhibits a design with n-fold rotational symmetry. A dihedral group presents a design having n distinct reflection axes and n-fold rotation. Designs from both categories are classified dependent upon the order of rotation and the number of reflection axes present. A design from the cyclic group is denoted as a finite design of class *cn*, where *c* stands for a cyclic group and *n* stands for the order of rotation. Designs from the dihedral group are denoted as finite design of class *dn*, where *d* stands for a dihedral group and *n* stands for the order of rotation as well as the number of reflection axes. It is noted that n can be any integer ranging from 1, 2,3,...,∞ The larger the number of reflection axes and the order of rotation employed the clearer the rotational centre is noticed. A circle may thus be a final case of designs from both class cn and dn, which exhibits an infinite order of rotation and an infinite number of reflection axes [Hann and Thomson, 1992, p.18].

2.3.2 Finite Designs Symmetry Class cn

A cyclic group of order n or a finite design of class cn admits only n-fold rotation about a fixed point. Schematic illustrations of finite designs of class cn where n is ranged from 1 to ∞ are shown in Figure 2.7.

Figure 2.7 Schematic illustrations of finite design of class cn, n = 1-∞

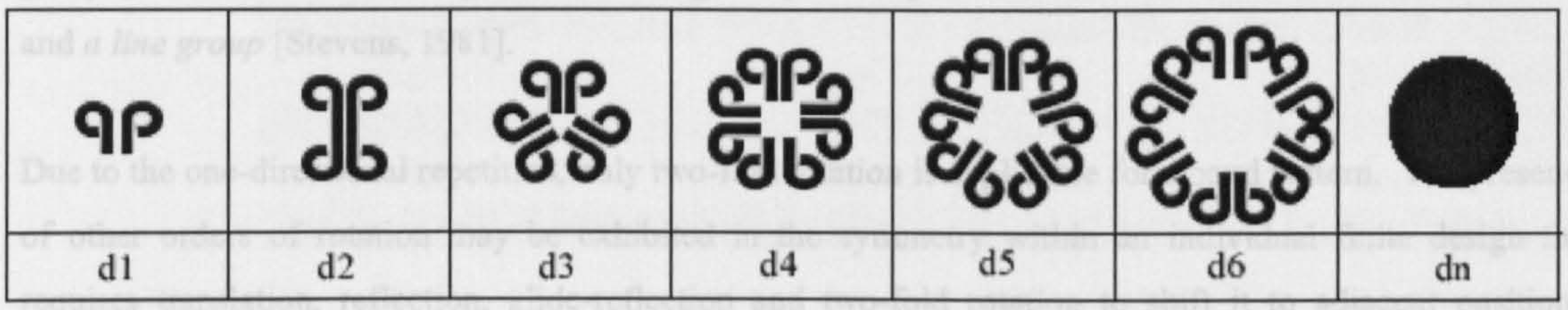


A finite design of class c1 is an asymmetrical motif which can repeat or coincide with itself after a full rotation of 360° [Hann and Thomson, 1992, p.10]. Symmetrical designs of the cyclic group cn are classified when n≥2. A finite design of class c2 exhibits two-fold rotational characteristics in which one fundamental region is turned 180° to coincide with its original fundamental region. Three-fold rotation produces a finite design of class c3 where three fundamental regions are spaced at 120°, 240° and 360° about a rotational centre. Four-fold rotation produces a finite design of class c4 where four fundamental regions are spaced at 90°, 180°, 270° and 360° about a rotational centre. Five-fold rotation produces a finite design of class c5 where five fundamental regions are spaced at 72°, 144°, 216°, 288° and 360° about a rotational centre. Six-fold rotation produces a finite design of class c6 where six fundamental regions are spaced at 60°, 120°, 180°, 240°, 300° and 360° about a rotational centre. There is no limitation of the magnitude of n until n=∞ which may be represented by a circle.

2.3.3 Finite Designs Symmetry Class dn

A dihedral group of order n or a finite design of class dn admits n distinct reflection axes together with n-fold rotation about a fixed point. Symmetrical designs of the dihedral group are classified when n≥1. Schematic illustrations of finite designs of class dn, where n is ranged from 1to ∞, are shown in Figure 2.8.

Figure 2.8 Schematic illustrations of finite design of class dn, n = 1-∞



A finite design of class d1 is a *bilateral design*, in which two fundamental regions exhibit reflection but not rotation. As mentioned previously when making reference to Figure 2.5, designs of the dihedral group where n>1 exhibit both reflection and rotation in that they may be produced through reflection of a fundamental region in a series of reflection axes intersecting at a fixed point, or on the other hand, may be produced by rotation of a bilateral design or a finite design of class.

In the case of a finite design of class d2, two-fold rotation of a bilateral design produces the intersection of two reflection axes at right angles. A fundamental region is thus one-fourth of a complete design. A finite design of class d3 contains three pairs of bilateral designs (six fundamental regions) located on three intersecting reflection axes intersected at 60°. A finite design of class d4 contains four pairs of bilateral designs (eight fundamental regions) located on four intersecting reflection axes intersected at 45°. A finite design of class d5 contains five pairs of bilateral designs (ten fundamental regions) located on five intersecting reflection axes intersected at 36°. A finite design of class d6 contains six pairs of bilateral designs (twelve fundamental regions) located on six intersecting reflection axes intersected at 30°. There is no limitation of the fraction of 360° to generate finite designs of class dn, that means n can be varied to infinity which may be represented by a circle.

2.4 The Classification of Band Patterns

2.4.1 Definition and Notation

A band pattern is generated by the translation of motif/motifs along a longitudinal axis between parallel sides of the band, known as a *translation axis*. It must present the translation of at least the original motif/motifs and one copy by translation [Washburn and Crowe, 1988, p.53]. Synonymous terms are *a one-dimensional pattern* [Washburn and Crowe, 1988], *a monotranslation design* [Horne, 1997], *a frieze group* [Schattschneider, 1978], *a border design or pattern* [Woods, 1935 and Hann and Thomson, 1992] and *a line group* [Stevens, 1981].

Due to the one-directional repetition, only two-fold rotation is applicable for a band pattern. The presence of other orders of rotation may be exhibited in the symmetry within an individual finite design that requires translation, reflection, glide-reflection and two-fold rotation to shift it to adjacent positions. Reflection axes may be either longitudinally or transversely located relative to the band edges. A glide-reflection axis can lie only in the longitudinal direction. A unit cell which admits successive translation along a band axis can be bounded in one of three parallelogram shapes, i.e., an ordinary parallelogram, a square or a rectangle, by the band-edges on two parallel sides and two congruent lines or reflection axes on the other two sides.

A total of seven symmetry classes of band patterns or seven distinct symmetry groups may be identified. Each group has a generally accepted notation based on a four-symboled notation of the form *pxyz*. As explained by Washburn and Crowe [1988, p.57], the first symbol *p* prefaces the notation for each of the seven band pattern classes. The remaining three symbols are associated with the symmetry characteristics obtained in a band, i.e., vertical reflection, horizontal reflection or glide-reflection and two-fold rotation respectively. The symbol *x* is *m* (for mirror) where a vertical reflection occurs; otherwise *x* is *1*. The symbol *y* is *m* where a horizontal reflection occurs, or *a* where a glide-reflection occurs; otherwise *y* is *1*. The symbol *z* is *2* where two-fold rotation occurs; otherwise *z* is *1*.

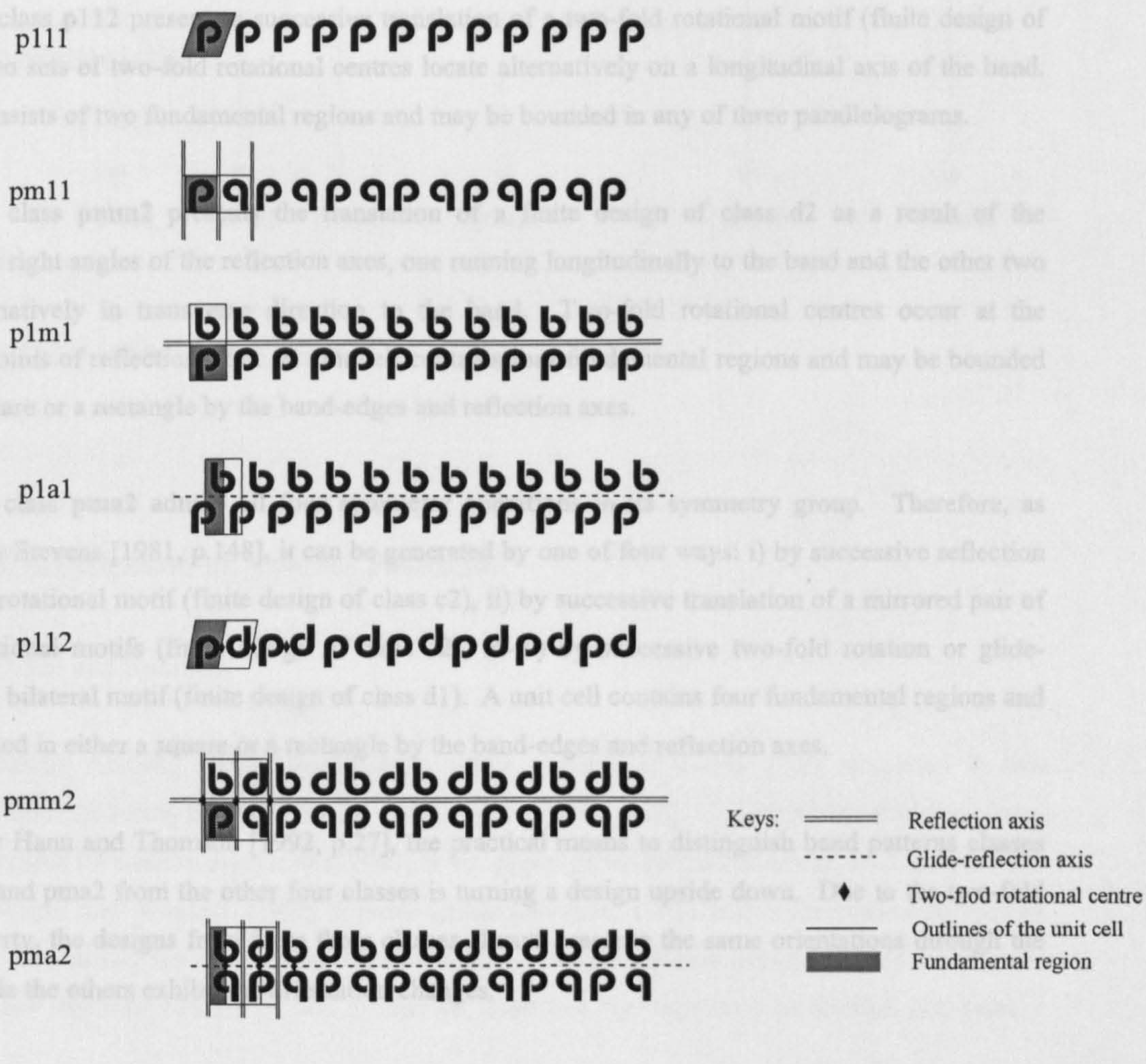
2.4.2 Seven Symmetry Classes of Band Patterns

A summary of the symmetry characteristics of the seven band pattern groups of classes p111, pm11, p1m1, p1a1, p112, pmm2 and pma2, is presented in Table 2.1. Further to this a schematic illustration of each class is presented in association with its fundamental region and unit cell in Figure 2.9. Further description of each class is presented below.

Table 2.1 Summary of symmetry characteristics of seven band pattern classes

Symmetry class	Possible unit-cell shape			Area of fundamental region/ unit-cell	Symmetry operation		
	Parallelogram	Square	Rectangle		The highest order of rotation	Reflection	Glide-reflection
p111	✓	✓	✓	1	1	No	No
pm11		✓	✓	1/2	1	Yes	No
p1m1		✓	✓	1/2	1	Yes	No
p1a1		✓	✓	1/2	1	No	Yes
p112	✓	✓	✓	1/2	2	No	No
pmm2		✓	✓	1/4	2	Yes	No
pma2		✓	✓	1/4	2	Yes	Yes

Figure 2.9 Schematic illustrations of the seven symmetry classes of band patterns



Band pattern class **p111** presents successive translation of an asymmetrical motif (finite design of class c1) along a longitudinal axis of the band. A fundamental region which has the same area as a unit cell is bounded by the band-edges on two parallel sides and two congruent lines on the other two parallel sides, which may be either a parallelogram, a square or a rectangle.

Band pattern class **pm11** is built up by the translation of a bilateral motif (finite design of class d1), where the reflection axes are perpendicular to the translation axis. There are two alternative sets of reflection axes, one bisecting a motif and the other one lying between motifs. A unit cell contains two fundamental regions and may be bounded in either a square or a rectangle by the band-edges and reflection axes.

Band pattern class **p1m1** exhibits reflection of translations [Stevens, 1981, p.126]. Reflection along an axis which lies longitudinally to the band produces the entire design bilaterally symmetrical along the translation axis. A unit cell consists of two fundamental regions and may be bounded in either a square or a rectangle.

Band pattern class **p1a1** exhibits a linear repetition of an asymmetrical motif (finite design of class c1) with glide-reflection whose axis lies longitudinally to the band. A unit cell consists of two fundamental regions and may be bounded in either a square or a rectangle.

Band pattern class **p112** presents a successive translation of a two-fold rotational motif (finite design of class c2). Two sets of two-fold rotational centres locate alternatively on a longitudinal axis of the band. A unit cell consists of two fundamental regions and may bc bounded in any of three parallelograms.

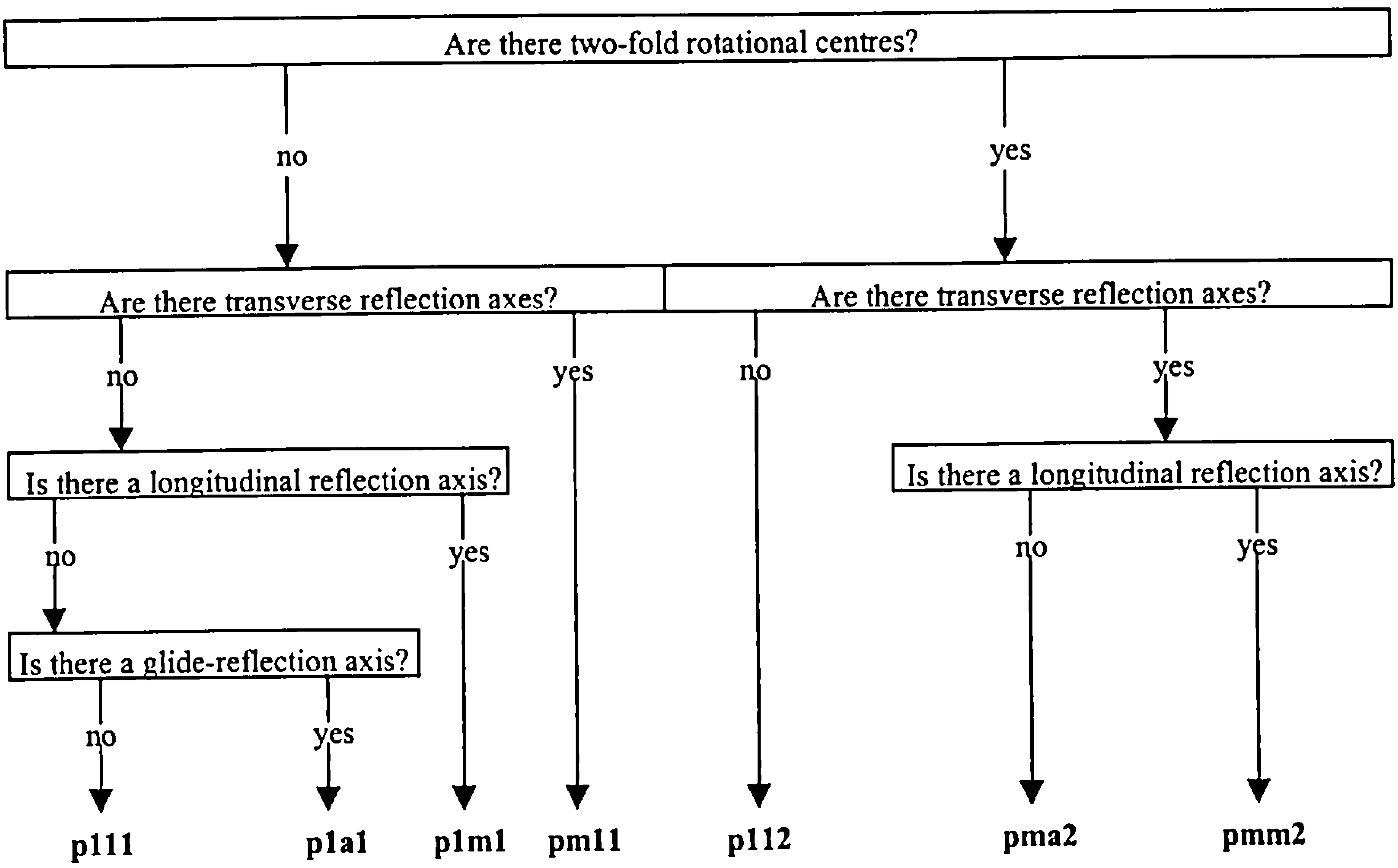
Band pattern class **pmm2** presents the translation of a finite design of class d2 as a result of the intersection at right angles of the reflection axes, one running longitudinally to the band and the other two running alternatively in transverse direction to the band. Two-fold rotational centres occur at the intersecting points of reflection axes. A unit cell contains four fundamental regions and may be bounded in either a square or a rectangle by the band-edges and reflection axes.

Band pattern class **pma2** admits all four symmetry operations in its symmetry group. Therefore, as pointed out by Stevens [1981, p.148], it can be generated by one of four ways: i) by successive reflection of a two-fold rotational motif (finite design of class c2), ii) by successive translation of a mirrored pair of two-fold rotational motifs (finite design of class c2), iii-iv) by successive two-fold rotation or glide-reflection of a bilateral motif (finite design of class d1). A unit cell contains four fundamental regions and may be bounded in either a square or a rectangle by the band-edges and reflection axes.

As noticed by Hann and Thomson [1992, p.27], the practical means to distinguish band patterns classes p112, pmm2 and pma2 from the other four classes is turning a design upside down. Due to the two-fold rotation property, the designs from these three classes always preserve the same orientations through the 180° turn while the others exhibit the orientation changes.

The flow-diagram adapted from Washburn and Crowe [1988, p.83] by Horne [1997, p.38] is presented in Figure 2.10 to aid the identification of seven symmetry classes of band patterns through a series of questions associated with symmetry operations contained within a design.

Figure 2.10 A flow-diagram aiding the identification of seven symmetry classes of band patterns.



Source: reproduced from Horne, 1997, p.38

2.5 The Classification of All-over Patterns

2.5.1 Definition and Notation

An all-over pattern exhibits regular repetition in which a motif or motifs is/are translated in two independent directions across the plane [Hann and Thomson, 1992, p.28]. Synonymous terms are *a two-dimensional pattern* [Washburn and Crowe, 1988], *a wallpaper group* [Schattschneider, 1978], *a periodic pattern* [Grünbaum and Shepard, 1987], *a plane group* [Stevens, 1981] and *a ditranslational design* [Horne, 1997]. To satisfy the minimal repetition condition, the pattern must present at least the original motif/motifs, one copy by translation and a copy of these two by translation in another non-parallel direction [Washburn and Crowe, 1988, p.53]. A total of seventeen distinct symmetry groups of all-over patterns may be generated from the combinations of one or more of the four symmetry operations.

There are various notations used by mathematicians and crystallographers to identify the seventeen classes of all-over patterns, as evidenced in Schattschneider's comparison chart of notation [Schattschneider, 1978, p.489]. The widely accepted crystallographic notation consists of four symbols which identify the conventional by chosen unit cell, the highest order of rotation and the axial symmetry obtained in two non-parallel directions. Further explanation of the four symbols was provided by Schattschneider [1978, p. 443], Washburn and Crowe [1988, pp.58-60] and Hann and Thomson [1992, p.30] and is summarised below.

The first symbol is either *p* or *c*, which identifies whether the unit cell associated with the pattern is primitive or centred. Fifteen of the seventeen classes are signified by *p*, while the remaining two generated from the rhombic lattices are denoted as *c*. The primitive cell is a parallelogram whose vertices are lattice points, no other lattice points inside or on its edges, and which may complete the pattern by translation only. A rhombus is denoted as centre-celled since its equal-sided shape is held within a rectangle where the vertices are located at the mid-sides of a rectangle. Each parallelogram-shaped unit cell admits rotation of the highest order at its vertices and its left side is called the *x-axis*.

The second symbol, the integer *n*, denotes the highest order of rotation. Taken into account is the crystallographic restriction that only two-, three-, four- and six-fold rotations are applicable to generate all-over patterns. Discussion relating to this restriction, including the absence of five-fold rotation, was provided by Stevens [1981, Appendix, pp.376-390].

The third symbol denotes a symmetry axis normal to the x-axis (i.e. perpendicular to one side of the unit cell): *m* (for mirror) indicates a reflection axis, *g* indicates a glide-reflection axis and *l* indicates no reflection and glide-reflection normal to the x-axis.

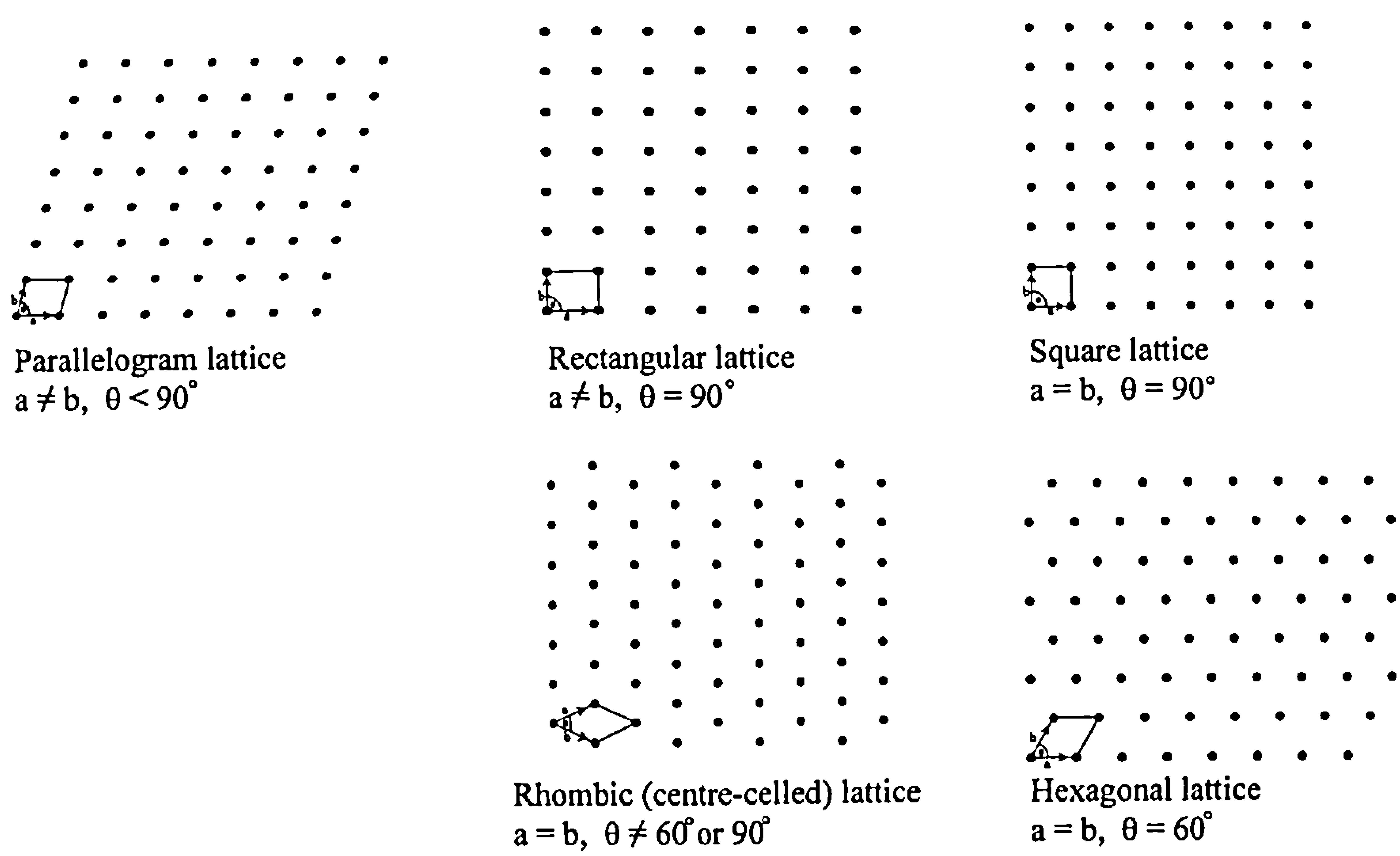
The fourth symbol denotes a symmetry axis at angle α to the x-axis, with α dependent on n, the highest order of rotation (shown by the second symbol): $\alpha=180°$ if n=1 or 2, $\alpha=45°$ if n=4 and $\alpha=60°$ if n=3 or 6. The symbols *m*, *g* and *l* are interpreted as in the third symbol.

No symbols in the third and fourth positions denote that the pattern admits no reflection and glide-reflection, as evidenced in classes p1, p2, p3, p4 and p6.

With the primary concern of the highest order of rotation exhibited by the pattern, seventeen all-over pattern classes are categorised into five groups, i.e., the patterns without rotation (i.e. classes p1, p1m1, p1g1 and c1m1), the patterns with two-fold rotation (i.e. classes p2, p2mm, p2gg, p2mg and c2mm), the patterns with three-fold rotation (i.e. classes p3, p31m and p3m1), the patterns with four-fold rotation (i.e. classes p4, p4mm and p4gm) and the patterns with six-fold rotation (i.e. classes p6 and p6mm). Each of the seventeen symmetry groups is basically constructed on one of five distinct types of geometrical lattices, i.e., parallelogram, rectangular, square, rhombic and hexagonal.

Crystallographers call these five lattices *Bravais lattices*, after Bravais who verified that lattices can be classified into five types [Grünbaum and Shepard, 1987, p.262]. Schematic illustrations of the five lattices are shown in Figure 2.11.

Figure 2.11 Five types of geometric Lattices

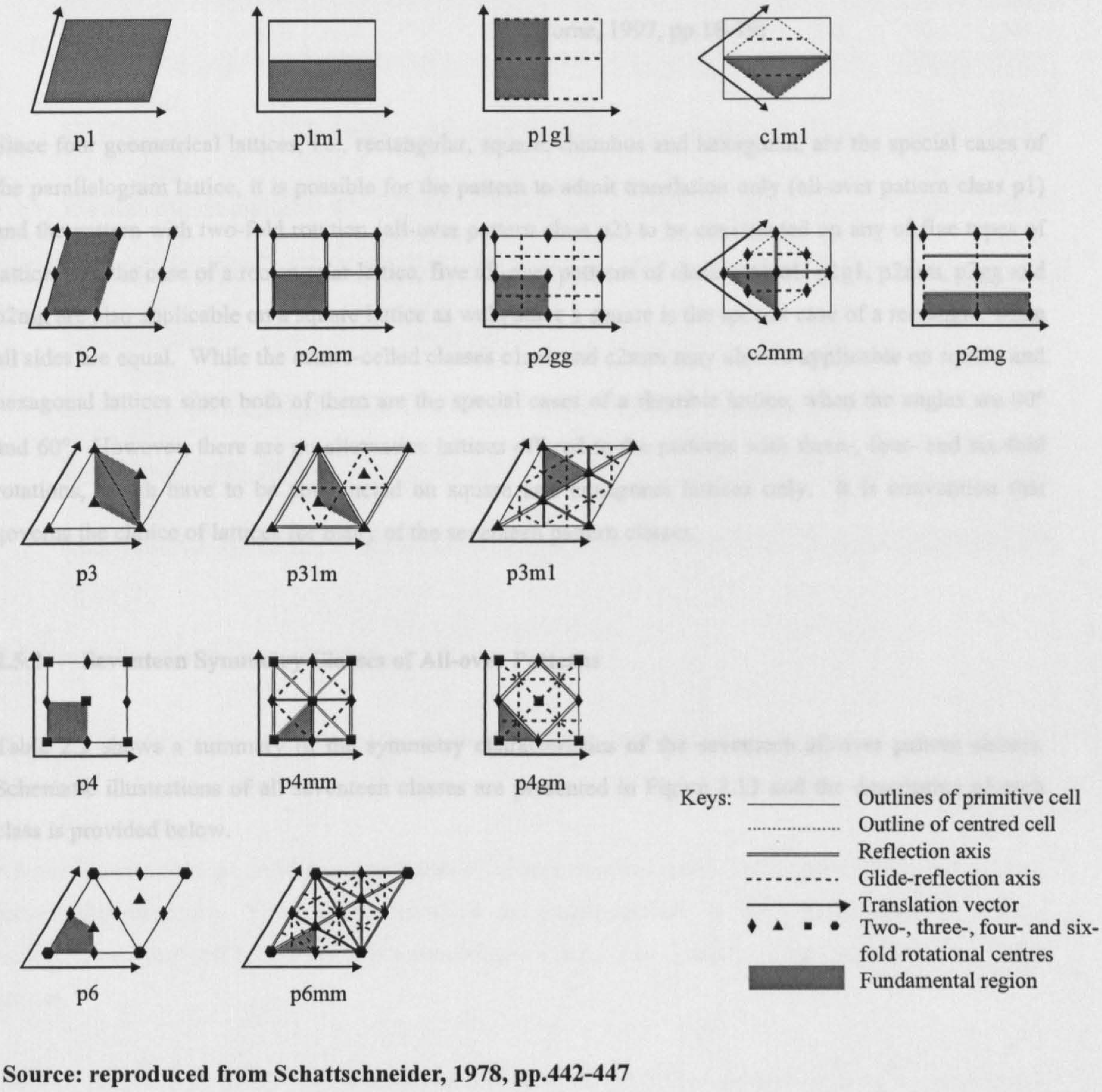


Source: reproduced from Horne, 1997, p.21

Each lattice type provides individual features to admit certain symmetry groups. In addition to a translation, a parallelogram lattice allows full rotation (360°) and two-fold rotation about the vertex of each cell, the mid-side of each cell-edge and the cell-centre, that is sufficient to generate all-over pattern classes p1 and p2. The rectangular lattice allows reflection and glide-reflection in either one or two perpendicular directions as well as two-fold rotation, that is applicable to generate all-over pattern classes p1m1, p1g1, p2mm, p2gg and p2mg. The square lattice offers four-fold rotation as the highest rotation together with two-fold rotation, reflection and glide-reflection at 45° and 90°, that is applicable to generate all-over pattern classes p4, p4mm and p4gm. Since its unit cell is formed by two equilateral triangles whose interval angles are 60°, the hexagonal lattice provides three- and six-fold rotation as the highest rotations together with reflection at 30°, 60° and 90°, that is applicable to generate all-over pattern classes p3, p31m, p3m1, p6 and p6mm. The rhombic lattice offers a special orientation as a centred rectangular lattice, in which a rhomboid-shaped unit cell is held within a larger rectangle. It is applicable to admit reflection in either one or two perpendicular directions and hence two-fold rotation, that is the particular feature of all-over pattern classes c1m1 and c2mm.

Schematic illustrations of the seventeen lattice units or unit cells containing symmetry groups and fundamental regions are shown in Figure 2.12.

Figure 2.12 Schematic illustrations of the seventeen lattice units (unit cells) of all-over patterns, each of which contains an individual symmetry group and fundamental region



Source: reproduced from Schattschneider, 1978, pp.442-447

Nonetheless, in practice, the geometric relation between the five parallelogram lattices implies the alternative application of more than one lattice type to some symmetry groups (whereby the contents contained in each unit cell, e.g., symmetry operations, a number and orientation of fundamental regions are accommodated). As Horne pointed out:

" A parallelogram has four straight sides: two parallel sides of length a and two parallel sides of length b. One of the angles, at which these two sets of lines intersect each other, is θ°. The specific type of parallelogram is determined by the conditions held by a, b and θ. The results of different combinations of these variables are given below:

1) *If $a=b$ and $\theta=90°$, the parallelogram is a square.*
2) *If $a=b$, the parallelogram is a rhombus.*
3) *If $a=b$ and $\theta=60°$, the parallelogram is a special kind if rhombus composed of two equilateral triangles. (These types of parallelogram are associated with the hexagonal lattice.)*
4) *If $a\neq b$, $\theta=90°$, the parallelogram is a rectangle.*
5) *If $a\neq b$ and $\theta\neq 90°$, the parallelogram is a just an ordinary parallelogram."*

[Horne, 1997, pp.18-19]

Since four geometrical lattices, i.e., rectangular, square, rhombus and hexagonal, are the special cases of the parallelogram lattice, it is possible for the pattern to admit translation only (all-over pattern class p1) and the pattern with two-fold rotation (all-over pattern class p2) to be constructed on any of five types of lattices. In the case of a rectangular lattice, five all-over patterns of classes p1m1, p1g1, p2mm, p2gg and p2mg are also applicable on a square lattice as well, since a square is the special case of a rectangle, when all sides are equal. While the centre-celled classes c1m1 and c2mm may also be applicable on square and hexagonal lattices since both of them are the special cases of a rhombic lattice, when the angles are 90° and 60°. However, there are no alternative lattices offered to the patterns with three-, four- and six-fold rotations, which have to be constructed on square and hexagonal lattices only. It is convention that governs the choice of lattices for many of the seventeen pattern classes.

2.5.2 Seventeen Symmetry Classes of All-over Patterns

Table 2.2 shows a summary of the symmetry characteristics of the seventeen all-over pattern classes. Schematic illustrations of all seventeen classes are presented in Figure 2.13 and the description of each class is provided below.

Table 2.2 Summary of symmetry characteristics of the seventeen all-over pattern classes

Symmetry class	Applicable lattice					Area of fundamental region/unit cell	Symmetry operation		
	Parallelogram	Rectangular	Square	Rhombic	Hexagonal		The highest order of rotation	Reflection	Glide-reflection
p1	✓	✓	✓	✓	✓	1	1	No	No
p2	✓	✓	✓	✓	✓	1/2	2	No	No
p1m1		✓	✓			1/2	1	Yes	No
p1g1		✓	✓			1/2	1	No	Yes
p2mm		✓	✓			1/4	2	Yes	No
p2gg		✓	✓			1/4	2	No	Yes
p2mg		✓	✓			1/4	2	Yes	Yes
c1m1			✓	✓	✓	1/2	1	Yes	Yes
c2mm			✓	✓	✓	1/4	2	Yes	Yes
p4			✓			1/4	4	No	No
p4mm			✓			1/8	4	Yes	No
p4gm			✓			1/8	4	Yes	Yes
p3					✓	1/3	3	No	No
p31m					✓	1/6	3	Yes	Yes
p3m1					✓	1/6	3	Yes	Yes
p6					✓	1/6	6	No	No
p6mm					✓	1/12	6	Yes	Yes

Patterns without rotation symmetry

There are four all-over pattern classes in which the highest order of rotation is 1 (full rotation 360°) or considered as having no rotation.

All-over pattern class **p1** exhibits a translation of an asymmetrical motif (finite design of class c1) in two non-parallel directions. There are no reflection and glide-reflection. A fundamental region having the same area as a unit cell is translated on a parallelogram lattice and possibly on the other four geometrical lattices.

All-over pattern class **p1m1** is constructed on a rectangular lattice and possibly also on a square lattice with two alternating and parallel reflection axes in one direction. A unit cell contains two fundamental regions, each of which has two parallel sides bounded by reflection axes.

All-over pattern class **p1g1** is possibly generated on either a rectangular lattice or a square lattice with two alternating and parallel glide-reflection axes. A fundamental region which is half the area of a unit cell admits glide-reflection normal to the x-axis and translation in the perpendicular direction.

Figure 2.13 Schematic illustrations of the seventeen symmetry classes of all-over patterns

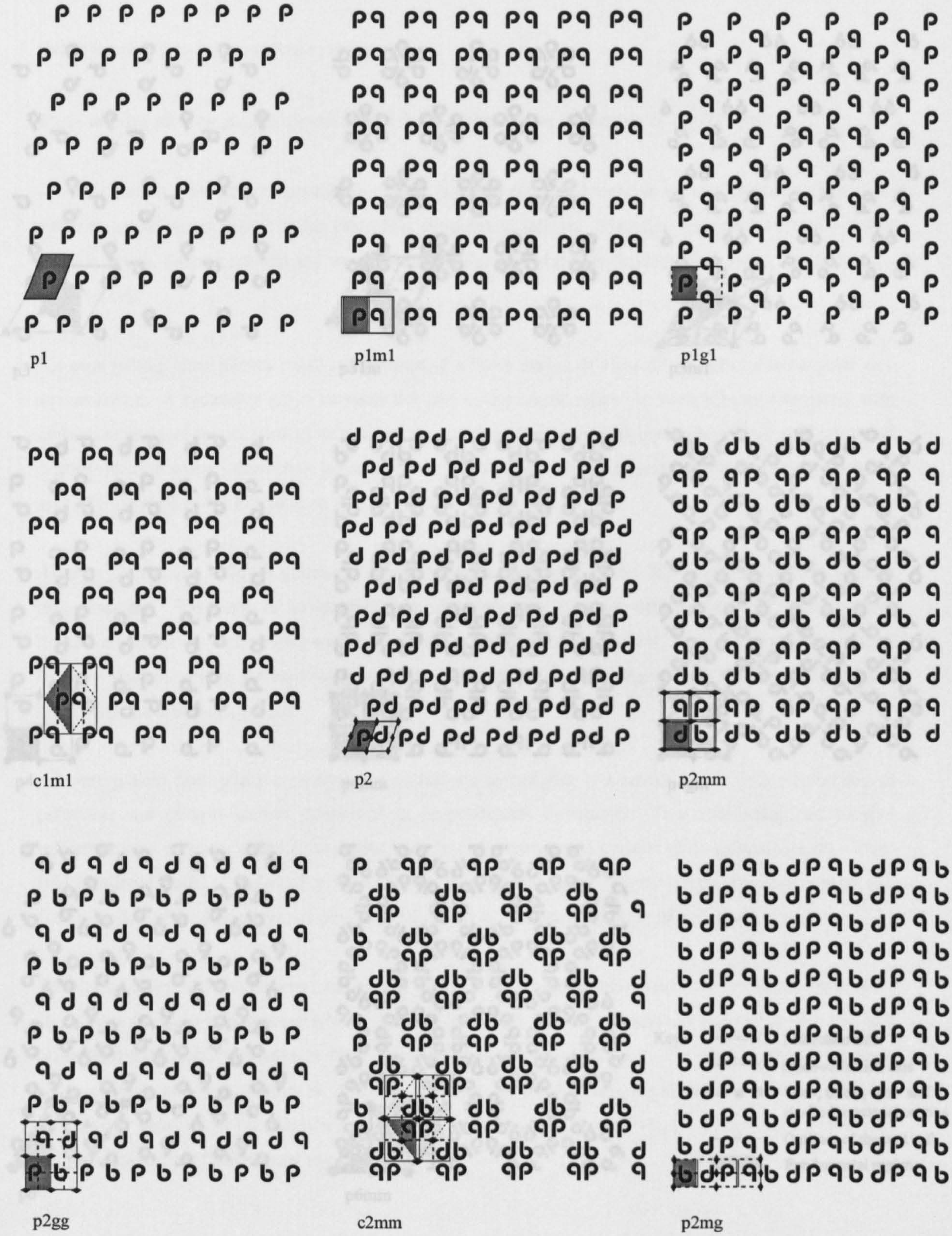
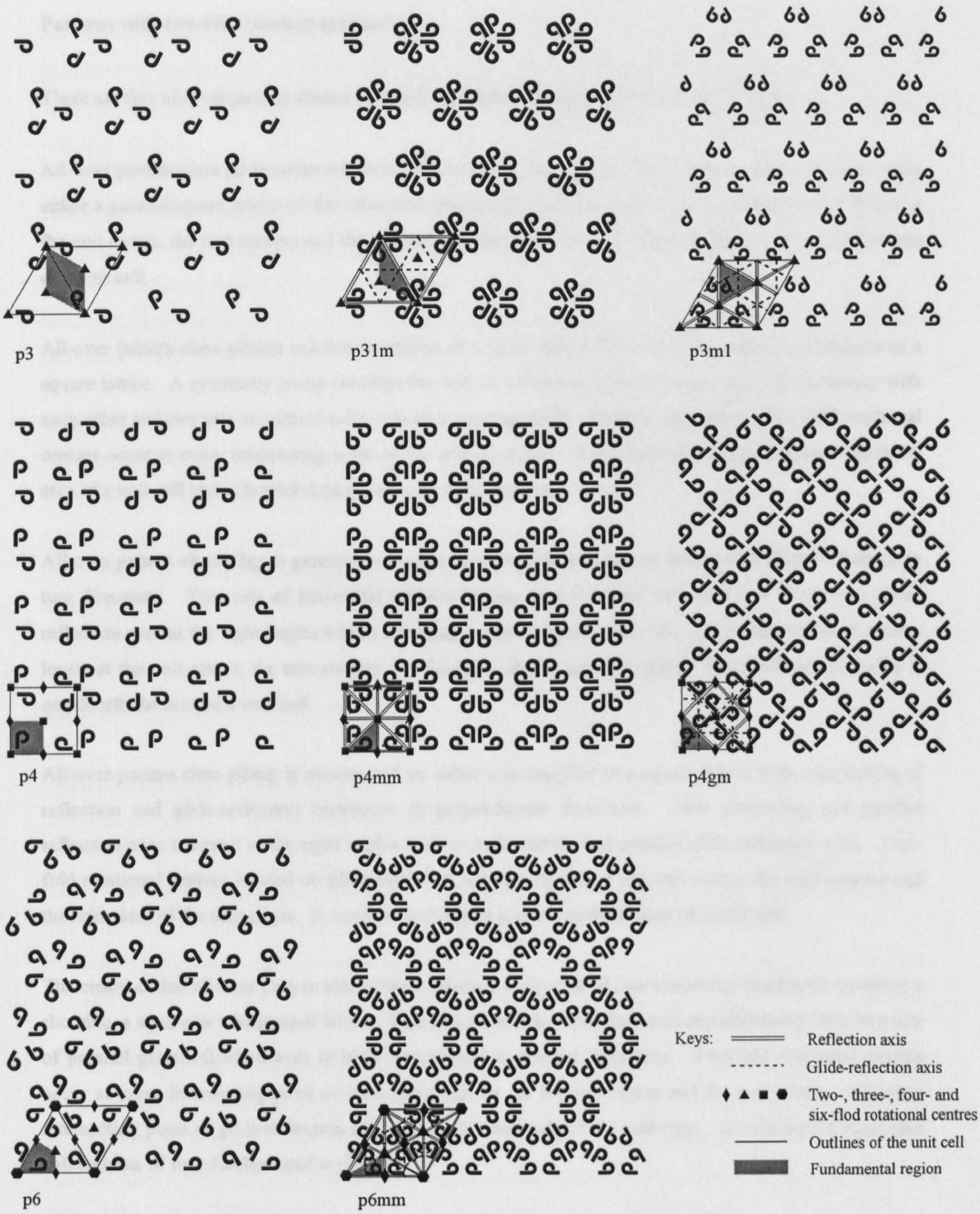


Figure 2.13 continued



The centre-celled all-over pattern class **c1m1** is constructed on a rhombic lattice or otherwise on a square or a hexagonal lattice with a symmetry group of parallel reflection and glide-reflection whose axes run alternately with each other and are normal to the x-axis. Reflection axis bisects a rhomboid-shaped unit cell into two triangle-shaped fundamental regions.

Patterns with two-fold rotation symmetry

There are five all-over pattern classes in which the highest order of rotation is 2 (180° turn).

All-over pattern class **p2** presents repetition of a two-fold rotational motif or a finite design of class c2 on either a parallelogram lattice or the other four geometrical lattices. Two-fold rotational centres locate at the unit centre, the unit corners and the mid-sides of the unit edges. A fundamental region is half the area of a unit cell.

All-over pattern class **p2mm** exhibits repetition of a finite design of class d2 on either a rectangular or a square lattice. A symmetry group contains two sets of horizontal reflection axes running alternately with each other and two sets of vertical reflection axes running alternately with each other. Two-fold rotational centres occur at every intersecting point of the reflection axes. The fundamental region is one-fourth the area of a unit cell and is bounded on all sides by reflection axes.

All-over pattern class **p2gg** is generated on either a rectangular or a square lattice with glide-reflections in two directions. Two sets of horizontal glide-reflection axes intersect with two sets of vertical glide-reflection axes at the right angles within a rectangle-/square-shaped unit cell. Two-fold rotational centres locate at the unit centre, the unit corners and the mid-sides of the unit edges. The fundamental region is one-fourth the area of a unit cell.

All-over pattern class **p2mg** is constructed on either a rectangular or a square lattice with a collection of reflection and glide-reflection operations in perpendicular directions. Two alternating and parallel reflection axes intersect at the right angles with two alternating and parallel glide-reflection axes. Two-fold rotational centres located on glide-reflection axes positioned at the unit centre, the unit corners and the mid-sides of the unit edges. A fundamental region is one-fourth the area of a unit cell.

The centre-celled all-over pattern class **c2mm** admits a collection of four symmetry operations on either a rhombic, a square or a hexagonal lattice. Two sets of parallel reflection axes run alternately with two sets of parallel glide-reflection axes in both vertical and horizontal directions. Two-fold rotational centres occur at every intersecting point of reflection axes, i.e., at the unit centre and the unit corner, and every intersecting point of glide-reflection axes, i.e., at the mid-side of the unit edge. A rhomboid-shaped unit cell consists of four fundamental regions.

Patterns with three-fold rotation symmetry

There are three all-over pattern classes in which the highest order of rotation is 3 (120° turn). All three-fold rotational varieties are constructed on a hexagonal lattice whose unit cell is bounded in a 60°-120°-60°-120° rhomboid or two equilateral triangles.

All-over pattern class **p3** admits a symmetry group of three-fold rotation on a 60°-120°-60°-120° rhomboid-shaped unit cell. There are three types of three-fold rotational centres, one locates at every unit corner and the other two locate at the centres of the triangular cells. A fundamental region is one-third the area of a unit cell.

All-over pattern class **p31m** exhibits a reflection of three-fold rotation. Reflection axes are positioned along all sides of a triangular cell. Not all three-fold rotational centres occur on reflection axes. Two of three types of three-fold rotational centres are the mirrored images of one another, and are located at the centres of two triangular cells. Glide-reflection axes run alternately with reflection axes in all three directions. The fundamental region is one-sixth the area of a unit cell.

All-over pattern class **p3ml** presents a combination of three-fold rotational centres and reflection axes. All three types of three-fold rotational centres are located at the intersecting points of reflection axes. A reflection axis runs along the longest diagonal of the unit cell, which bisects the opposite side of the triangular cell at the right angle. Glide-reflection axes run alternately with reflection axes in all three directions. The fundamental region is one-sixth the area of a unit cell and is bounded on all sides by reflection axes.

Patterns with four-fold rotation symmetry

There are three all-over pattern classes in which the highest order of rotation is 4 (90° turn). All four-fold rotational varieties are constructed on a square lattice.

All-over pattern class **p4** admits a symmetry group of four- and two-fold rotations on a square-shaped unit cell. Four-fold rotational centres are located at the centre and four corners of the unit cell. Two-fold rotational centres occur at the mid-sides of the unit edges. The fundamental region is one-fourth the area of a unit cell.

All-over pattern class **p4mm** admits four- and two-fold rotations at the same time as reflection. Reflection axes occur in horizontal, vertical, 45° clockwise and 45° anti-clockwise of diagonal directions to connect all four- and two-fold rotational centres. A network of reflection axes divides a square-shaped unit cell into eight 45°-90°-45° triangle-shaped fundamental regions. Glide-reflection axes intersect at the right angles on two-fold rotational centres located at the mid-sides of the unit edges.

All-over pattern class **p4gm** presents a reflection of four-fold rotation. Reflection axes intersect at the right angles on two-fold rotational centres located at the mid-sides of the unit edges. Four-fold rotational centres at the unit centre and the unit corners are thus the mirrored images of one another. Glide-reflection axes intersect the reflection axes at 90° and 45°. A fundamental region is one-eighth the area of a unit cell and is bounded in a 45°-90°-45° triangle by connecting two two-fold rotational centres and one four-fold rotational centre.

Patterns with six-fold rotation symmetry

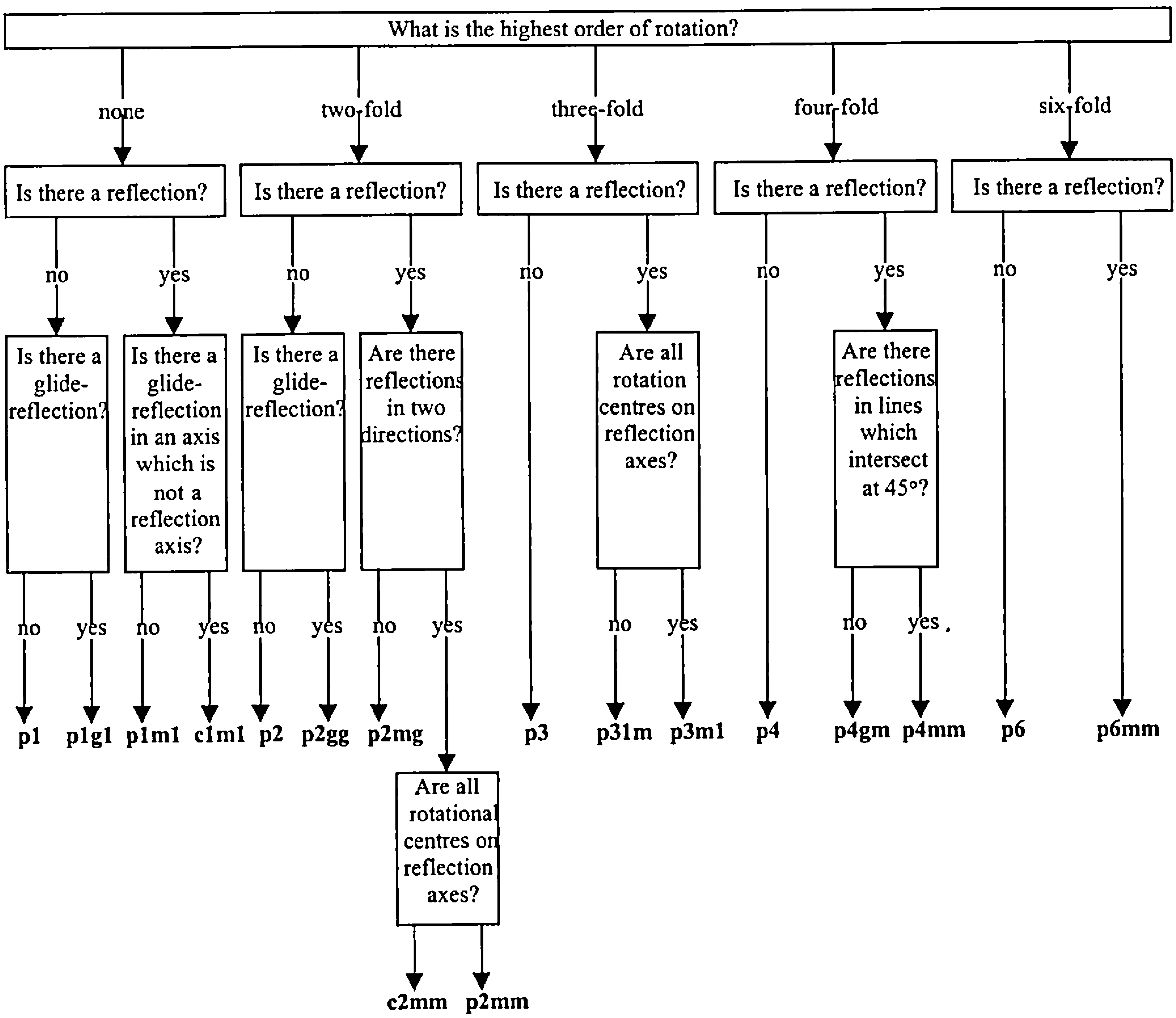
There are two all-over pattern classes in which the highest order of rotation is 6 (60° turn). All six-fold rotational varieties are constructed on a hexagonal lattice whose unit cell is bounded in a 60°-120°-60°-120° rhomboid or two equilateral triangles.

All-over pattern class **p6** admits a symmetry group of six-, three- and two-fold rotations on a 60°-120°-60°-120° rhomboid-shaped unit cell. Six-fold rotational centres locate at the unit corners. Three-fold rotational centres locate at the centres of triangular cells. Two-fold rotational centres locate at the mid-sides of the triangular edges. A fundamental region if one-sixth the area of a unit cell.

All-over pattern class **p6mm** exhibits a combination of six, three- and two-fold rotations with reflection. Based on the positions of six-fold rotational centres at the unit corners, reflection axes connect each corner with the other three corners and run through each corner to bisect the opposite side at the right angle. There are six-reflection axes intersecting at a six-fold rotational centre, three reflection axes intersecting at a three-fold rotational centre and two-reflection axes intersecting at a two-fold rotational centre. The fundamental region is one-twelfth the area of a unit cell and is bounded on all sides by reflection axes, which connect three different n-fold rotational centres, i.e., two-, three- and six-fold.

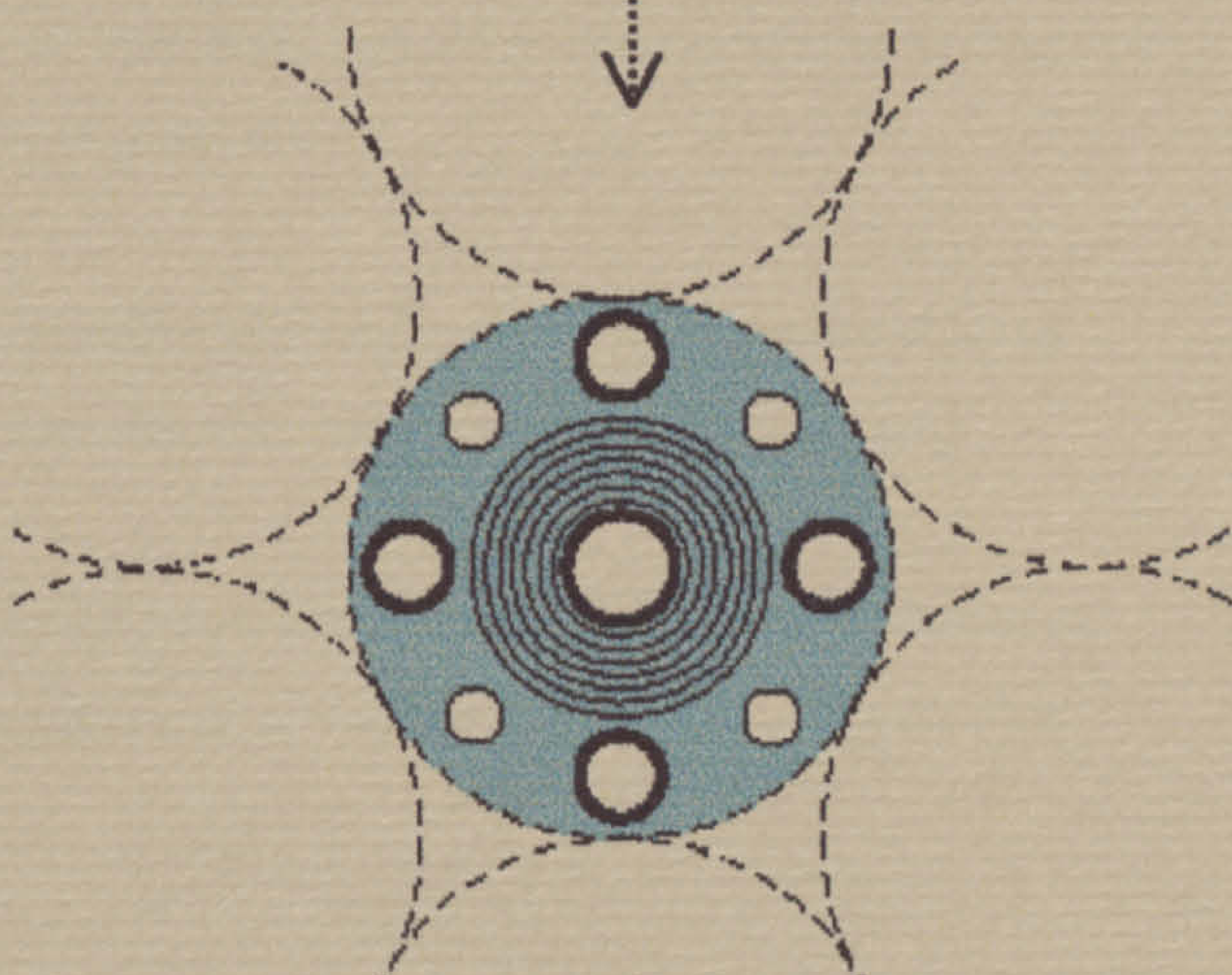
The flow-diagram adapted from Washburn and Crowe [1988, p.128] by Horne [1997, p.45] is presented in Figure 2.14. This can aid the identification of seventeen all-over pattern classes through a series of questions involving the symmetry operations contained in each symmetry group.

Figure 2.14 A flow-diagram aiding the identification of seventeen symmetry classes of all-over patterns



Source: reproduced from Horne, 1997, p.45

All-over pattern symmetry class p1m1



Chapter 3 Symmetry Classification and Analysis of Thai Textile Patterns

3.1 Introduction

This chapter explores symmetry characteristics in representative samples of traditional Thai textiles produced domestically or imported from aboard during the Ayuthaya (1351-1767) and early Ratanakosin periods (1782-1910) [Gittinger and Lefferts, 1992, pp.19-20]. The textiles chosen are of one of two types: court textiles used by the royal family and other aristocratic people, and village textiles for everyday use by rural peoples. Each group can be classified further into sub-groups dependent on end-use, patterning techniques used, places of production and the ethnic origin of the weavers, as evidenced in Leesuwan [1987], Fraser-Lu [1988, chapter 6], Conway [1992] and Prangwatthanakun and Naenna [1990/94]. Many kinds of cloths including court textiles, have, with the passage of time, fallen into disuse, and are found today in museums and private collections around the world*. However some are still used during ceremonial occasions by provincial villagers. Several patterning techniques were employed, e.g. supplementary-weft and -warp techniques, weft-ikat and tapestry weaving**.

Six categories of Thai textiles were selected for analysis. Three were village textiles, i.e., *tung*, *muon khit* and *pha zin*. The other three were court textiles, i.e., *pha lai-yang*, *pha phuum* and *pha yok*. A set of flow-diagrams shown in Figure 2.10 and 2.14 was employed as a means of readily identifying a pattern's constituent geometrical characteristics and its resultant symmetry class (one of seven classes for band patterns and one of seventeen classes for all-over patterns).

While there is great diversity apparent in the decoration of Thai textiles, it would seem that there are certain common characteristics, which in some way are manifestations of Thai identity. In order to ascertain what these characteristics are, selections of Thai textile patterns were subjected to symmetry classification. The intention was to test the following hypotheses:

(i) While varieties of Thai textiles may fulfil different functions and were produced using different patterning techniques, symmetry characteristics are nonetheless broadly shared.

(ii) The symmetry characteristics exhibited may be closely associated with and explained in the context of Thai culture and may in some way be manifestations of traditional Thai beliefs and Buddhist philosophy.

The findings of this chapter have been published [Tantiwong et.al., 2000] in a refereed academic journal, ARS Textrina.

3.2 Sources of Data

Six categories of traditional textiles were selected in order to explore pattern symmetry in the Thai context. The first three categories were domestically produced and used by local villagers particularly in the northeast of Thailand. The second three categories were court textiles. Each category is further described and illustrated below.

3.2.1 Village Textiles (*Tung, Muon Khit and Pha Zin*)

Tung are long-vertical banners in variable proportions (width: 45-50 cm., length: 5-7 m.) with supplementary-weft patterns. In occasional rituals, they are hung vertically from the temple ceilings or on bamboo poles along the pathways to the temples. Each *tung* contains a number of horizontal-patterned strips arranged continuously along a vertical axis. Patterns can be created by multi-coloured yarn or bamboo slats. Only the items patterned by yarns were selected as samples. The total sample size was 134 patterns (from eight textile pieces). Examples of this category are shown in Figure 3.1a,b.

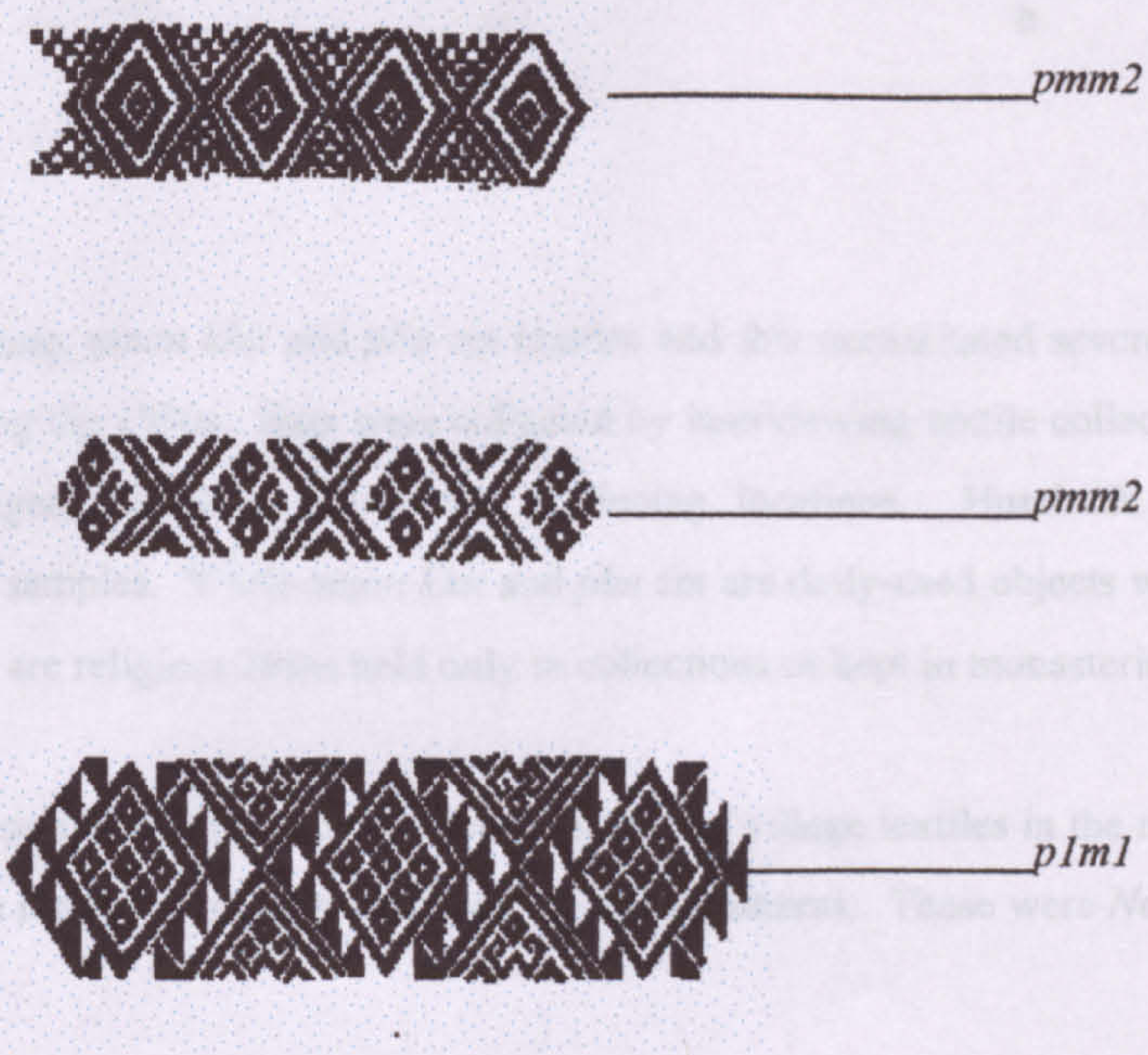
Figure 3.1 a) A variety of *tung* patterned by bamboo slats (item on the far left) and multi-coloured thread (five remaining items)
Source: Tungsawang temple in Ubon Rajatanee
b) Symmetry classification of some *tung* patterns which exhibit a finite design of class d1 (emboldened text) and four band patterns (italicised text)

a b

Muon khit are pillow-cases with supplementary-weft patterns arranged along central bands. They are used as household objects and also on occasions as traditional offerings [Gittinger and Lefferts, 1992, pp.50-51]. They are typically produced in a variety of sizes and in three forms, i.e., a box with a triangular section, a box with a square section and a box with a rectangular section. Sometimes they are named after the number of divided compartments or internal structures as is the case with *muon-khao-luk* (translated as "pillow with nine components") which means there are nine compartments in its square structure. The total sample size was 100 patterns (from 100 textile pieces). Examples of this category are shown in Figure 3.2a,b.

Figure 3.2 a) Three sizes of *muon-khao-luk*, nine-compartmented pillows; large size: 25x25x50 cm., medium size: 22.5x22.5x41.5 cm. and small size: 20x20x37 cm. Source: Pa-til district in Yasothorn
b) Symmetry classification of the *muon khit* patterns. The illustration exhibits three band patterns each of which was derived from the central band of the *muon khit* textiles displayed in Figure 3.2a

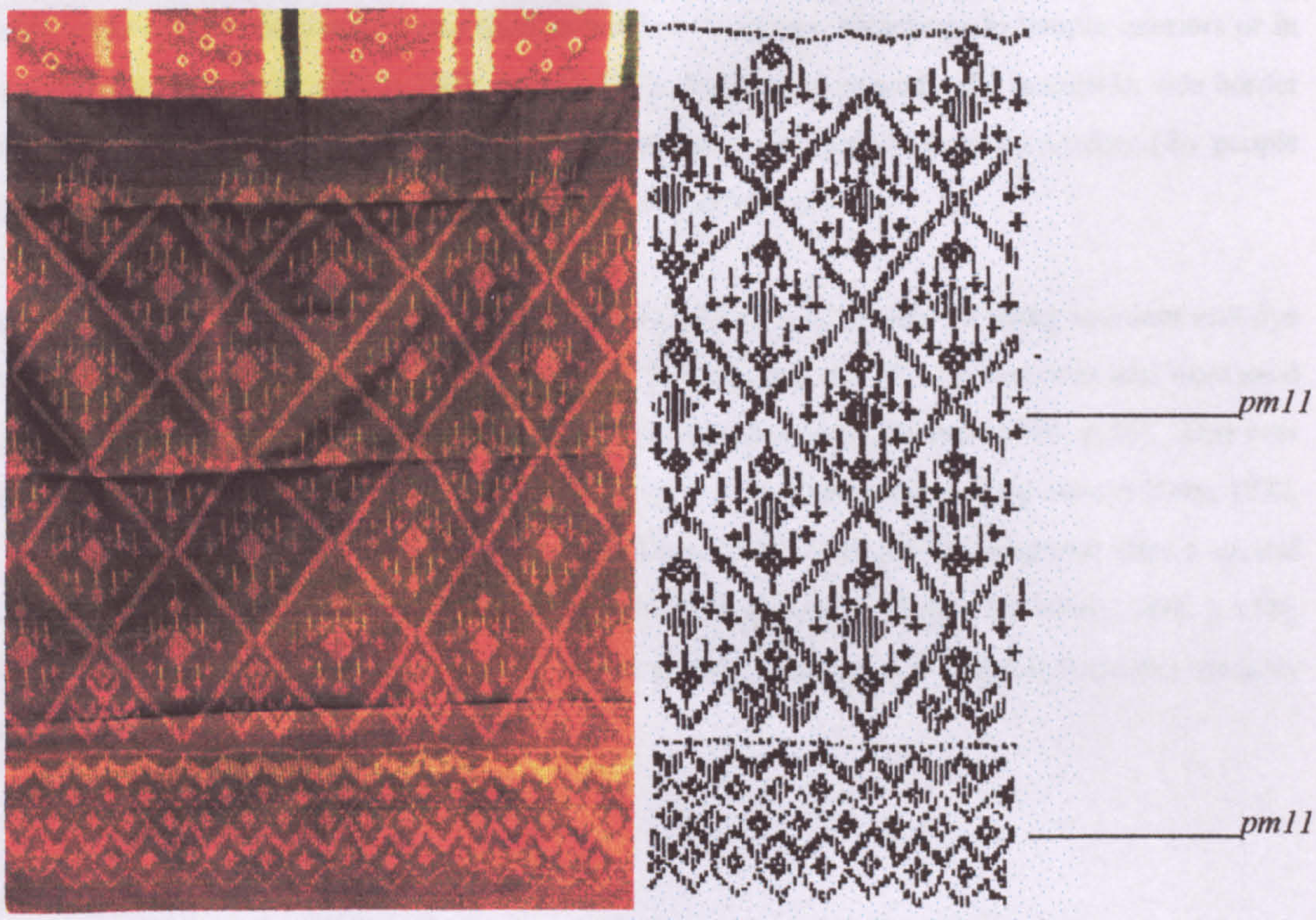
a



b

Pha zin are tubular skirts worn by women of the Thai-Laos in the northeast. Each item contains three parts, i.e., a waistband, a main body and a hem piece [Prangwatthanakun and Naenna, 1994, p.52]. The main body exhibits weft-ikat patterns, while the waistband and the hem piece are plain or decorated with simple geometric patterns obtained by weaving or tie-dyeing. In this case, only patterns in the main bodies were considered for classification. The total sample size was 100 patterns (from 100 textile pieces). Examples of this category are shown in Figure 3.3a,b.

Figure 3.3 a) Pha zin textiles; width: ~69 (x 2) cm., height: 95 cm. (from waist to hem)
Source: Lomkaol district in Petchaboon
b) Symmetry classification of the *pha zin* patterns which exhibit two band patterns class pm11



a b

A survey was made of *tung*, *muon khit* and *pha zin* textiles and this necessitated several fieldtrips to the north and northeast during the 1990s. Data were collected by interviewing textile collectors and weavers, and visiting local villages, museums and textile producing locations. Hundreds of textiles were photographed as pattern samples. While *muon khit* and *pha zin* are daily-used objects which are found in nearly every home, *tung* are religious items held only in collections or kept in monasteries.

Three publications by researchers who had conducted surveys of village textiles in the northeast provided readily available sources for supplementary-weft and weft-ikat patterns. These were *Northeast Cloth* (in

Thai: *Pha Thai Lai Esarn*) by S. Wanamas [1991], and *Supplementary-weft Cloth* (in Thai: *Pha Thai Lai Khit*) and *Weft-ikat Cloth* (in Thai: *Pha Mat-mi*) published by the Department of Industrial Promotion.

Representative samples were collected from the sources mentioned above. A total of 134 *tung* patterns, 100 *muon khit* patterns and 100 *pha zin* patterns were considered.

3.2.2 Court Textiles (*Pha Lai-yang*, *Pha Phuum* and *Pha Yok*)

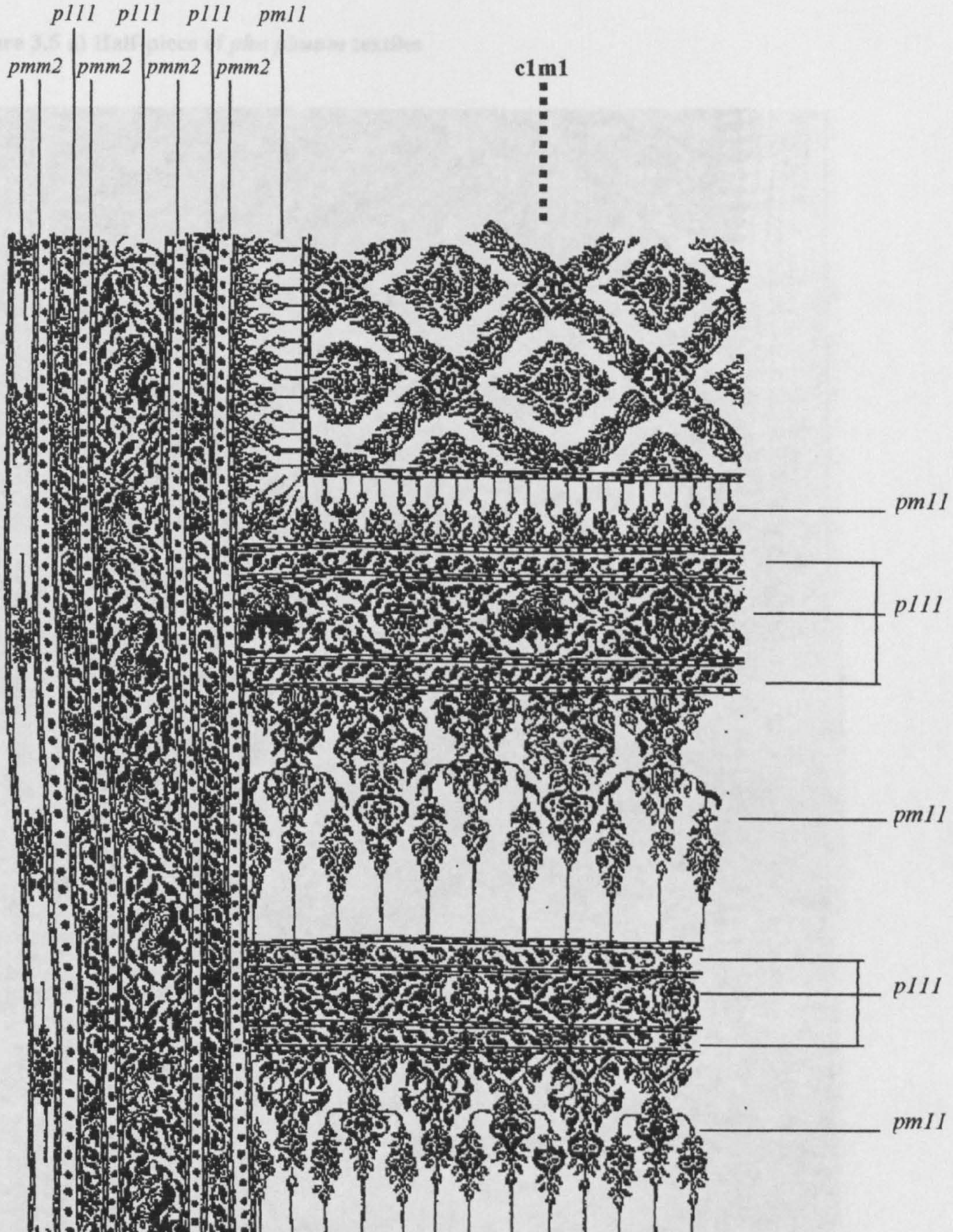
Three categories of court textiles were considered, i.e., *pha lai-yang*, *pha phuum* and *pha yok*. Basically these are framed rectangular cloths which are used either as hip-wrapping costumes worn by either males or females, or as interior decorative items used as window hangings, backdrops to temple interiors or in court settings [Gittinger and Lefferts, 1992, pp.148-159]. Each piece consists of a mainfield, side border and end border. Though they share similar functions and all-over features, they are produced by people in different cultural settings employing different patterning techniques.

Pha lai-yang textiles are cotton cloths which have been printed and/or painted using mordant and dye resist techniques. They were originally imported from India during the 17-18th centuries and were used by kings and other members of the aristocracy [Prangwatthanakun and Naenna, 1994, p.28]. However sometimes the kings presented them as royal gifts to foreign ambassadors and visiting envoys [Guy, 1992, p.92]. Parallel to its name (*pha lai-yang* means the fabric whose patterns are produced after a special design.), the Thai court supplied manufacturers in India with the required patterns [Guy, 1998, p.130]. The total sample size was 195 patterns (from 13 textile pieces). Examples of patterns from this category are shown in Figure 3.4a,b.

Figure 3.4 a) Half-piece of *pha lai-yang* textiles

Source: the National Museum in Bangkok, photographic collection of P. Aun-siri

b) Symmetry classification of the *pha lai-yang* patterns. The sample contains an all-over pattern (emboldened text) in the mainfield which exhibits a rhombic lattice filled with divinities and stylised foliage. Twenty-four band patterns (italicised text) embellish the side and end borders with *khuaycheong* (long-vertical-triangular motifs), mythical features and stylised foliage.

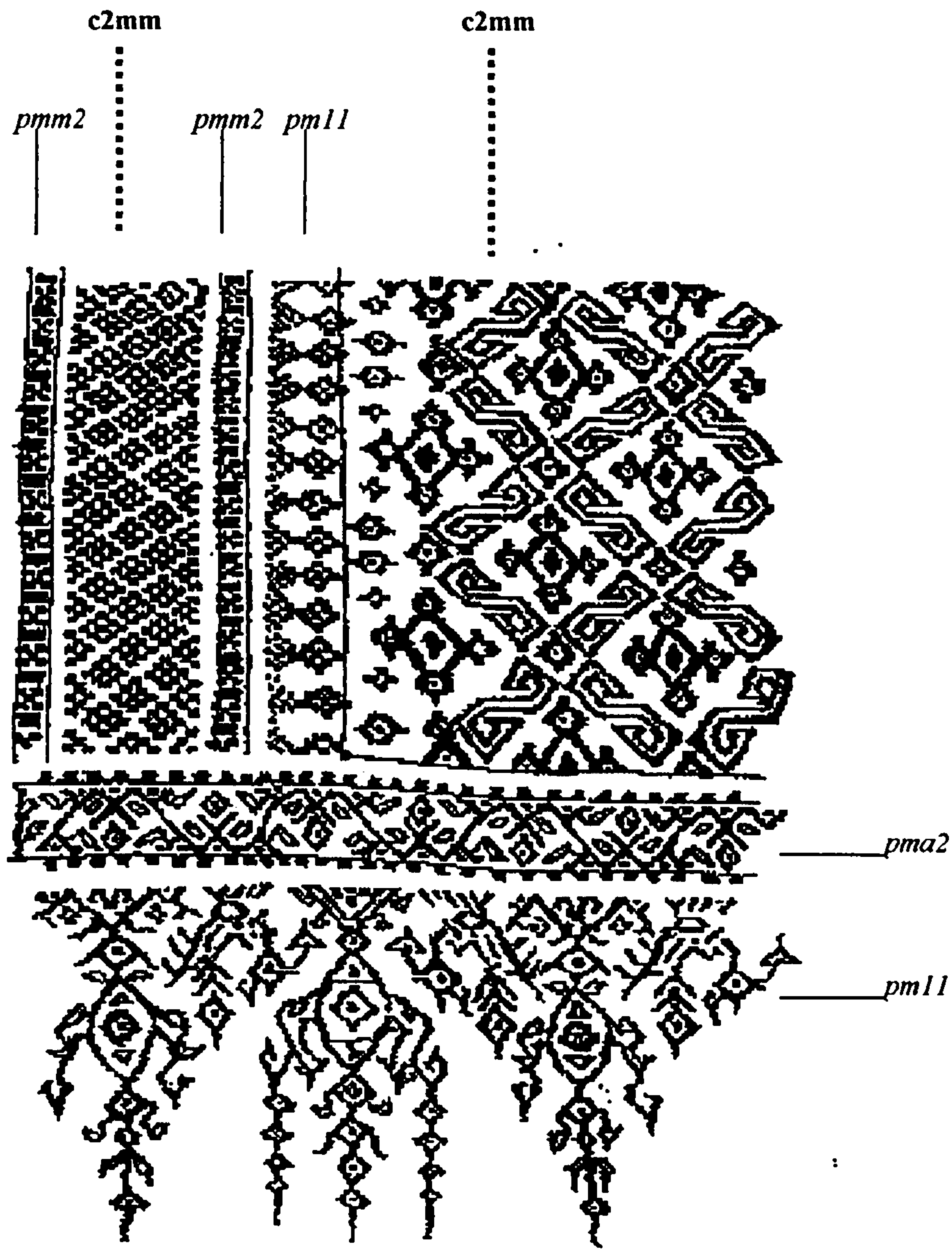


Pha phuum or *som-pat-phuum* are weft-ikat silk cloths made by the Khmer people. During the Ayuthaya and early Ratanakosin periods the kings distributed these cloths annually as uniforms to aristocrats [Dachatiwong Na Ayuthaya, p.36]. The total sample size was 126 patterns (from 13 textile pieces). Examples of this category are shown in Figure 3.5a,b.

Figure 3.5 a) Half-piece of *pha phuum* textiles

Source: the National Museum in Bangkok, photographic collection of P. Aun-siri

b) Symmetry classification of the *pha phuum* patterns. The sample contains two all-over patterns (emboldened text); one is in a mainfield which exhibits hook and star-like motifs in a rhombic lattice and the other is a band within the side border. Five band patterns (italicised text) embellish the side and end borders with *khuaycheong*, stylised foliage and fine geometric shapes.

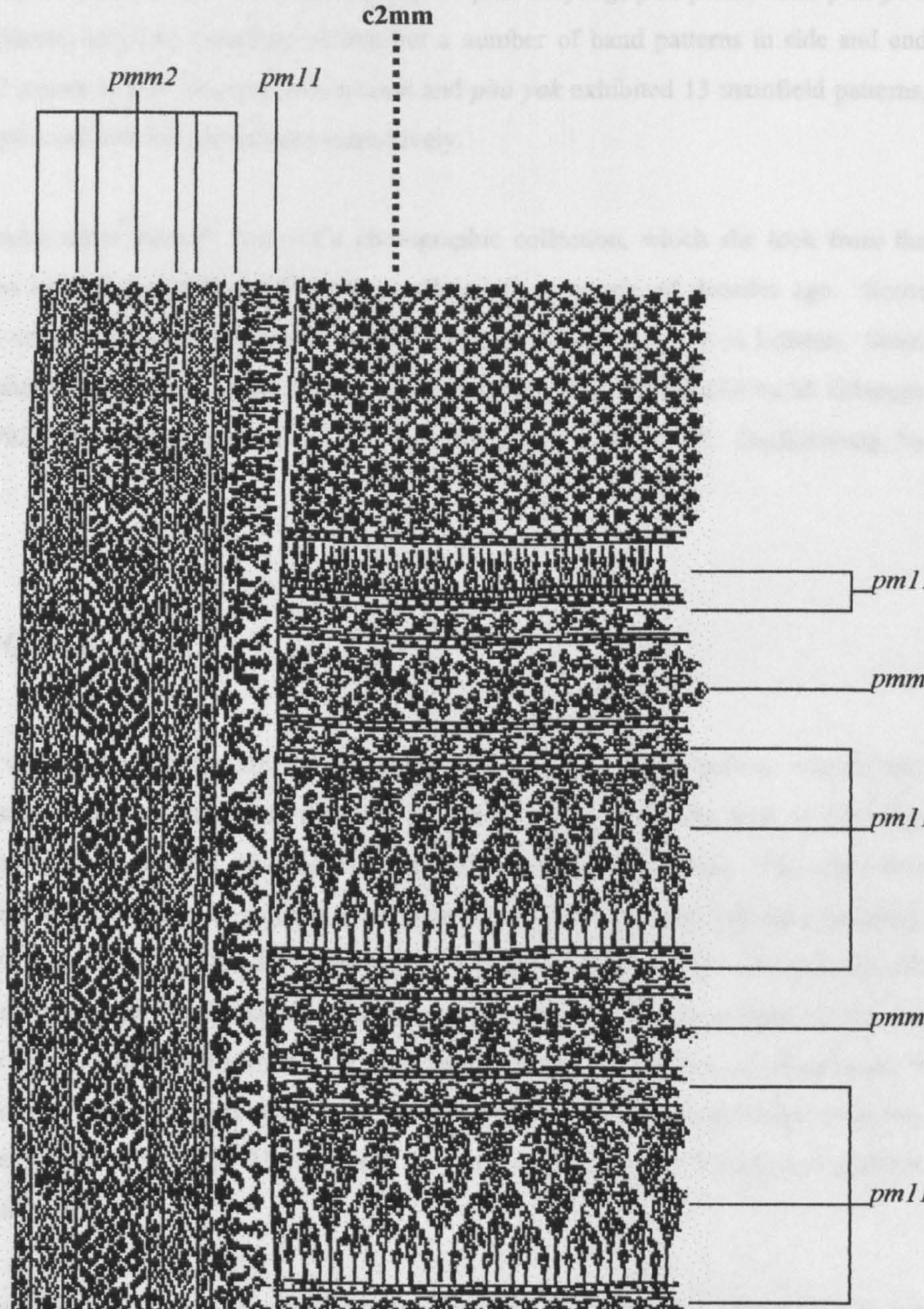


Pha yok are supplementary-weft silk cloths usually patterned with gold or silver thread. Some of these were imported from India while the others were produced by the Thai-Malays in the southern part of Thailand particularly in Nakhon Sri-thammaraj [Gittinger and Lefferts, 1992, pp.158-159, and Prangwatthanakun and Naenna, 1994, p.97]. The total sample size was 124 patterns (from 13 textile pieces). Examples of this category are shown in Figure 3.6a,b.

Figure 3.6 a) Half-piece of *pha yok-thong*, gold-thread supplementary-weft textiles

Source: the National Museum in Bangkok, photographic collection of P. Aun-siri

b) **Symmetry classification of the *pha yok-thong* patterns. The sample contains an all-over pattern (emboldened text) in a mainfield which exhibits a fine floral design in a rhombic lattice. Twenty-one band patterns (italicised text) embellish the side and end borders with *khuaycheong* and stylised foliage.**



In the case of court textiles, owing to their prestige and rareness, samples were limited in number and origin. Some samples are held in museums both in Thailand and abroad, while a great deal are scattered in private collections around the world. In this case a random-sampling technique was not applicable.

Thirteen textiles were the representatives from each group of *pha lai-yang, pha phuum* and *pha yok.* Each textile piece contained only one mainfield pattern but a number of band patterns in side and end borders. Therefore, 13 pieces of *pha lai-yang, pha phuum* and *pha yok* exhibited 13 mainfield patterns, and 182, 113 and 111 side-and-end-border patterns respectively.

Most of the samples were taken from P. Aun-siri's photographic collection, which she took from the ancient cloth collection held at the National Museum in Bangkok, a couple of decades ago. Some samples came from an un-displayed collection held at the Victoria & Albert Museum in London. Some were taken from published sources, i.e., *Textiles and the Tai Experience in Southeast Asia* by M. Gittinger and H.L. Lefferts [1992], and *Thai Textiles* (in Thai: *Sing-tor Thai*) edited by K. Dachatiwong Na Ayuthaya.

3.3 Symmetry Classification

Symmetry inspection was applied to the six categories of Thai textiles. Each pattern sample was identified by its geometrical characteristics and resultant symmetry class. From the total of 134 *tung* pattern samples, 9 were bilateral designs of class d1, while 125 were band patterns. The other five categories exhibited both band and all-over patterns in different ratios (band patterns : all-over patterns), i.e., *muon khit* (67 : 33), *pha zin* (40 : 60), *pha lai-yang* (182 : 13), *pha phuum* (104 : 22) and *pha yok* (111 : 13). For court textile categories, all mainfield patterns were classified as all-over patterns, but not every side-and-end-border pattern was identified as a band pattern class. In the case of *pha phuum,* 9 side-and-end-border patterns were classified as all-over patterns (because each showed translation in two directions), which when included with the 13 mainfield patterns, gave a total of 22 all-over patterns contained in 13 *pha phuum* textiles.

The relevant data are presented in Table 3.1 and in histogram form in Figures 3.7-3.10 which identify the percentage incidence of each of the seven classes of band patterns and seventeen classes of all-over patterns employed in the three categories of village textiles and the three categories of court textiles.

Table 3.1 The classification of bilateral design class d1, seven band pattern classes and seventeen all-over pattern classes exhibited by three categories of village textiles and three categories of court textiles

		Bilateral design class d1	Band pattern classes								All-over pattern classes																	
			p111	p112	pm11	p1m1	pmm2	p1a1	pma2	**Total**	p1	p211	p1m1	p2mm	p1g1	p2gg	p2mg	c1m1	c2mm	p3	p3m1	p31m	p4	p4mm	p4gm	p6	p6mm	**Total**
Village textiles	**Tung**	9	77	-	39	7	2	-	-	125	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
	Muon Khit	-	10	4	11	3	30	-	9	67	1	6	1	5	1	2	1	-	16	-	-	-	-	-	-	-	-	33
	Pha Zin	-	-	2	28	-	8	-	2	40	3	1	17	7	-	-	6	7	17	-	-	-	-	-	2	-	-	60
Court textiles	**Pha Lai-yang**	-	49	9	91	-	33	-	-	182	1	-	-	-	-	-	-	7	-	-	-	-	-	5	-	-	-	13
	Pha Phuum	-	-	3	56	-	37	1	7	104	-	-	-	-	-	-	-	2	20	-	-	-	-	-	-	-	-	22
	Pha Yok	-	9	9	55	11	25	1	1	111	-	-	-	2	-	-	-	4	7	-	-	-	-	-	-	-	-	13

3.3.1 Band Pattern Symmetries Exhibited in *Tung*, *Muon Khit* and *Pha Zin* Patterns

The percentage frequency of the seven band pattern classes exhibited by 125 *tung* patterns, 67 *muon khit* patterns and 40 *pha zin* patterns is shown in Figure 3.7.

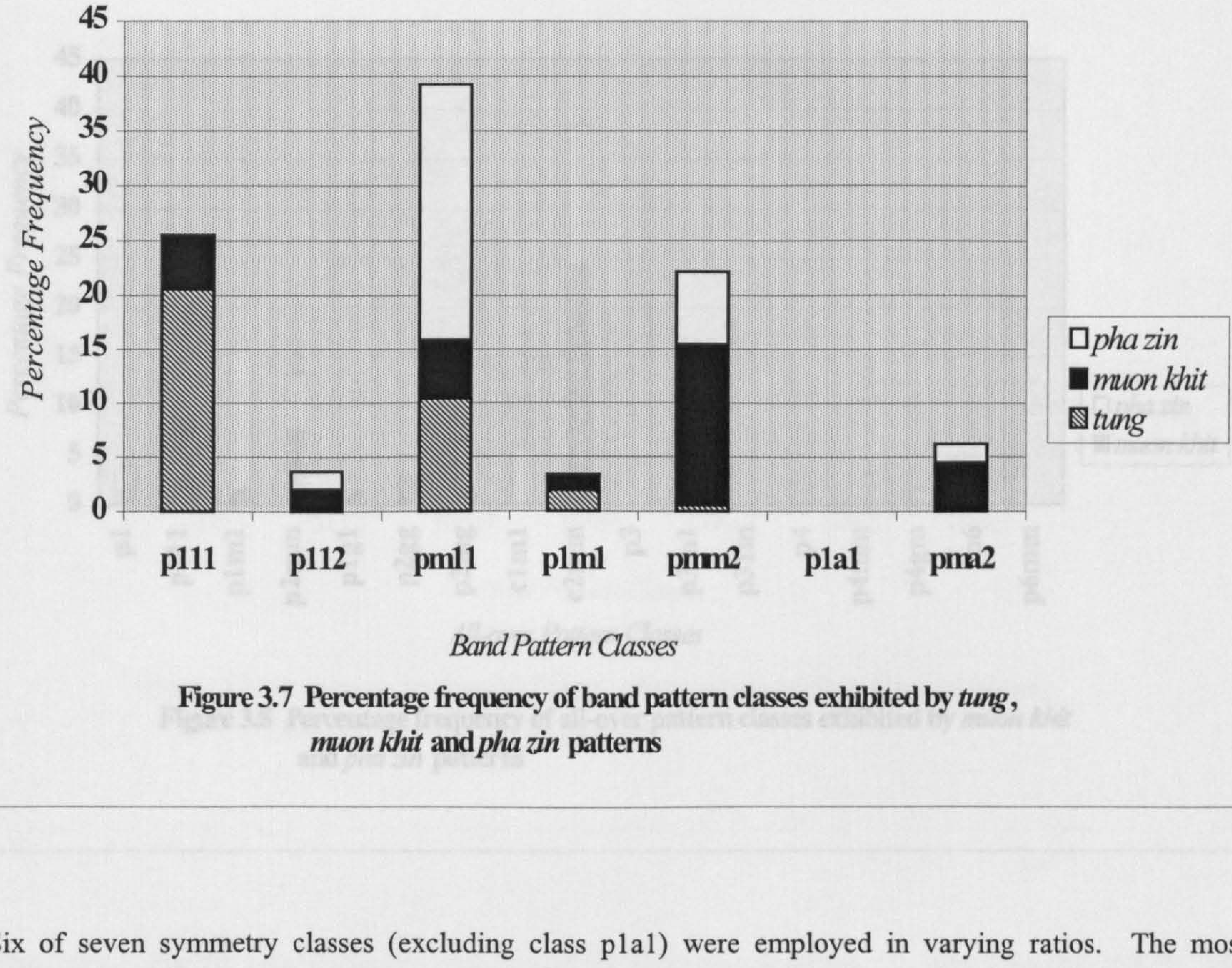


Figure 3.7 Percentage frequency of band pattern classes exhibited by *tung*, *muon khit* and *pha zin* patterns

Six of seven symmetry classes (excluding class p1a1) were employed in varying ratios. The most dominant class was class pm11 which accounted for 10.4% in *tung* patterns, 5.47% in *muon khit* patterns and 23.33% in *pha zin* patterns. Meanwhile class p111 was employed mostly in *tung* patterns (20.53%) and in *muon khit* patterns (4.98%). Two-fold reflectional symmetry class pmm2 was exhibited in 0.53% of *tung* patterns, 14.93% of *muon khit* patterns and 6.67% of *pha zin* patterns. The remaining classes obtained percentage frequencies in varying ratios, i.e., class pma2 accounted for 4.48% in *muon khit* patterns and 1.67% in *pha zin* patterns; class p112 accounted for 1.99% in *muon khit* patterns and 1.67% in *pha zin* patterns; class p1m1 accounted for 1.87% in *tung* patterns and 1.49% in *muon khit* patterns.

3.3.2 All-over Pattern Symmetries Exhibited in *Muon Khit* and *Pha Zin* Patterns

A total of 33 *muon khit* patterns and 60 *pha zin* patterns employed ten symmetry classes from the seventeen all-over pattern classes. The percentage frequency is shown in Figure 3.8.

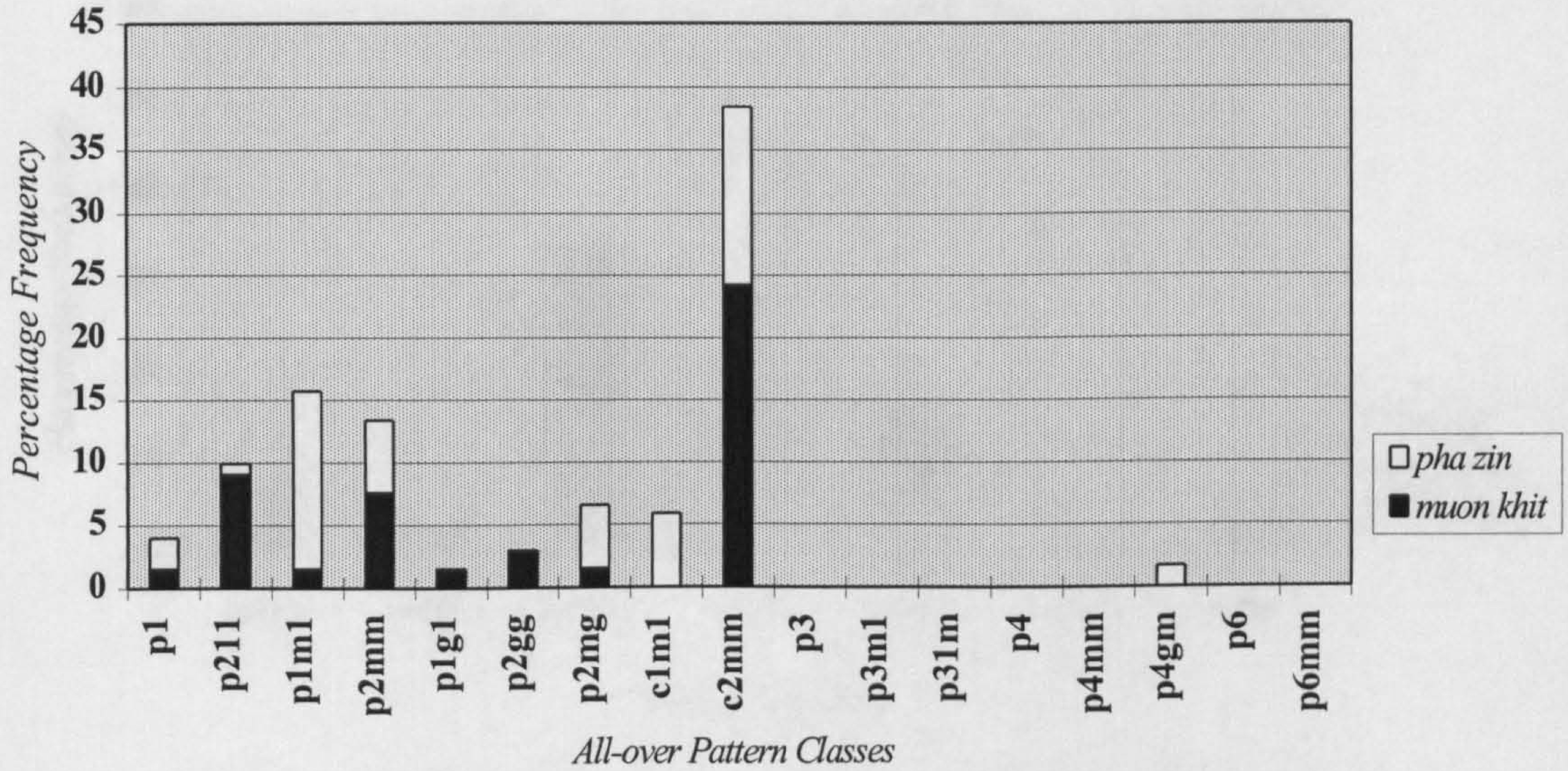


Figure 3.8 Percentage frequency of all-over pattern classes exhibited by *muon khit* and *pha zin* patterns

Class c2mm was the most dominant symmetry class and accounted for 24.24% in *muon khit* patterns and 14.17% in *pha zin* patterns. Class p1, p211, p1m1, p2mm and p2mg were also employed in *muon khit* and *pha zin* patterns at different percentage ratios (*muon khit* : *pha zin*), i.e., 1.52 : 2.5, 9.08 : 0.84, 1.52 : 14.16, 7.57 : 5.84, and 1.52 : 5 respectively.

However class p1g1 and p2gg were exhibited only by *muon khit* patterns and accounted for 1.52% and 3.02% respectively. Meanwhile classes c1m1 and p4gm were exhibited only by *pha zin* patterns and these accounted for 5.84% and 1.66% respectively.

3.3.3 Band Pattern Symmetries Exhibited in *Pha Lai-yang*, *Pha Phuum* and *Pha Yok* Patterns

All seven symmetry classes of band patterns were employed by 182 *pha lai-yang* patterns, 104 *pha phuum* patterns and 111 *pha yok* patterns. The percentage frequency of each class is shown in Figure 3.9.

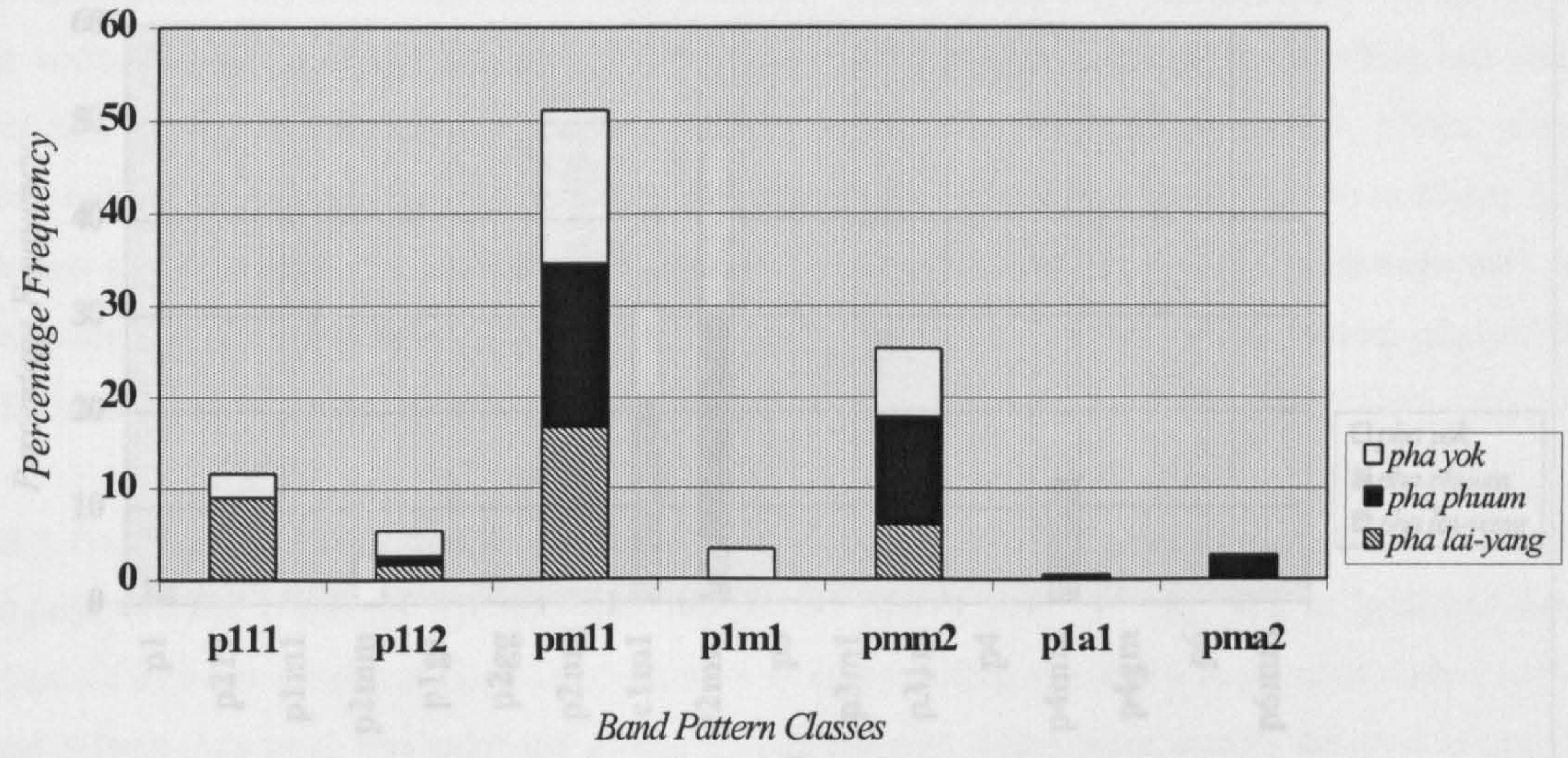


Figure 3.9 Percentage frequency of band pattern classes exhibited by *pha lai-yang*, *pha phuum* and *pha yok* patterns

Class pm11 was the most dominant class shared by all three categories, i.e., *pha lai-yang* 16.67%, *pha phuum* 17.95% and *pha yok* 16.52%. The second most dominant class was class pmm2, which accounted for 6.04% in *pha lai-yang* patterns, 11.86% in *pha phuum* pattrerns and 7.51% in *pha yok* patterns. Class p112 was obtained in 1.65% of *pha lai-yang* patterns, 0.96% of *pha phuum* patterns and 2.7% of *pha yok* patterns. Only *pha lai-yang* and *pha yok* patterns exhibited class p111 patterns at 8.97% and 2.7% respectively. Two glide-reflectional symmetry classes were employed in *pha phuum* and *pha yok* patterns, i.e., class p1a1 was obtained in 0.32% of *pha phuum* patterns and in 0.3% of *pha yok* patterns. Class pma2 was obtained in 2.36% of *pha phuum* patterns and in 0.3% of *pha yok* patterns. However only 3.3% of *pha yok* patterns exhibited symmetry class p1m1.

3.3.4 All-over Pattern Symmetries Exhibited in *Pha Lai-yang*, *Pha Phuum* and *Pha Yok* Patterns

Only five of the seventeen all-over pattern classes were employed in the 13 *pha lai-yang* patterns, the 22 *pha phuum* patterns and the 13 *pha yok* patterns. The percentage frequency is shown in Figure 3.10.

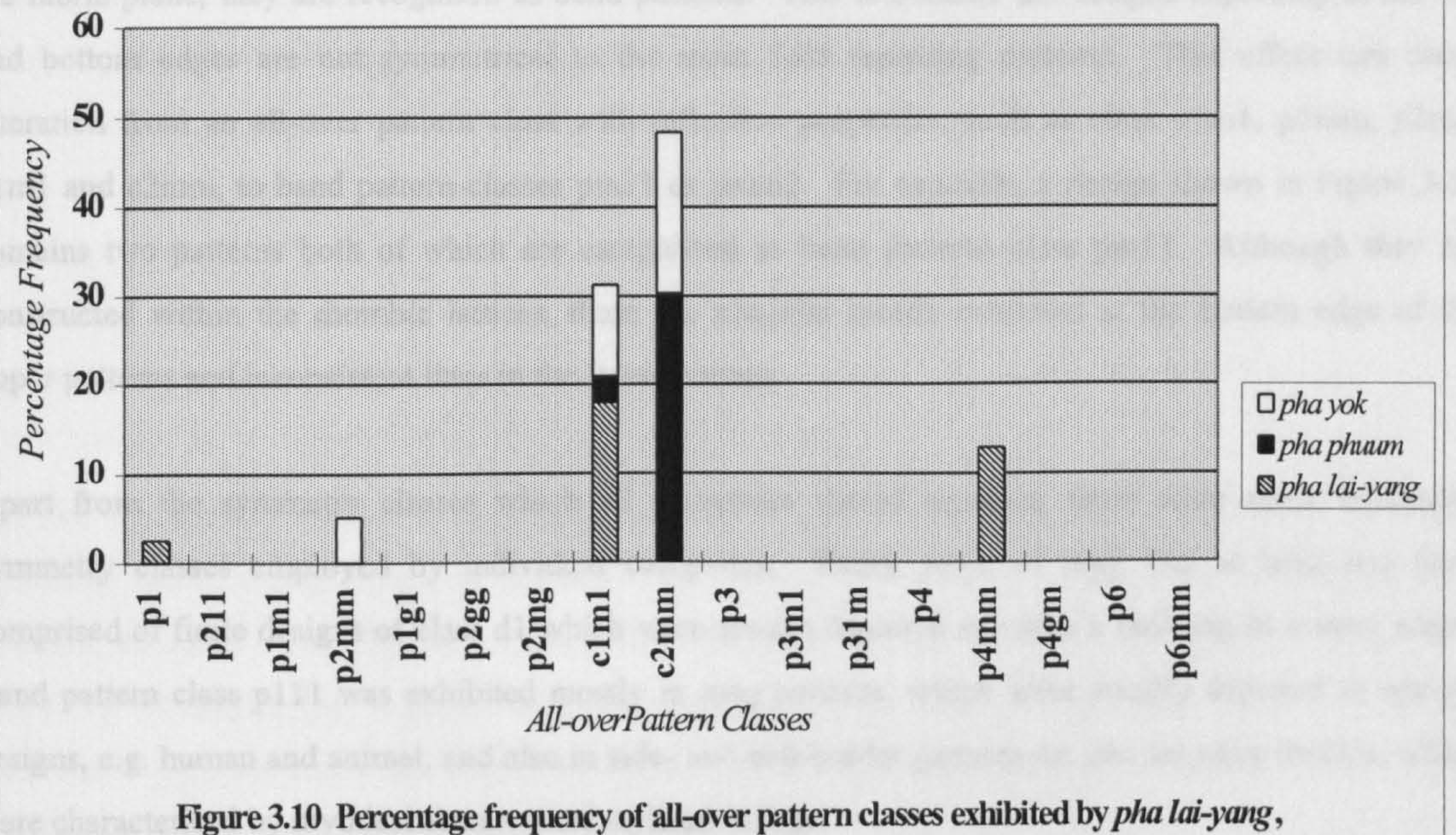


Figure 3.10 Percentage frequency of all-over pattern classes exhibited by *pha lai-yang*, *pha phuum* and *pha yok* patterns

The most dominant symmetry was class c2mm, which accounted for 30.3% in *pha phuum* patterns and 17.95% in *pha yok* patterns. Class c1m1 was shared by all three categories, i.e., *pha lai-yang* 17.95%, *pha phuum* 3.03% and *pha yok* 10.26%. The three remaining classes were class p4mm and p1 each of which was employed only in *pha lai-yang* patterns and accounted for 12.82% and 2.56% respectively, and class p2mm which was employed in 5.13% of *pha yok* patterns.

3.4 Summary of Findings

Dominant symmetry classes were common across all categories of all-over patterns; these classes (classes c1m1 and c2mm) were characterised by a variety of the rhombic lattices***. Meanwhile the dominant features of band patterns exhibited bilateral symmetry perpendicular to the band axis (class pm11).

In fact there are some links between the symmetry characteristics of some band and all-over patterns. For example, when the centre-celled all-over pattern class c2mm is cropped into bands perpendicular to either the x or the y axis, band patterns of class pmm2 or pma2 may be produced. However, if there are reflection axes in only one direction as in class c1m1 patterns, cropping into bands will produce pm11 bands when reflection axes are perpendicular to the band axis, or class p1m1 when a reflection axis is parallel to the band axis.

In some cases of *pha zin* textiles, although the patterns are governed by the lattices extending throughout the fabric plane, they are recognised as band patterns. This is because the designs especially at the top and bottom edges are not symmetrical to the main field repeating patterns. This effect can cause alteration from an all-over pattern class with reflection properties, such as class p1m1, p2mm, p2mg, c1m1 and c2mm, to band pattern classes pm11 or pmm2. For example, a design shown in Figure 3.3b contains two patterns both of which are categorised as band patterns class pm11. Although they are constructed within the rhombic lattices, there are irregular motifs exhibited at the bottom edge of the upper patterns and inconsistent lines in the lower pattern.

Apart from the symmetry classes which all categories shared together, there were some noticeable symmetry classes employed by individual categories. Every piece of *tung* had at least one band comprised of finite designs of class d1 which were always depicted as either a building or a story scene. Band pattern class p111 was exhibited mostly in *tung* patterns, which were usually depicted as upright designs, e.g. human and animal, and also in side- and end-border patterns on *pha lai-yang* textiles, which were characterised by mythical features and stylised foliage.

Patterns from classes p3, p3m1, p31m, p4, p6 and p6mm were absent. All-over pattern class p4gm of *pha zin* patterns always displayed a variety of key-like designs, while class p4mm of *pha lai-yang* patterns revealed centre-celled square lattices filled with four-directional divinities and star-like motifs.

Khuaycheong, a long-vertical-triangular motif, was the predominant design representing a band pattern of class pm11 in each of the three categories of court textiles.

Design components of the various textile categories differ, but the geometry within pattern structures remains broadly similar. Based on the rhombic lattices, *pha lai-yang* textiles exhibit fine and rich designs of divinities, mythical features and stylised foliage, while *muon khit* textiles display rigid geometrical designs inspired by nature and folk lore.

Some dominant features were found among the six categories of Thai textiles studied. Configuration designs, in which a variety of fine details were presented together to form particular shapes, were the main characteristic exhibited in the patterns of all categories. A simple example can be seen from the *muon khit* pattern class p1m1 (the bottom one in Figure 3.2b). Each rhomboid located at a band axis consists of nine small rhomboid inside and is bounded with a repetition of black and white triangles along its edges. The half-shaped rhomboids at both edges are filled with bilateral hook-like motifs, one small rhomboid and two half-shaped rhomboids. In the case of *pha lai-yang* textiles in Figure 3.4, *kanok* (a flame-like motif) is used as a basic element incorporated with floral and figurative motifs (e.g. divinity and mythical features) to form configuration designs of a curvilinear pattern based on a rhombic lattice within the mainfield and band patterns within side and end borders.

Among the three categories of court textiles it was common to find multi-layered bands in which a series of band patterns were embellished with different kinds of configuration motifs. In the case of village textiles, *naga* (underwater serpent) was the most predominant motif.

3.5 Discussion of Results

The two hypotheses presented in the introduction to this paper were supported by the dominance of a few symmetry classes among the six categories of Thai textiles. When the patterns from textiles employed in different end-uses (i.e. religious, household or costume textiles) and produced by three different patterning techniques (i.e. supplementary-weft, weft-ikat or printing and painting) were classified by reference to their symmetry features, it was found that each category exhibited some similarities in their symmetry preferences. Symmetry characteristics of the preferred symmetry classes as well as their design content were examined in order to ascertain their cultural significance. Symbolism derived from Buddhist concepts and traditional beliefs appeared to be of importance in the context of pattern construction. Further consideration of these matters is presented below.

3.5.1 Significance of Patterning Techniques

One of three patterning techniques was used in the production of each of the six categories of textile samples. *Tung*, *muon khit* and *pha yok* textiles were patterned by the supplementary-weft technique. *Pha zin* and *pha phuum* textiles were patterned by the weft-ikat technique. *Pha lai-yang* textiles were produced by printing and painting techniques.

The emergence of certain symmetry characteristics were considered in the context of the various patterning techniques. The use of traditional block-printing and hand painting allows repeats to be constructed employing any of four symmetry operations. However, *pha lai-yang* textiles exhibited only a few symmetry classes of band patterns and all-over patterns.

The presence of all-over patterns class p4mm found only in the *pha lai-yang* category suggests a possible link to all-over pattern classes p2mm and c2mm exhibited in the other four categories, i.e., *muon khit, pha zin, pha phuum* and *pha yok* textiles. It is possible to produce the patterns based on four equal sides and right angles of a square lattice by block-printing. But for the textiles whose patterns are associated with weaving, the square lattice is capable of being extended in one direction to produce a rectangular or a rhombic lattice (depending on the orientations of the square lattices) due to a non-symmetrical density of warp and weft.

In the case of the supplementary-weft and weft-ikat patterning techniques, which were employed in patterning the other five textile categories, certain symmetry classes predominated, i.e., band pattern classes pm11 and p2mm, and all-over pattern classes c1m1, c2mm, p1m1 and p2mm. It should be noted that all of these symmetry classes have reflection properties.

Reflection symmetry along the weft direction is the significant feature co-existing within the supplementary-weft and weft-ikat patterning techniques. A reflection axis automatically occurs between every last row of each repeat and the first row of the next repeat.

Beyond certain symmetry characteristics, different patterning techniques also provide different design features. Rigid geometric designs are applicable by the supplementary-weft technique, whereas the weft-ikat technique allows a diagrammatic use of fragile broken lines to create fluidity and iridescent patterns. A wide range of patterns, especially fine, curved and pictorial designs, can be presented by printing and painting.

3.5.2 Symbolism and Geometry

Symmetry classification and analysis indicated a predominance of the rhombic lattice across nearly every sample group, bilateral long-vertical-triangular motifs (*khuaycheong*) within multi-layered bands of the three categories of court textiles, the presence of all-over pattern class p4mm in *pha lai-yang*, and bilateral designs class d1 and band patterns class p111 of *tung*. Certain motifs were also found frequently, e.g. various shapes of *naga* motifs from village textiles, stylised foliage, mythical features and divinities from court textiles. All of these similarities and diversities are discussed below.

Looking back to the cultural function of *muon khit*, they are served as traditional offerings in occasional rituals and ceremonies performed by the Tai Buddhists in the north and north-east of Thailand. In a service of social hierarchy, young people offer the textiles to the elders as a sign of respect and as an act of apology at weddings or new year ceremonies [Gittinger and Lefferts, 1992, pp.58-73]. They were also presented to monks during *Bun Kathin*, a yearly religious ceremony at the close of the rainy season in order to earn merit [Gittinger and Lefferts, 1992, p.116]. The patterns thus exhibit auspicious motifs, e.g., *bai-sri* (an offering object made of banana leaf), *tham-mas* (a pulpit in the form of an elaborately carved seat), *dok jun* (a four-petal flower) and *naga* (Figure 3.11).

Figure 3.11a-c Three auspicious motifs of *muon khit* patterns: a) *bai-sri*, b) *dok jun*, and c) *tham-mas*

a

b c

Sources: (a) and (c) were from Ko-wang district in Yasothorn, and (b) was from Mueng district in Mahasarakam

As a religious object, *tung* were suspended inside and outside the temple during special occasions. They were produced primarily to earn merit for dead people and symbolised the maker's wish for a prosperous future [Gittinger and Lefferts, 1992, p.130]. In the *Bun Phrawet* ceremony, they were hung vertically on the poles located at the eight principle compass directions, thus representing the Buddhist wishing tree of life [Gittinger and Lefferts, 1992, p.136]. In association with the *Bun Phrawet* ceremony, which celebrates the story of the Buddha, the patterns were thus associated with the ten-births prior to being Buddha, generally known as *thotsachat*. The prime focus is on the greatest virtue of giving that has been communicated through every birth scene, especially in the great birth scene of Prince Vessantara, known as *mahachat* [Ginsburg, 1989, p.44]. Each life-scene is depicted by various figures of humans, animals, plants and buildings along consecutive panels. Every *tung* piece usually has at least one band showing a building motif which Gittinger and Lefferts [1992, p.130] identified as *hau prasat*, a temple or a stupa flanked by banners and occasionally trees.

From the village textile viewpoint, it seems that nature and religious belief are the major sources of inspiration. Most motifs are simplified into geometric shapes and repeated in pattern form. Nevertheless they are still named after their sources and have their own meanings.

Naga or underwater serpents are the most popular features among all categories of village textiles. They appear on textile patterns in a variety of designs, e.g., a bilateral *naga*, a seven-headed *naga*, and a repetition of the *naga's* head along the geometric frames (Figure 3.12). In agriculturally-based countries, such as Thailand, water is considered to be of great cultural significance and the *naga* is chosen as a water symbol [Jumsai, 1997, pp.16-23]. According to Buddhist mythology it is not only an underwater serpent, which guards *amarit*, sacred ambrosia, but it is also a water-distributor. Additionally while the Buddha was meditating by the riverside, the *naga* coiled itself as his throne over flooded water and spread its seven hoods to shield him from rain [Archambault, 1988, p.54].

Figure 3.12a,b A variety of *naga* designs in a) *pha bieng* (shawl) and b) *pha zin*

a **b**

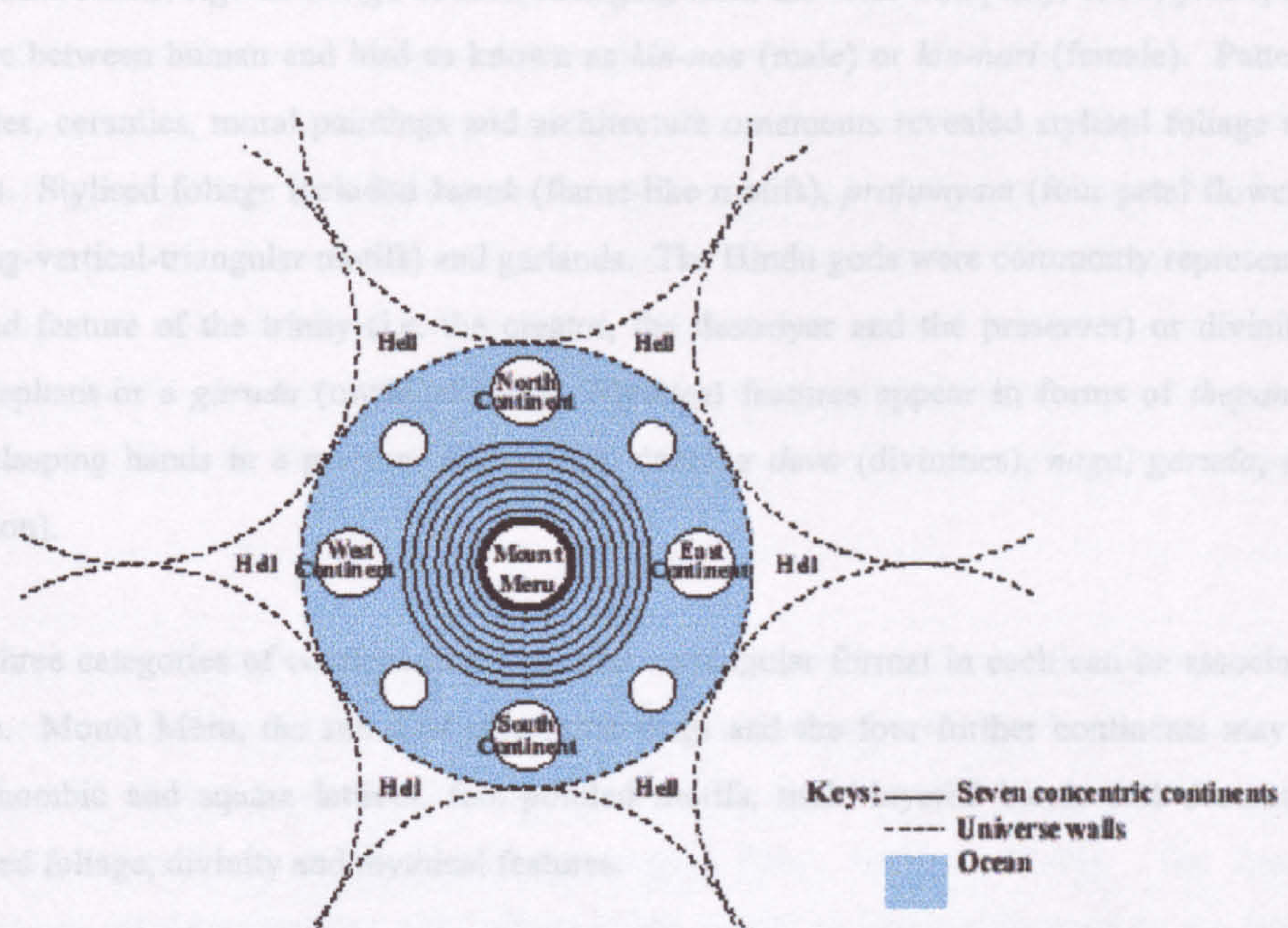
Sources: (a) was from Chiang Khong district in Chiang Rai, and (b) was from Ban Phon district in Kalasin

Another noticeable feature derived from the village textiles is a rhombic configuration design, which is widely produced in many different cultural settings. The rhomboid or diamond is denoted as the symbol for the Buddha's mind or sometimes symbolises a house or a temple [Archambault, 1988, p.54]. When it is used as an outline shape enclosing other motifs such as the *naga* in a diamond shape then the design is known as a *naga*'s house. A concentric-diamond motif at one end of a Tai Neua shawl is considered to be a third-eye symbol in spirit-invoking ritual [Cheeseman, 1988, p.88]. In addition, when the similar motif is exhibited on a small rectangular-formatted cloth, it is considered to aid meditation by Buddhists [Gittinger, 1989, p.228].

Buddhism plays a significant role in Thai culture. Artists glorify Buddhism by applying Buddhist ethics, beliefs and also philosophy on various kinds of crafted items. Jumsai [1982, p.176] maintained that there were no ritual events which did not relate to water and the cosmos.

The Hindu-Buddhist cosmological model is applied widely on many kinds of media, i.e., politics [Chuintaranond, 1990], paintings, architecture, royal artefacts and decorative patterns throughout the Southeast Asia region. Kalyanamitra [1982, pp.315-317] explained the Thai cosmos model according to the description in the *traiphum phra ruan* (a collection of ancient Thai literature). A helpful illustration is provided in Figure 3.13. It is believed that at the centre of the universe is the location of Mount Meru, which is encircled by a series of concentric rings representing seven continents. All these are surrounded by the ocean extending indefinitely in every direction. There are four continents each located at each of the four compass directions. The continent at the south is called *chomphuthawip* and is the human earth where the Buddha was born. On the top of Mount Meru, Indra, the king of the divine world, is enthroned at *daowadung* heaven guarded by four divinities in four directions [Ginsburg, 1989, p.16].

Figure 3.13 A cosmological model showing Mount Meru at the centre encircled with seven concentric rings of continents, ocean and four further continents at the north, south, east and west.



Source: derived from Kalyanamitra, 1982, p.316

As Ginsburg [1989, pp.13-26] pointed out the Thais clarified the cosmos model through text and illustrations of the three worlds which were known as *traiphum*, and which were generally interpreted as heaven, earth and hell. Buddhist philosophy emphasised three principles (i.e. *dukkha* (suffering), *annica* (impermanence), *anatta* (non-self)) and rebirth, and held that *krama*, the good or bad actions in this life, determined modes of birth in the next life [Taylor, 1994, pp.15-16]. A hierarchy of birth was classified in different levels of existence ranging from the bottom-most of hell to the highest position of heaven. *Nirvana* or extinction is the ultimate goal and the highest achievement to aid the abolition of suffering.

Architects and craftsmen developed the cosmos model through plans and multi-tiered conical forms of three-dimensional objects where the summit is recognised as *nirvana*. This can be detected in large-scaled architecture, e.g., pyre, pagoda, stupa and chariot, as well as in small-scale artefacts, e.g. *mahapichai mongkut* (the king's step-conicle crown), *chat* (a tiered parasol) and *bai-sri* (an offering object made of banana leaf) [Jumsai, 1997, pp.10-43, and Kalyanamitra, 1982, pp.310-364].

The concept of the cosmos model was also developed on two-dimensional surface decoration. A great variety of Thai classical patterns have mostly been derived from nature**** and myth. As Taylor pointed out Thai artists hardly portrayed nature in a realistic form, but instead embellished the image of nature with a touch of fantasy [Taylor, 1994, p.xix]. Imaginative figures were usually created by merging plant with human or animal forms, e.g. the design of *deva* emerging from the lotus-bud [Guy, 1998, p.134] and a combined figure between human and bird as known as *kin-non* (male) or *kin-nari* (female). Patterns depicted on textiles, ceramics, mural paintings and architecture ornaments revealed stylised foliage and figurative designs. Stylised foliage included *kanok* (flame-like motifs), *prajumyam* (four-petal flowers), *khuaycheong* (long-vertical-triangular motifs) and garlands. The Hindu gods were commonly represented by the triple-faced feature of the trinity (i.e. the creator, the destroyer and the preserver) or divinities riding a white elephant or a *garuda* (mythical bird). Mythical features appear in forms of *thepanom* (angelic beings clasping hands in a gesture of worship), dancing *deva* (divinities), *naga*, *garuda*, and *singa* (mythical lion).

Considering the three categories of court textiles, the total rectangular format in each can be associated with the universe. Mount Meru, the series of concentric rings and the four further continents may be represented by rhombic and square lattices, four-pointed motifs, multi-layered bands and decorative elements of stylised foliage, divinity and mythical features.

The three samples of *pha lai-yang, pha phuum* and *pha yok* textiles, shown in Figure 3.4, 3.5 and 3.6 respectively, exhibited rhombic lattices within their mainfields. The rhombic lattice offered three possible positions for the motif placement, i.e., at the interval, the intersecting point and along the lattice side. The main motifs were in all cases placed at the interval guarded by the motifs at four intersecting points and along four sides of the lattice. Four-directional motifs such as four-petal flowers and four-point-star-like motifs were found commonly within the rhombic lattices of *pha phuum* and *pha yok* textiles, while the configuration designs usually filled with divinities, mythical features and stylised foliage were found only on *pha lai-yang* textiles. The rhombic lattice were sometimes presented by a centre-celled square lattice or developed by inward and outward curves to generate a curvilinear lotus-bud lattice known as a *phum* lattice. The lotus-bud-like shape is probably derived from an iconic lotus symbol, which is associated with the birth of the Buddha [Taylor, 1994, p.17].

The side and end borders of the three categories of court textiles revealed multi-layered bands embellished with assorted kinds of stylised foliage and figurative designs. A number of bands and special types of motifs were indicated according to the hierarchical distinction of the users. As Gittinger pointed out the textiles with three tiers of *khuaycheong* motifs in the end border were usually worn by men, and only rarely by women who commonly wore two-tiered end borders [Gittinger and Lefferts, 1992, p.153]. Special designs were additionally restricted in use, e.g., the textiles patterned with gold or depicting Indra riding on the multi-head white elephant (Erawan) within the mainfield was presumably reserved for the kings and the royal family, while plain white cloths with decorative frames were generally worn by the aristocrats [Guy, 1998, p.127].

The cosmological concept also related to the old Sai-lane belief that the kings from Ayuthaya to the present Ratanakosin period were recognised as the god-kings or as Siva [Pramoj, 1982, p.27]. The *pha lai-yang* textiles of the type illustrated in Figure 3.4 might be reserved for royal use. In this case, the cosmos model was applied on a curvilinear rhombic lattice. The pattern exhibited a configuration *phum* (lotus-bud shape) design containing the *thepanom* emerging from a lotus-bud at every interval, which was guarded by mythical protector's faces at four intersecting points and dancing divinities along four lattice sides. Assorted types of stylised foliage and mythical features were packed in multi-layered bands framing around the mainfield. However, there were no Hindu gods or mythical features appearing on the patterns of *pha phuum* of the type illustrated in Figure 3.5, since it was officially used as the aristocrat's uniform.

The analysis of the patterns from six categories of Thai textiles indicated some particular features, which were not only shared within Thai culture but probably also shared among certain other Asian cultures. The *khuaycheong* motif within a band pattern was denoted as the distinctive Thai *tumpal* or saw-tooth-like patterns found commonly in the Southeast Asia region [Guy, 1998, pp.58-60]. The framed rectangular format was cited by Gittinger and Lefferts [1992, p.168] as being influenced by the Indian double-ikat cloth, known as *patola* (*patolu* in singular), in which the Khmer and Thai most frequently employed a rhombic lattice in the centrefield region.

A close link between Thai court textiles and two traditional Indian textiles, i.e., *patola* and *sarasa,* was also suggested by Guy [1989/92]. *Patola* are double-ikat-patterned silk cloths originally produced in

Gujarat, western Indian. *Sarasa* are cotton cloths with printed and/or painted patterns by mordant and dye resist methods. Basically both textiles share similar features in terms of motifs and pattern composition. Due to the high-valued materials and sophisticated production methods of *patola*, they were reasonably imitated either by lower-valued material such as cotton or less-complex patterning such as weft-ikat or printing and/or painting techniques. Consequently there were many kinds of cloth in India and throughout Southeast Asia which are deemed to be *patola* imitations or developments [Buhler, 1959, and Larsen, 1976].

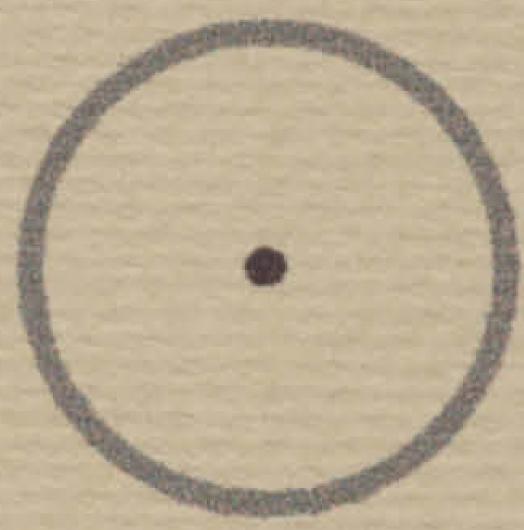
Endnotes

*A number of Thai textile collections are held both in Thailand and elsewhere. In Thailand, these are at the National Museum in Bangkok, Ubon Rajathani and Nakhon Sri-thammaraj; regional craft/textile collections, e.g. the Promotion Centre of Arts and Culture at Chiang Mai University. Other locations include Ban Lai Pai Ngam (in Chiang Mai), Sathon (in Sukhothai), Khum-pun (in Ubon Rajathani), and Songkram Mai Thai (in Khonkaen). Important collections outside Thailand include those held in the Textile Museum in Washington D.C., the National Museum of Design (Smithsonian Institute), the Victoria & Albert Museum (un-displayed collection), the Ethnographical Museum of Basle in Switzerland and the Australian National Gallery.

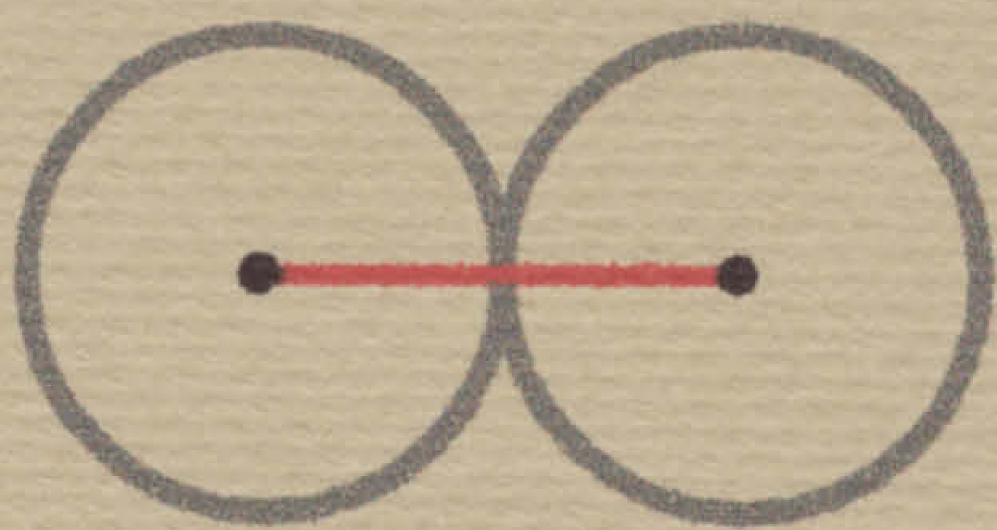
** It can be assumed that weaving culture in Thailand was originally practised by the Tai race whose sub-groups include the Tai Daeng, Tai Phuan and Tai Lue who settled scatteringly throughout the north, northeast and in some central areas. Prangwatthanakun and Naenna [1990, pp.21-39] distinguished various traditional patterning techniques. These included *yok dok* in which patterns were made during the weaving process itself was a form of patterning traditionally employed throughout the country. *Mat-mi* or ikat, especially weft-ikat, was extensively used by Thai-Laos, the majority group in the north and northeast, and Thai-Cambodian in the southern part of the northeast. Supplementary-weft technique, known as *khit* is when continuous supplementary-weft is woven to form the patterns across the cloth's width. Alternatively *chok* is when discontinuous supplementary-weft is used instead; both techniques were widely employed by Tai peoples in the north, northeast and central areas. Meanwhile the supplementary-warp technique, known as *muk*, was employed particularly by the Tai Daeng and the Tai Phuan in the north. *Ko* or *luang* or tapestry weave was practised only by the Tai Lue in Nan, Chiang Rai and Payao provinces.

*** The rhombic lattices involve a wide range of the centre-celled patterns which possibly exhibit symmetry classes c1m1, c2mm, p1m1 or p2mm depending on the contents within unit cells and between two adjacent unit cells. Moreover, when inward and outward curves are applied instead of the straight lines, we will obtain the traditional lotus-shaped patterns known as *phum-thong-khao-bin* patterns.

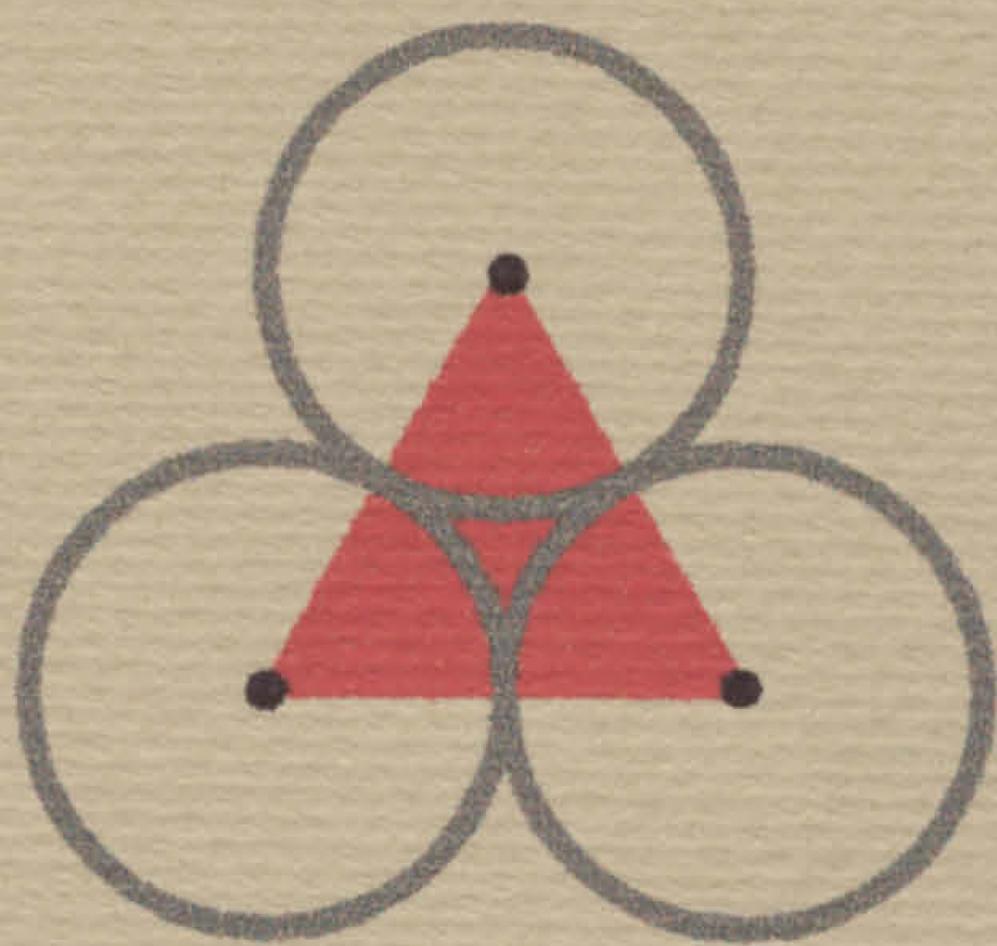
**** According to Phra Devabhinimmit [1994, pp.i-iii], the author of the *Thai Pattern Book* (*Samud Tham-ra Lai Thai*), it is believed that basic Thai classical patterns are derived from lotus shapes, which are evident in both pattern outlines and embellished details. *Khuaycheong*, for instance, is a motif representing a long shape of *sattabud* lotus, while the *krajung* motif is drawn on an equilateral triangle, which is reminiscent of a lotus petal. When four *krajung* motifs are placed at four cardinal directions, a finite design of class d4, called *prajumyam*, is created.



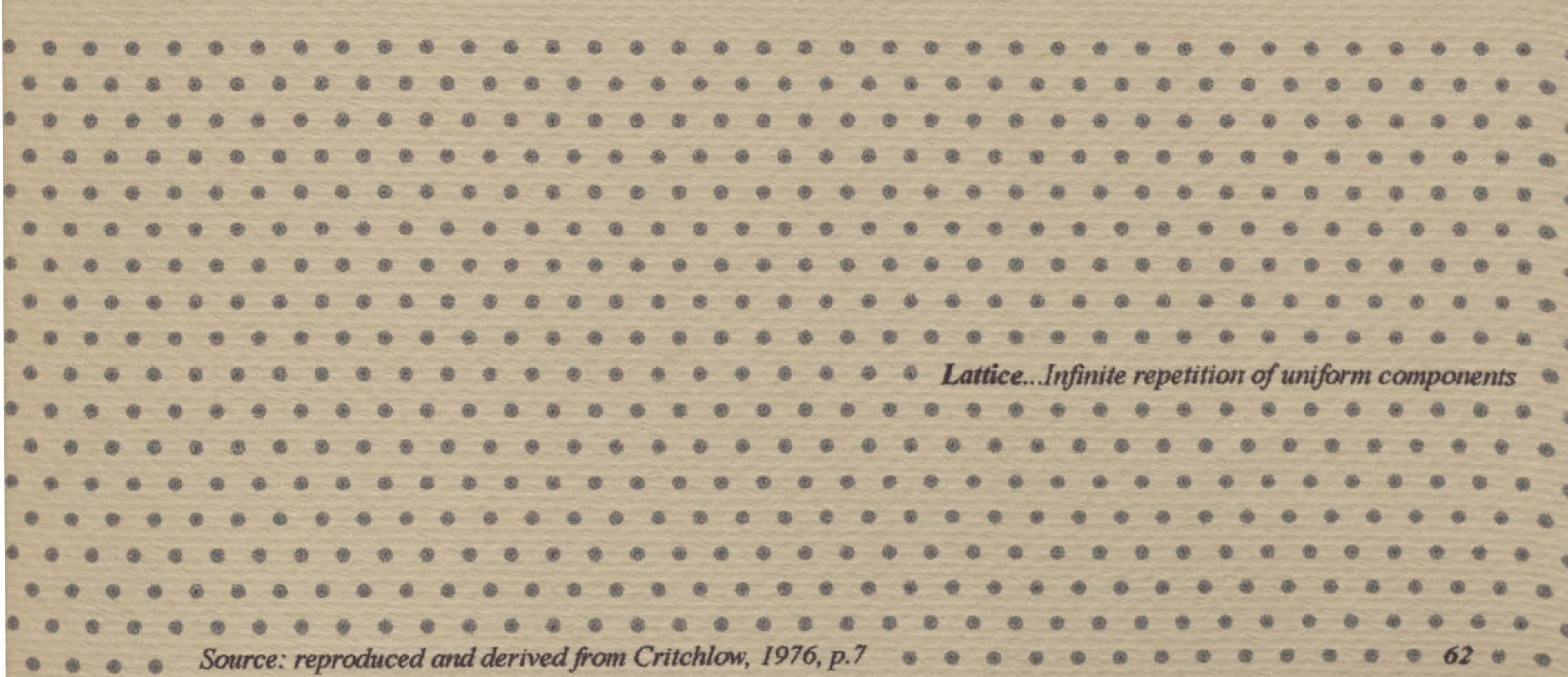
***Point**...Primary and unity*



***Line**...First move, the twoness*



***Plane**...Second move, the threeness*
The triangle: three points,
three lines and three contacts



***Lattice**...Infinite repetition of uniform components*

Source: reproduced and derived from Critchlow, 1976, p.7

Chapter 4 Analysis of Pattern Construction

4.1 Introduction

Repeating units and underlying structures are two crucial components in the construction of pattern. Day highlighted the relationship between both elements when he observed:

> *" Close up the waved lines and they give you an ogee diaper. Open out the ogee diaper and it gives you waved lines."*
>
> [Day, 1903, reprinted 1979, p.39]

The intention of this chapter is to provide an understanding of pattern construction through the two-fold approaches of space sub-dividing and space filling. The geometrical basis governing the regular division of the plane is discussed with reference to the point connection technique on two fundamental lattices, i.e., a square lattice and an isometric lattice. An investigation is made of the Islamic inner space sub-division and the seventeen geometric symmetry constructions as the unit construction approaches in comparison with the linear construction approach. Designers' construction means are examined as a hybrid approach through the use of certain customarily repeating formats and weave structure formats. Case studies from four categories of designs, i.e., two-dimensional graphics, three-dimensional objects, computer-generated images and contemporary household products, are reviewed in terms of their individual development from the repeating pattern concepts.

4.2 Pattern Structures

The concept of pattern construction basically arises from the two-fold approaches of space sub-dividing and space filling. Space sub-division involves an entire space which is divided systematically, while space filling emphasises how each individual sub-divided part can form a repeating unit which can fill the plane in a regular manner.

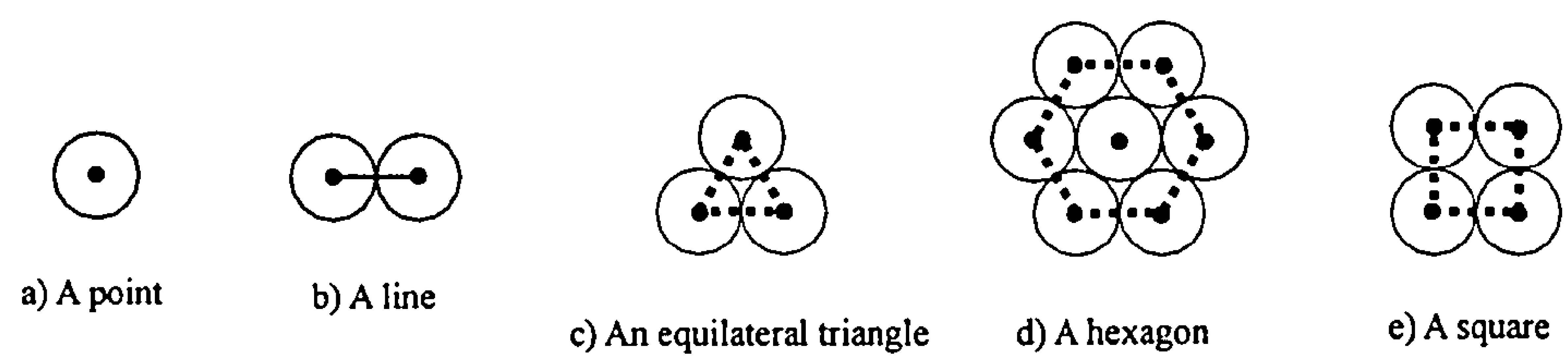
4.2.1 Points, Lines and Planes: The Origin of Geometric Lattices

The perception of order in space involves regular division of the Euclidean plane with respect to the geometric principles governing three fundamental components, i.e., points, lines and the enclosed plane. Albarn examined the emergence of these three components through individual dimensions as follow:

> *" As the moving point leaves a trace we call a line, so the moving line leaves a trace we call a plane. The nature of the plane is determined primarily by its line cross-section. So a straight line leaves a flat plane, an un-straight line an un-flat plane."*
>
> [Albarn 1974, p.14]

Points, lines and enclosed space can be understood on consideration of the principles of circle-packing in which tangential circles of the same radius are constructed with their outermost points just touching (Figure 4.1a-e). In Figure 4.1a, one circle represents one point with its centres. In Figure 4.1b, a line is generated by connecting the centres of two circles. To build up a plane, there are two possibilities to construct a group of identical circles, either in contact with axial alignment of 60° or 90°.

Figure 4.1a-e The construction of point, line and three fundamental polygons on the circle-packing with axial alignment of 60° and 90°



Source: reproduced from Critchlow, 1969, p.7

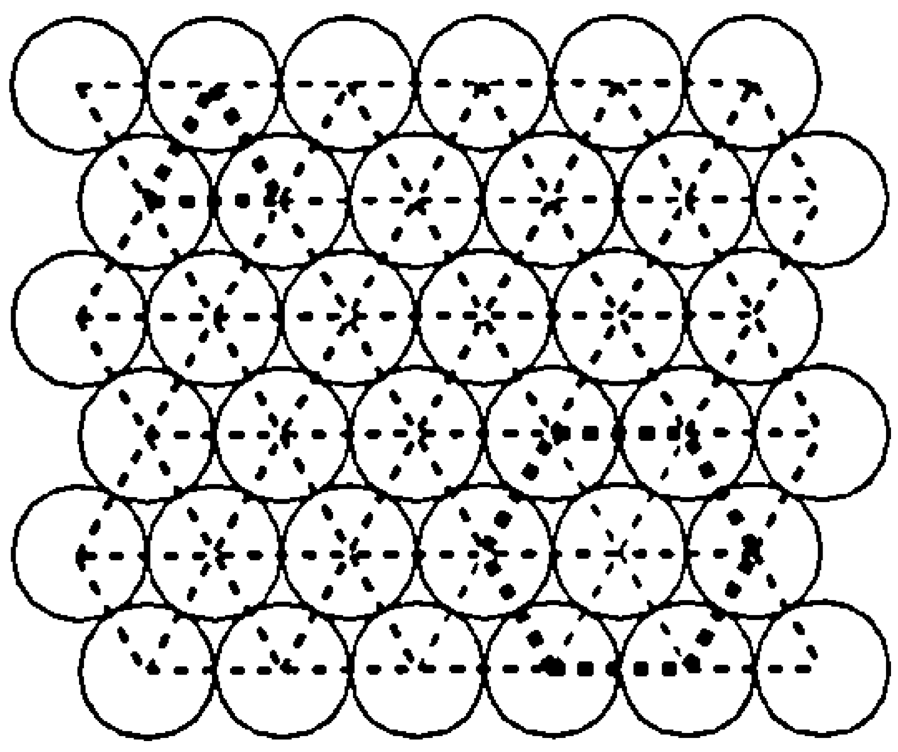
In the case of circle-packing with an axial alignment of 60°, an equilateral triangle is established by joining the centres of three circles, as illustrated in Figure 4.1c. Since a hexagon and an equilateral triangle are dual and complementary, the one co-existing within the other, six circles around a central circle of the 60° circle-packing can complete a hexagon which consists of six equilateral triangles, as illustrated in Figure 4.1d [Critchlow, 1976, p. 24]. In Figure 4.1e, when four circles are constructed with axial alignment of 90°, which is less close than the circle-packing with axial alignment of 60°, a square is established by four equal lines joining the centres of four circles.

The equilateral triangle, the square and the hexagon, whose internal angles are 60°, 90° and 120° respectively are three primary polygons which can individually cover the plane without leaving space between the meeting points of their vertices; these are known as regular tilings [Critchlow, 1969, p.60]. Repetition of four circles with the axial alignment of 90° produces a square lattice (Figure 4.2b). Whereas repetition of three or seven circles with axial alignment of 60° generated an isometric lattice (Figure 4.2 a). The centres of the circles act as the corresponding points of the lattices.

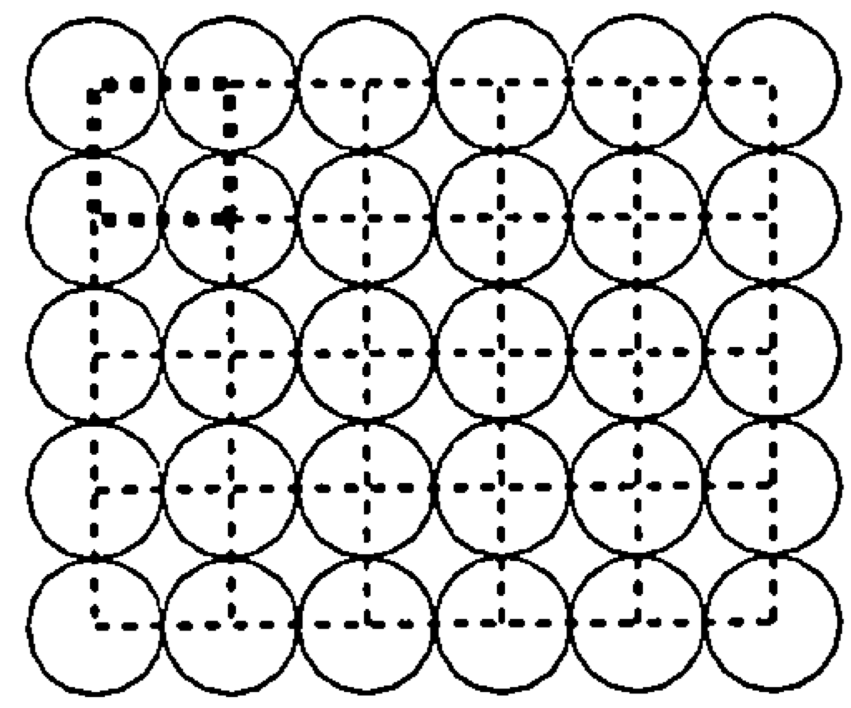
Different circle-packing in terms of angle and distance between two centres causes position changes of the correspondent points. The other three (of the five) geometric lattices, i.e., parallelogram, rectangular and rhombic lattices, can thus be developed on the 60° and 90° circle-packing since there is a geometrical interrelationship between these three shapes with the square and the equilateral triangle (Figure 4.2 c-h). As noted by Horne [1997, p.19], a square is a special form of a rectangle if all sides are equal, and also a special form of a rhombus when the internal angles are 90°.

In Figure 4.2c, a rectangular lattice is generated on a 90° circle-packing by setting regular intervals between each adjacent row of tangential circles either vertically or horizontally. It still preserves a right angle as a square lattice, but its perpendicular sides are not equal. In the case of a rhombic lattice, if the interval angles of the polygon is 60° or 90°, there are two possible constructions. Firstly, a rotation of a 90° circle-packing through 45° produces a right-angled rhombic lattice in Figure 4.2d. Secondly, we can obtain a 60°-120°-60°-120° rhombic lattice by merging two adjacent equilateral triangles on an isometric lattice in Figure 4.2e. But if we consider that the interval angles of the rhomboid can be any angle, but not 90° or 60°, we can also re-arrange an isometric circle-packing either by setting regular horizontal intervals between horizontal circle rows in Figure 4.2f or overlapping horizontal circle rows regularly in Figure 4.2g. In the case of a parallelogram in which the angles are not 90° and all sides are not equal, the parallelogram lattice can be generated on a 60° circle-packing by adding regular intervals between 60° circle rows as shown in Figure 4.2h.

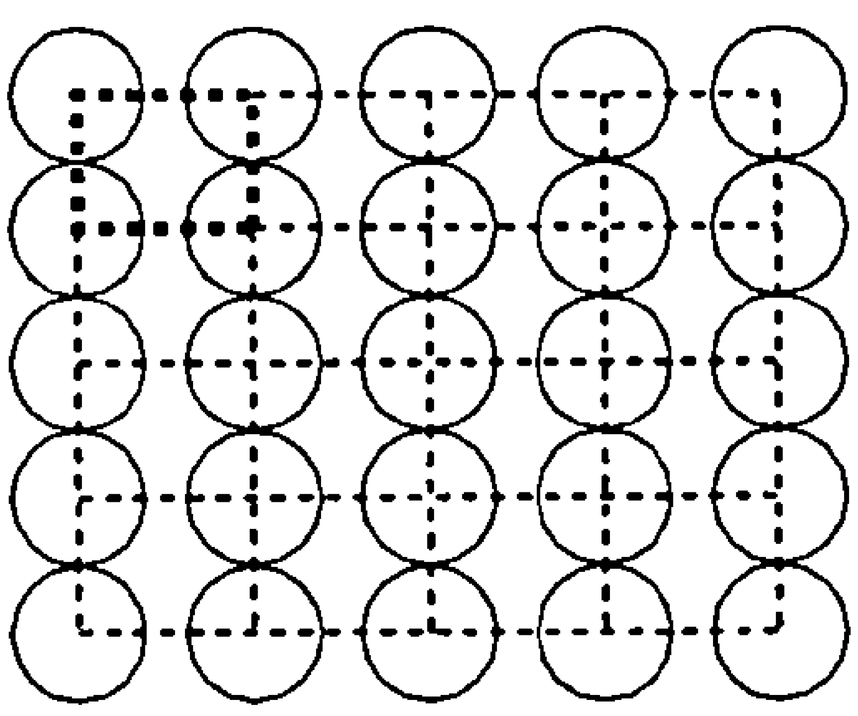
Figure 4.2a-h Eight geometric lattices generated on the circle-packing with axial alignment of 60° and 90°



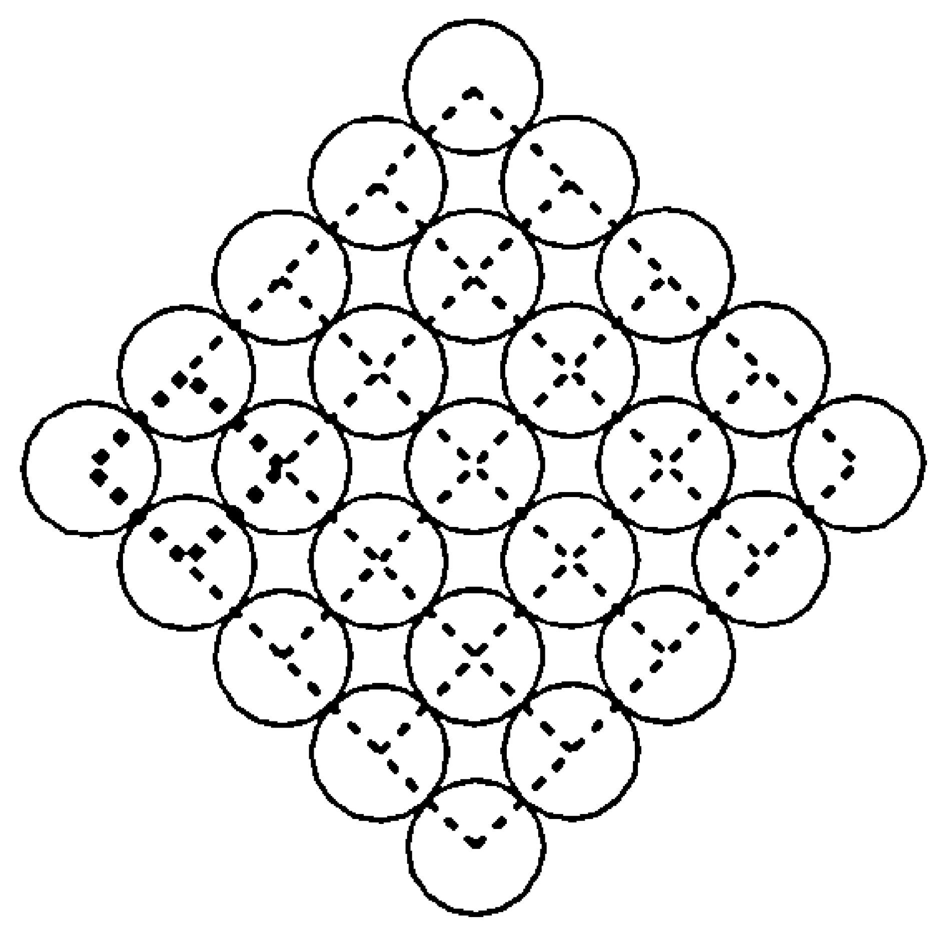
a) An isometric lattice



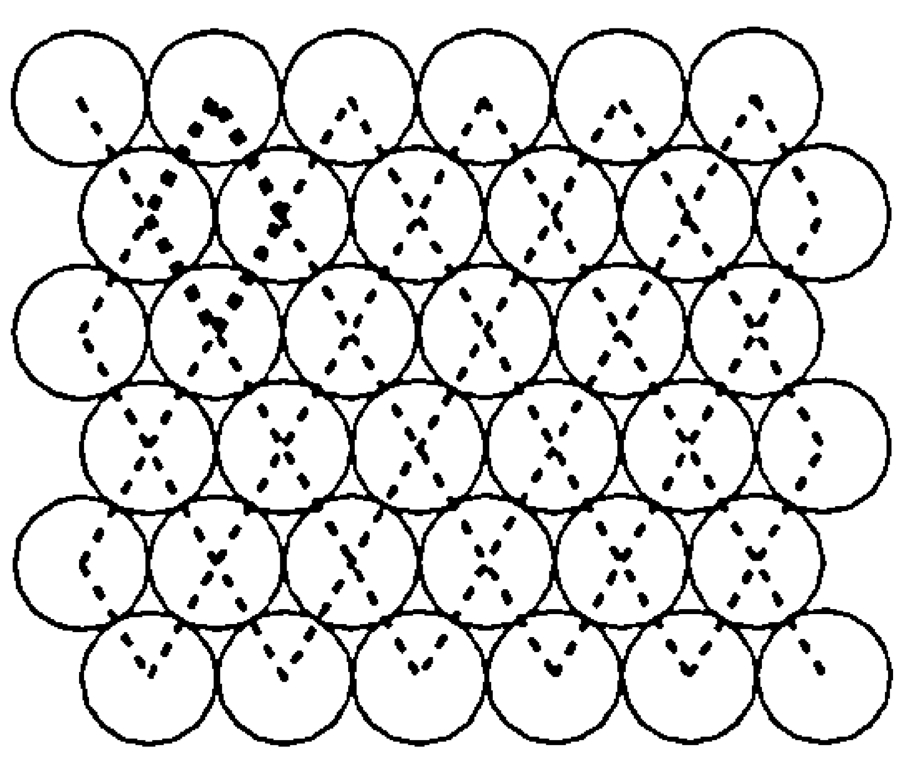
b) A square lattice



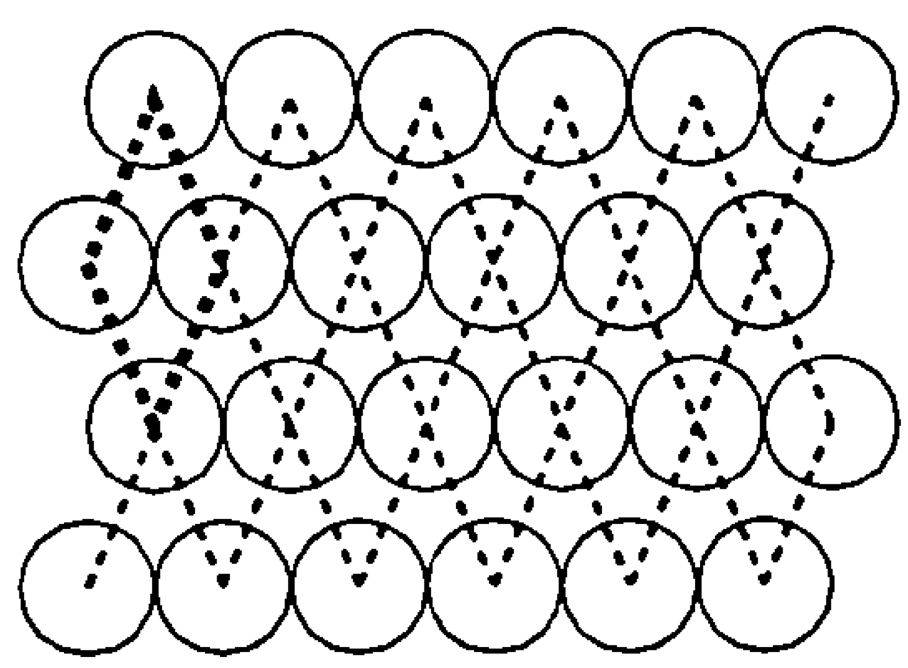
c) A rectangular lattice



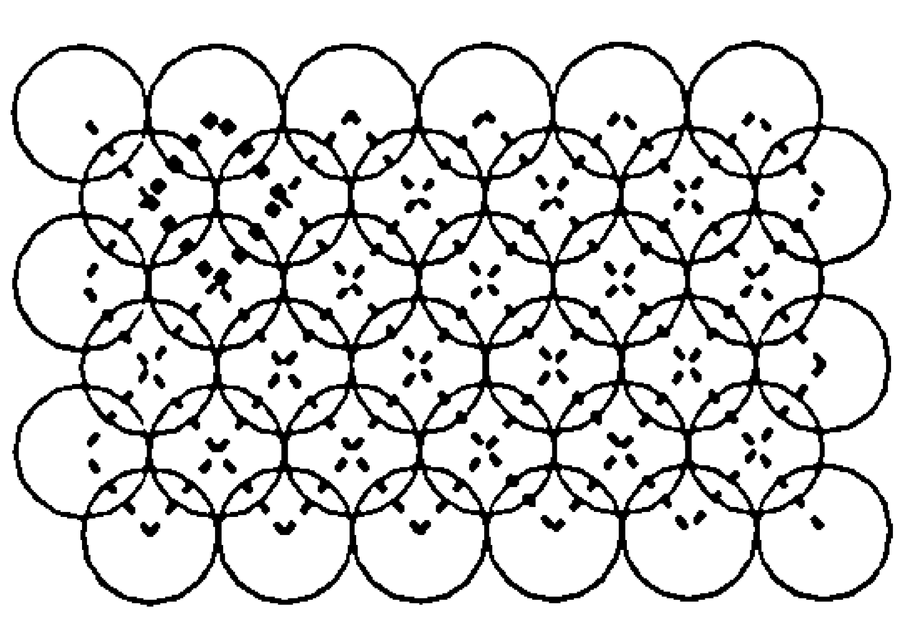
d) A right-angled rhombic or centre-celled square lattice



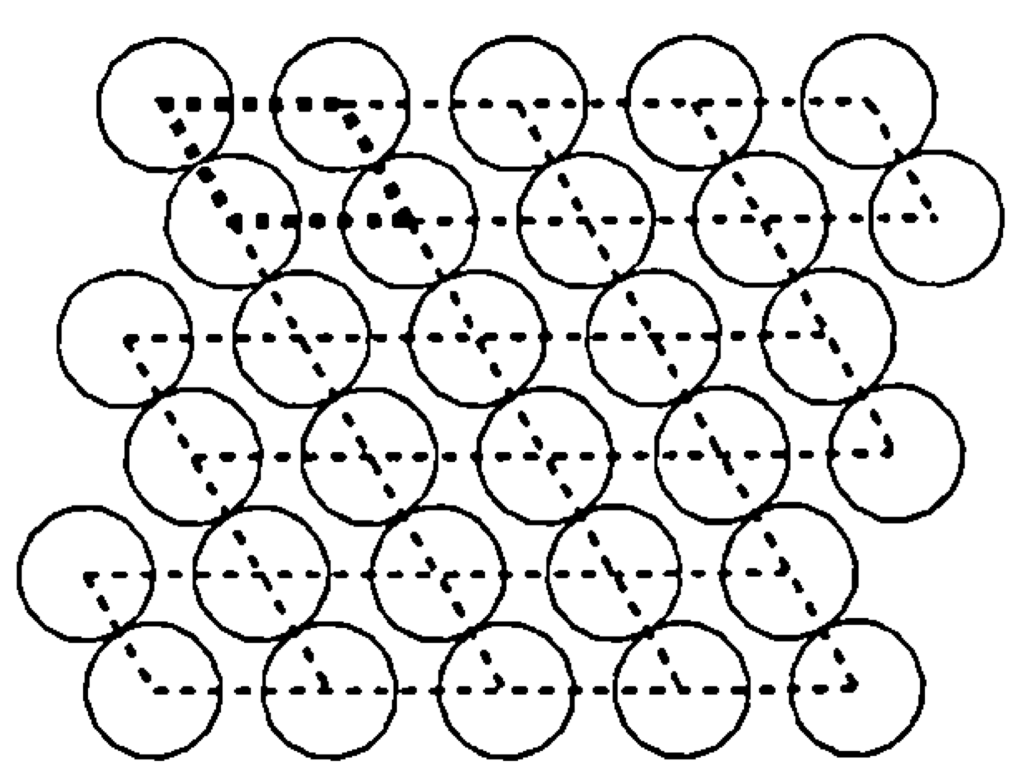
e) A 60°-120°-60°-120° rhombic lattice



f) A rhombic lattice generated on 60° circle-packing by leaving space between two horizontal circle rows



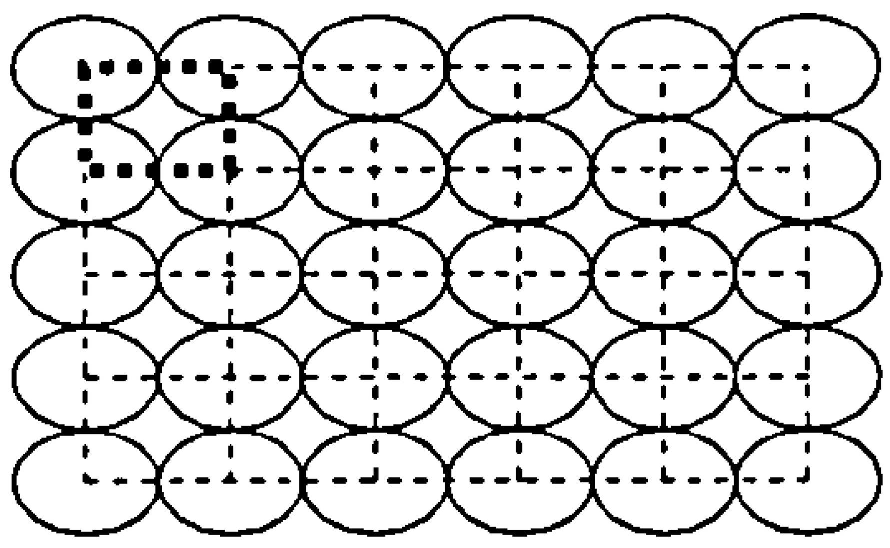
g) A rhombic lattice generated on 60° circle-packing by overlapping horizontal rows regularly



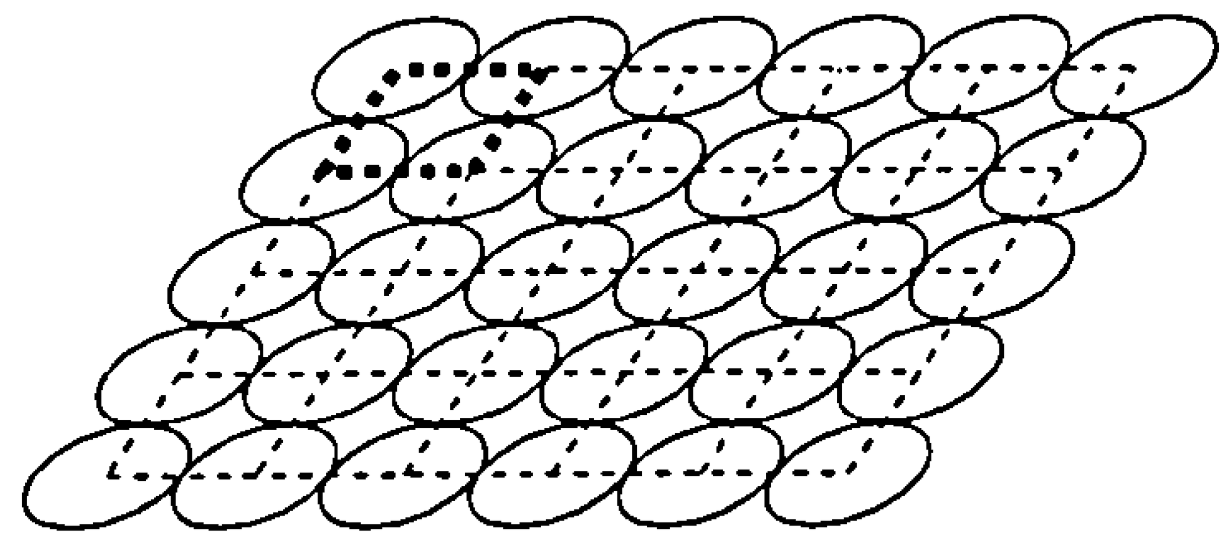
h) A parallelogram lattice

Computer technology can assist in the transformation of the square and isometric lattices into three further geometric lattices by extending or contracting the original lattices through one particular direction. Circle-packing is then replaced with ellipse-packing at the same time as positions of the corresponding points are changed. Examples are provided in Figure 4.3a-d.

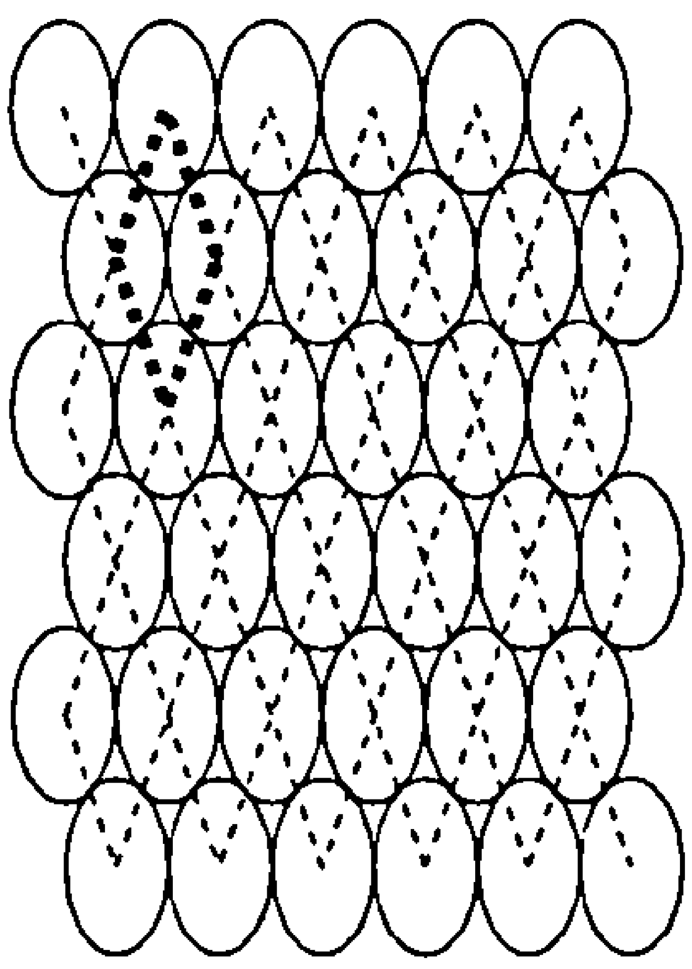
Figure 4.3a-d Variations of geometric lattices derived from the modification of the circle-packing with axial alignment of 60° and 90° (all are produced from Adobe Illustrator)



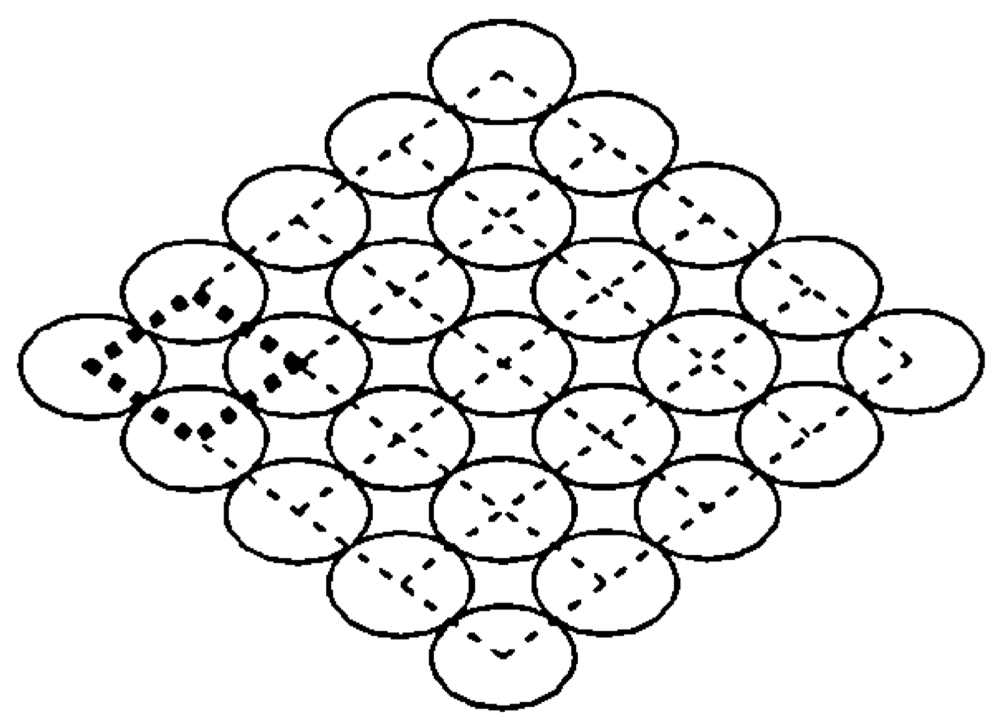
a) A rectangular lattice generated on 90° circle-packing with non-uniformed scaling at horizontal 140%, vertical 100%



b) A parallelogram lattice generated on 90° circle-packing with shearing at -153.343 (angle -2.527°) and non-uniformed scaling at horizontal 140%, vertical 100%



c) A rhombic lattice generated on 60° circle-packing with non-uniformed scaling at vertical 170%, horizontal 100%



d) A rhombic lattice generated on 90° circle-packing with non-uniformed scaling at horizontal 140%, vertical 100%

4.2.2 Point Connection: A Means to Construct Underlying Structures

It is found that both isometric and square lattices derived from the circle-packing with axial alignment of 60° and 90° are compatible guidelines for pattern construction. They are used widely by Wade [1976, 1982], Day [1903, reprinted 1979] and Wiltshire [1989] as the fundamental elements to illustrate how pattern variations have been created.

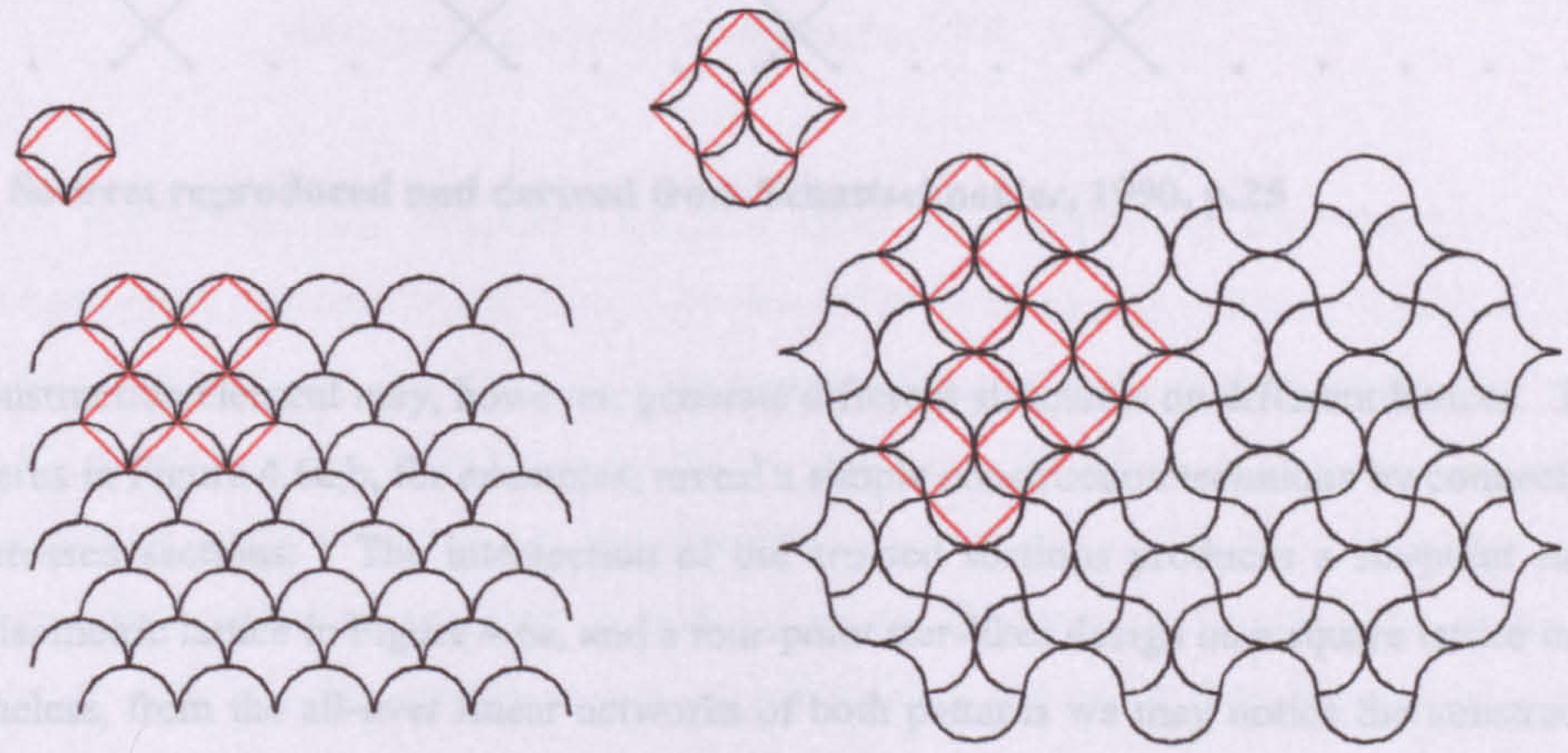
After all circumferences of the circle-packing are removed we will obtain a dotted lattice as an array of corresponding points. These points are used not only as the vertices to fit repeating units in a regular manner but also as guidelines to divide space proportionally with compatible angles and distance in the same way as using graph paper. Different sizes of lattices may be required to fit different design features. The finer the underlying lattice used, the more sophisticated the space sub-division can be made.

The principle of point connection has been developed as a means to divide space systematically either within a repeating unit or throughout the overall structure. A line connecting between two points may represent a side sharing by two constituent shapes. Repetition of constituent lines or sections produces a continuous line. A series of lines in a parallel direction or the intersection of series of lines from two or more than two non-parallel directions generates a lattice whose intervals may exhibit a repetition of identical shapes known as monohedral tilings or a combination of multiple shapes of constituent parts.

Although the patterns are built up on the geometrical lattices, the boundary of repeating unit may not necessarily be confined to one of parallelogram shapes relevant to its underlying structures [Horne, 1997, pp.23-32]. As Wade points out due to the fact that these geometric lattices are deformable, the lattice outlines may not be evident in the final designs but they are determining factors for the unit arrangement [Wade, 1976, p.11]. The motif whose entire region is equal to an area of the square unit can be regarded as the repeating unit of the square lattice. The unit edge connecting from one corresponding point to another one can thus be a curved or a zigzag line instead of a straight line. The parallelogram shape can thus be replaced with any shape such as a curvilinear or a free-formed or a figurative design.

A scale shape shown in Figure 4.4a, for example, could be the case of a curvilinear design derived from the right-angled rhomboid, as noted by Day [1903, reprinted 1979, pp.33-35]. It consists of inverse and outverse quarter-arcs along four sides of a rhomboid. Compound structures can be subsequently constructed with more than one unit such as an ogee which is comprises of four scale-shaped units (Figure 4.4b). As an individual unit repeated by translation on a right-angled rhombic lattice the scale pattern preserves symmetry class c1m1. The ogee pattern has symmetry class p4gm due to the combination of a square-based structure with three symmetry operations, i.e., perpendicular reflection and glide-reflection axes, and four-fold rotation centres located at the mid-sides of the unit edges.

Figure 4.4a,b Two curvilinear varieties generated from right-angled rhomboids



a) A scale-shaped unit and scale pattern **b) An ogee-shaped unit and ogee pattern**

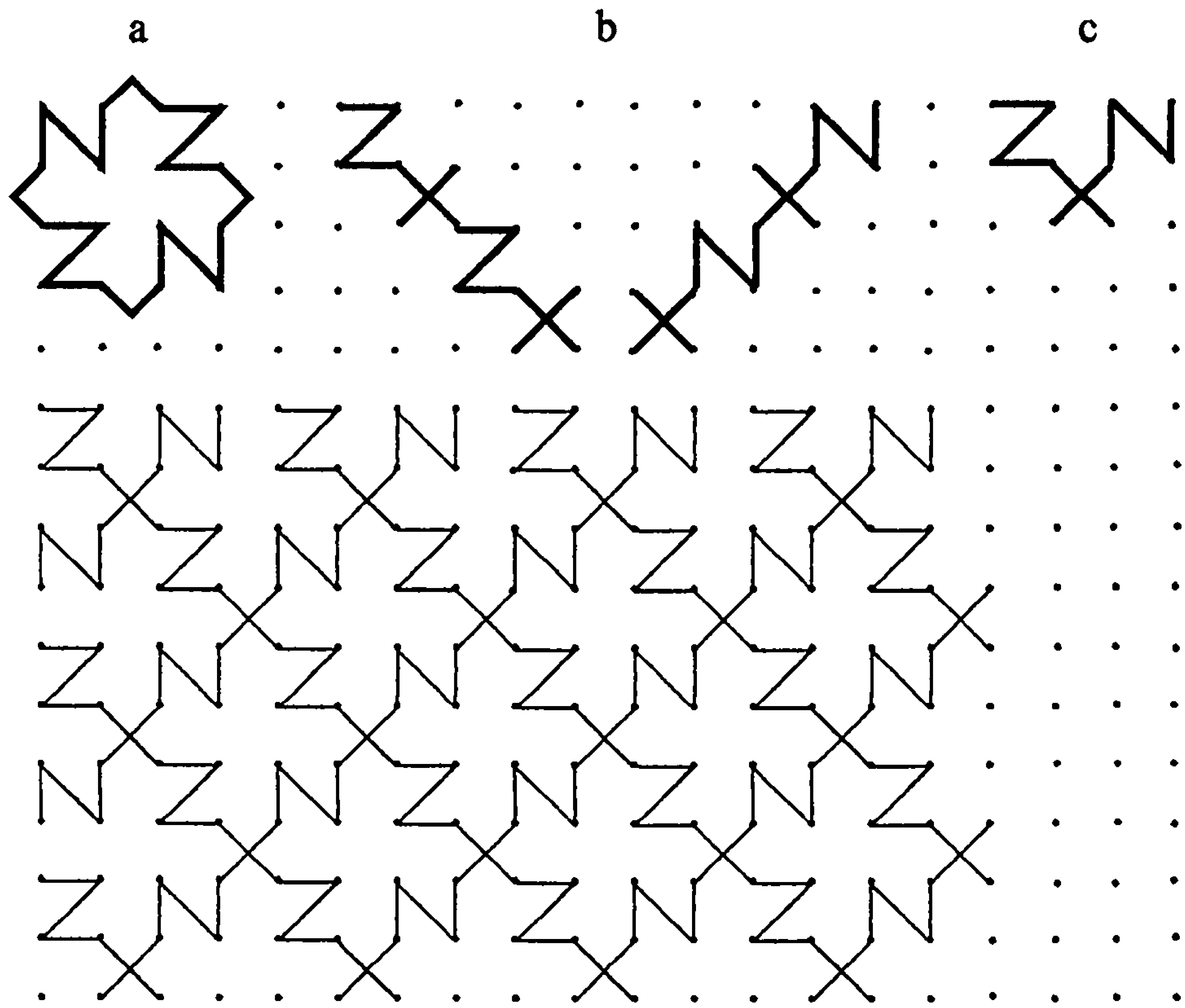
Source: reproduced and derived from Day, 1903, reprinted 1979, p.34

4.3 Construction Techniques

It is not necessarily the case that one pattern can be built up by only one definite means. Repetition of an identical section, a series of lines or a closed shape, for instance, becomes a possible means to fit space between two points in association with one of four symmetry operations.

The pattern shown in Figure 4.5a-c, for example, suggests three possible construction means based on a point connection on a square lattice, i.e., a) the translation of a multi-sided unit, b) the intersection of two series of identical zigzag lines in perpendicular directions: both lines are four-fold rotational figures of each other, or c) the distribution of three letters: Z, X, N.

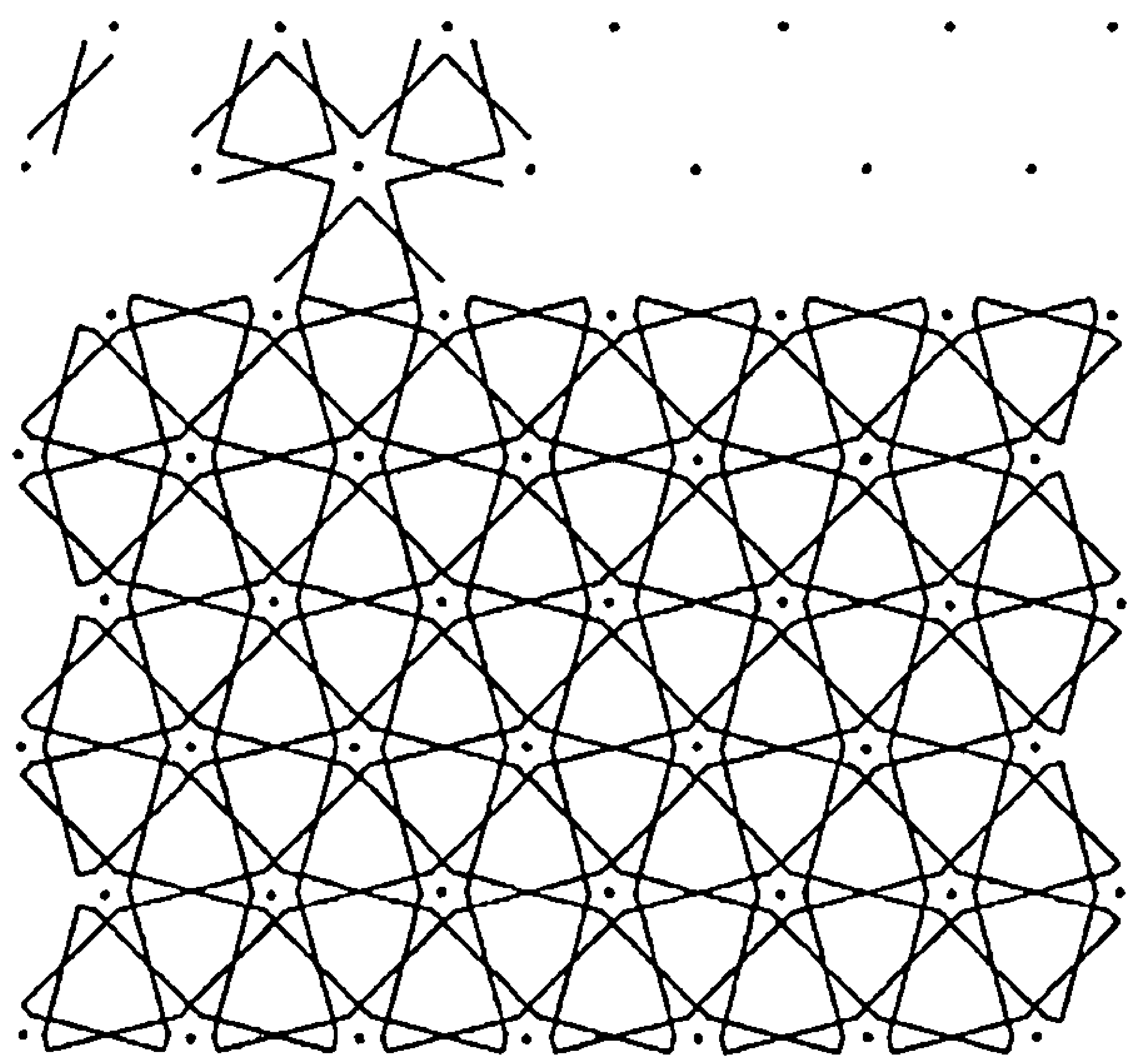
Figure 4.5 A square-based pattern generated from three possible construction elements (a-c)



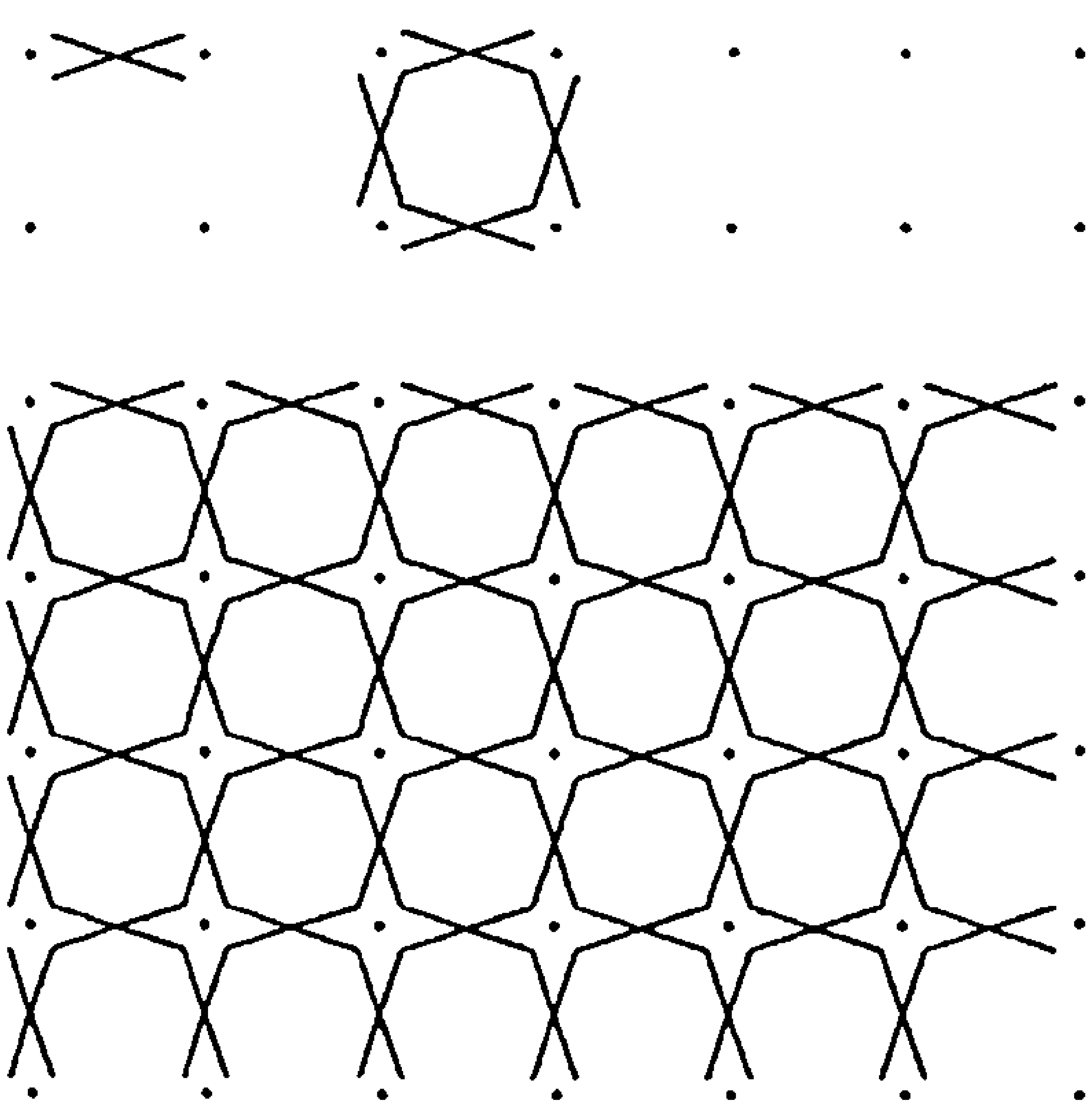
Source: reproduced and derived from Schattschneider, 1990, p.25

A uniform construction element may, however, generate different structures on different lattices. The two complex patterns in Figure 4.6a,b, for examples, reveal a simple construction technique by connecting two points with crossed sections. The intersection of the crossed sections produces a six-point star-liked design on an isometric lattice in Figure 4.6a, and a four-point star-likes design on a square lattice in Figure 4.6b. Nonetheless, from the all-over linear networks of both patterns we may notice the construction of series of lines in three and two non-parallel directions.

Figure 4.6a,b The construction of crossed sections on an isometric and a square lattice



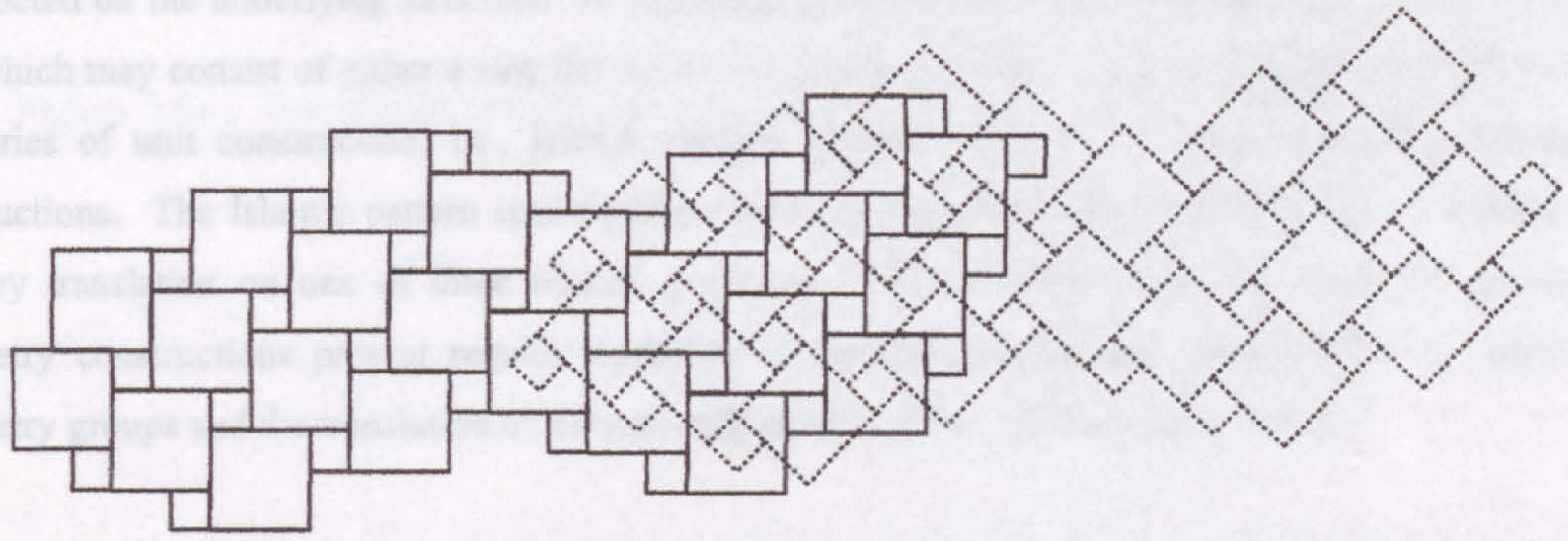
a) A six-point-star design generated on an isometric lattice



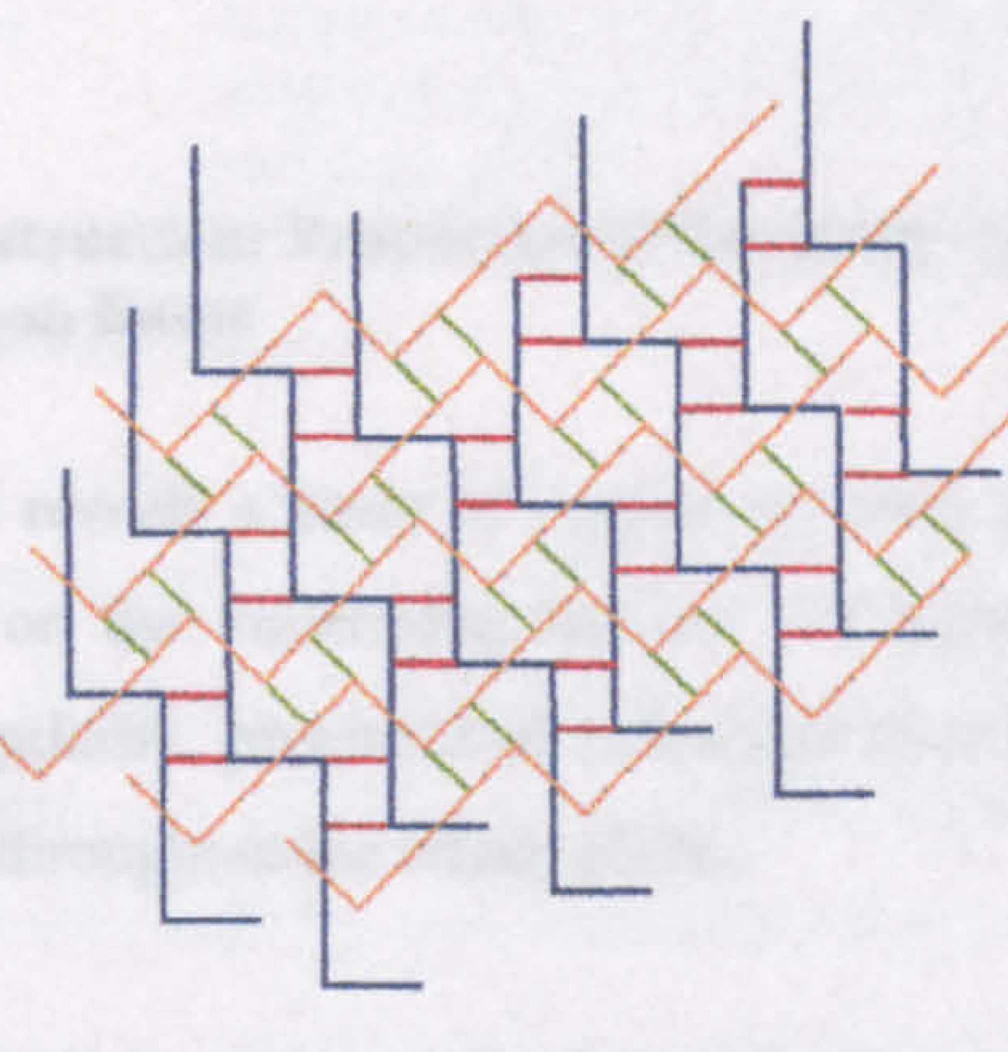
b) A four-point-star design generated on a square lattice
Source: reproduced from Wade, 1976, p.71

Certain patterns exhibit intricate structures resulting from the interplay of multi-layered structures. Overlaying is a means to combine two or more than two linear structures together, as suggested by Wade [1976, pp.42-47]. The pattern in Figure 4.7a, for example, consists of two overlaid square networks, one of which is a 45° rotation of the mirrored image of the other one. However, it may have an alternative construction means by using two series of identical lines intersecting at 45° and two sets of identical sections each of which connects two lines in each series, as illustrated in Figure 4.7b. The overlaid structure in Figure 4.7a may be developed into an interlocking pattern of four 90°-rotated arrow-like shapes in Figure 4.7c by applying a cross to every small-sized square.

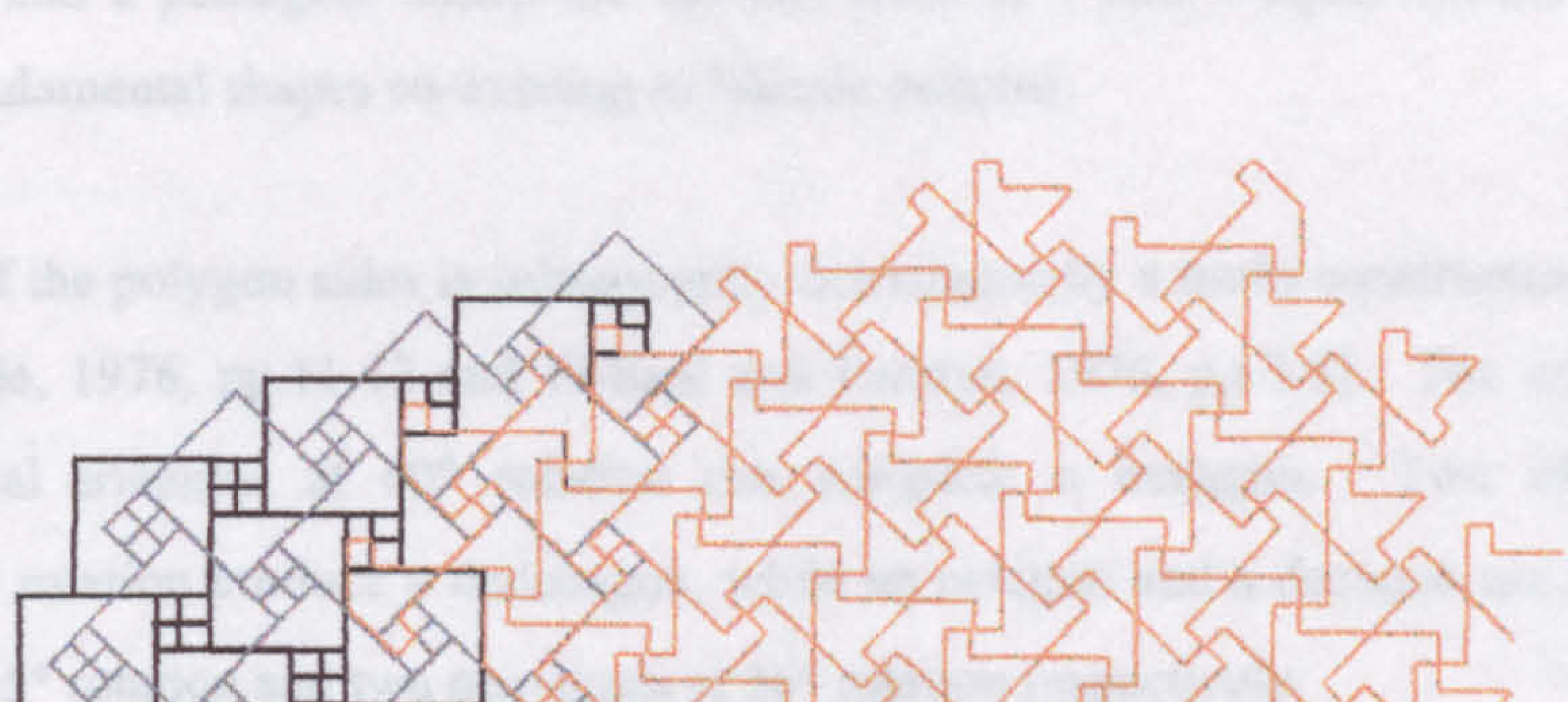
Figure 4.7a-c Illustrations which show alternative construction means to generate a pattern of symmetry class p4gm in (a) and (b), and the transfromation into an interlocking pattern containing 90°-rotated arrow-like shapes in (c).



a) An arrangement of a square structure overlaid on its 45°-rotated mirror image
Source: reproduced from Wade, 1976, p.42



b) An arrangement of two series of identical lines intersecting at 45° and two additional pairs of identical sections.



c) An interlocking pattern containing 90°-rotated arrow-like shapes
Source: reproduced from Wade, 1976, p.22

The interchange view between a linear structure and its internal units as mentioned previously by Day [1903, reprinted 1979, p.39] implies the basis of two construction techniques, i.e., an additive approach and a subtractive approach.

An additive approach involves the basis of space filling by which repeating units are modified and constructed on the underlying structure. A repeating unit refers to either a fundamental region or a unit cell, which may consist of either a singular motif or a group of motifs. An investigation is made on two categories of unit construction, i.e., Islamic pattern construction and seventeen geometric symmetry constructions. The Islamic pattern construction exhibits regular repeating system of space-divided unit cells by translation on one of three related geometric lattices. Meanwhile, the seventeen geometric symmetry constructions present regular repetition of the fundamental regions governed by individual symmetry groups and the translation of the unit cells on one of five parallelogram lattices.

A subtractive approach reveals the basis of space sub-division by which the all-over structure is primarily considered before the modification of the repeating unit takes place. The principles of successive point connection by series of lines in one or more than one direction is used as a means to divided infinite space into a regular linear structure.

4.3.1 Islamic Pattern Construction: Proportional Harmony of Space Sub-division within Square, Hexagon and Pentagon Bases

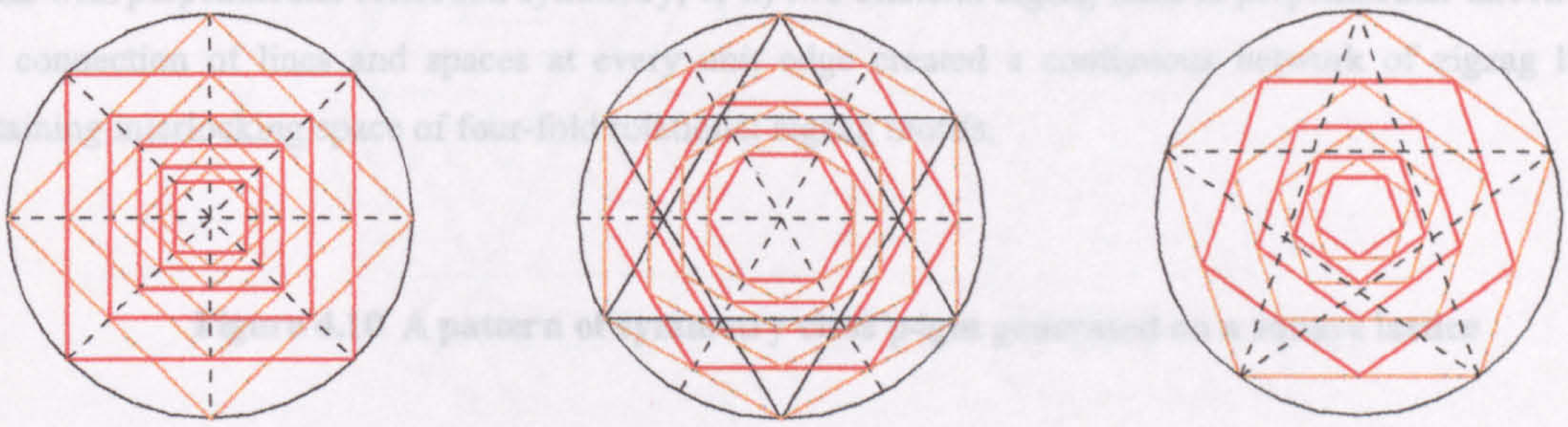
The study of Islamic patterns reveals a basis of regular division of space within individual geometric shapes before packing them on the underlying lattices. A network of polygonal sub-divided units possesses the properties of regularity, proportional ratios and inter-relationship of every constituent part not only within a unit but also throughout the whole plane.

To generate n-gon shapes, a circle is used as a fundamental element where space between the centre and the circumference is divided equally to the required number of sections, in which we obtain a regular polygon by joining points on the circumference with straight lines. Three polygons, i.e., an equilateral triangle, a square and a pentagon, which are derived from 3, 4 and 5 equal sectors respectively, are regarded as the fundamental shapes co-existing in Islamic patterns.

A larger number of the polygon sides is subsequently determined by a multi-construction of each of these three shapes [Wade, 1976, pp.11-12 and El-Said and Parman, 1976, pp.3-5]. The overlapping of two identical equilateral triangles at 60° rotation can complete a hexagon. Two identical hexagons overlapping at 30° rotation produce a dodecagon, while an octagon and a decagon are self-superposition of two squares at 45° rotation and two pentagons at 36° rotation respectively.

The process of space sub-division may continue further within each polygon with reference to sub-divided points on the polygonal sides and within the polygonal regions. Geometrical constraint imposed by the inner logic of proportions within each polygon underlies the systematic construction of space with a series of nested polygons [Abas and Salman, 1995, p.18]. As shown in Figure 4.8a-c three proportional ratios involve three fundamental shapes, i.e., a) the root two proportion system within the square, b) the root three proportion system within the equilateral triangle/ hexagon, and c) the golden ratio within the pentagon.

Figure 4.8a-c Illustrations of three proportional ratios within three fundamental shapes



a) A series of concentric squares progressively generated from the root two system

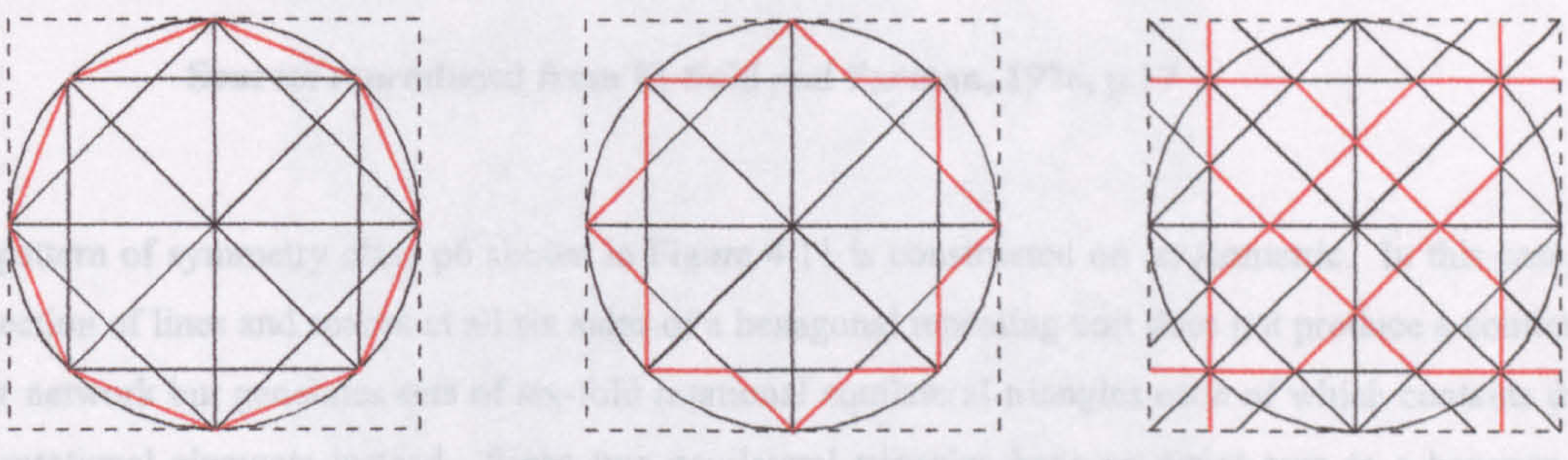
b) A series of concentric hexagons and co-existing equilateral triangles progressively generated from the root three system

c) A series of concentric pentagons progressively generated from the golden ratio

Source: derived from Wade, 1976, p.11

The sub-divided structure of each polygonal unit is then used as a guideline to generate a variety of designs by regular connecting points or emphasising certain sets of linear sections. Examples in Figure 4.9 a-c illustrate three possibilities to join eight points of circle division, i.e., a) an eight-sided polygon or an octagon, b) an eight-pointed star, and c) a cross with four squares at each corner.

Figure 4.9a-c Illustrations showing three possibilities to join eight points of circle division



a) An eight-sided polygon (octagon)

b) An eight-pointed star

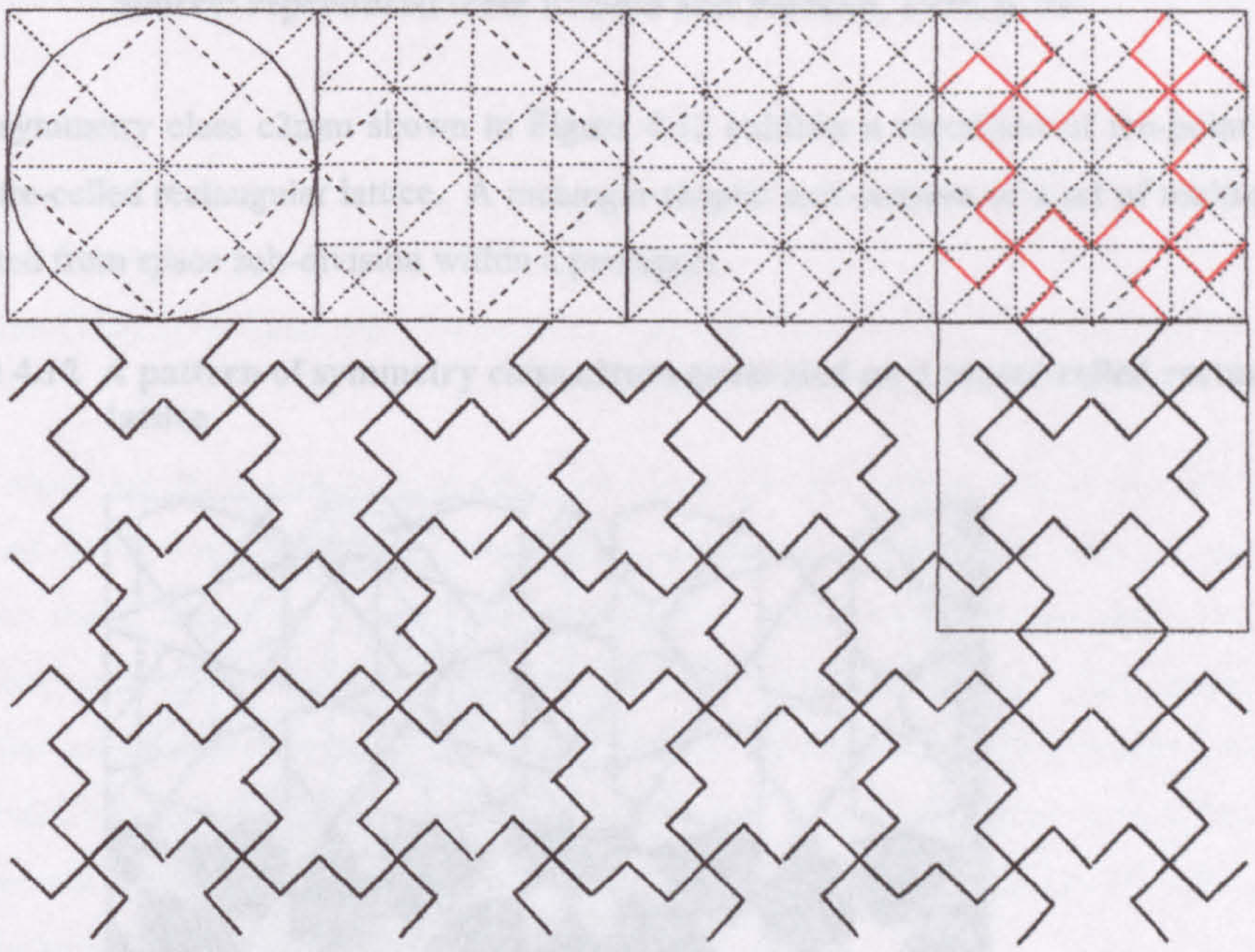
c) A cross with four squares at each corner

Source: reproduced from El-Said and Parman, 1976, pp.11,13,21

The square-based repeating units including 4-, 8- and 16-fold rotational designs are packed on the square lattices. The equilateral triangle-/hexagon-based repeating units including 6-, 12- and 24-fold rotational designs are packed on the isometric lattices. The pentagon-based repeating units including 5-, 10- and 20-fold rotational designs are packed on the rectangular or centre-celled rectangular lattices.

The pattern symmetry class p4gm shown in Figure 4.10 is built up from the translation of a square-shaped unit on a square lattice. Space sub-division within a square associated with the root two system provides a lattice consisting of squares and 45°-90°-45° triangles. A 90°-stepped design is generated by emphasising sets of certain sections which might be recognised as i) four "swastika" symbols or four-fold rotational motifs with perpendicular reflection symmetry, or ii) two bilateral zigzag lines in perpendicular directions. The connection of lines and spaces at every unit edge created a continuous network of zigzag lines containing interlocking space of four-fold rotational zigzag motifs.

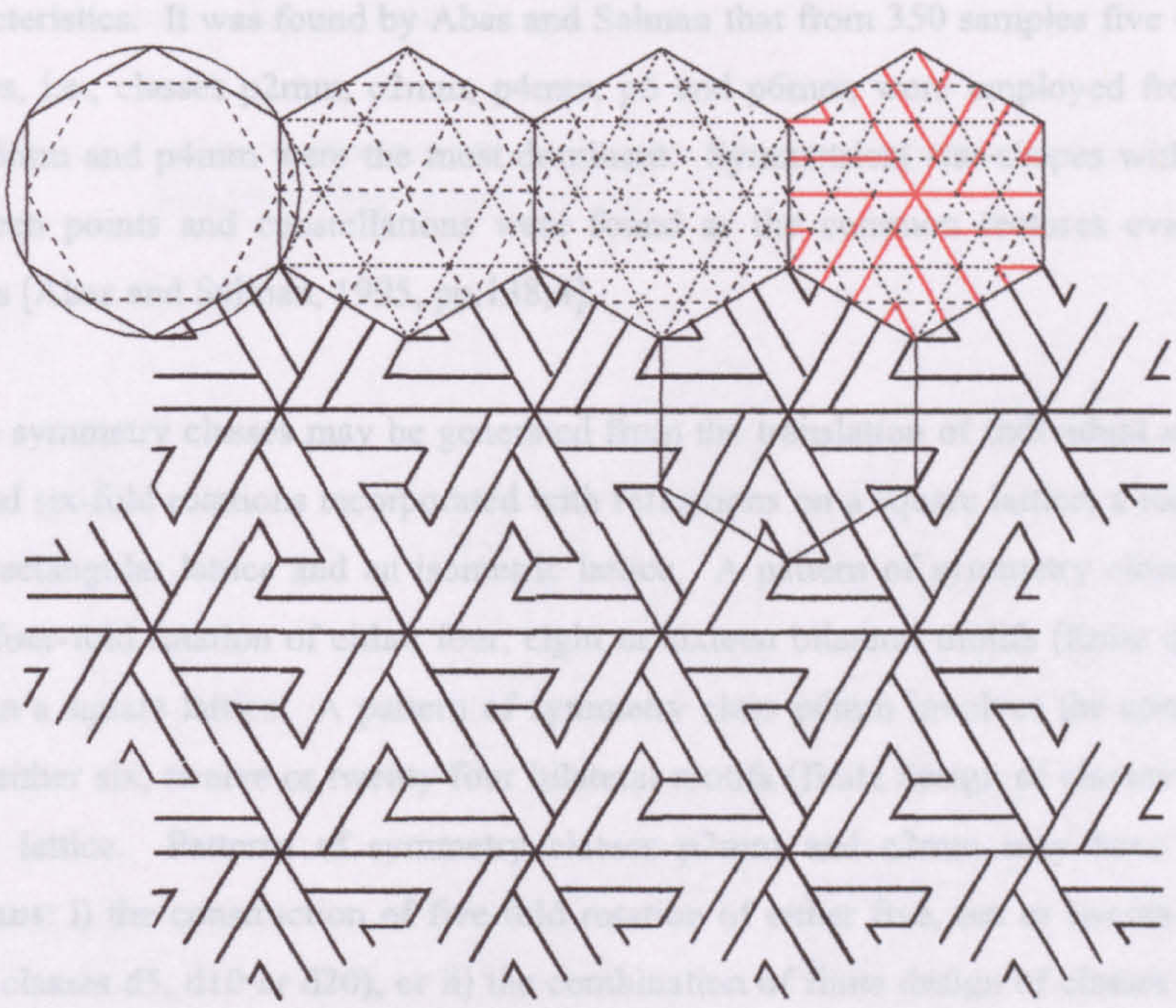
Figure 4.10 A pattern of symmetry class p4gm generated on a square lattice



Source: reproduced from El-Said and Parman, 1976, p.17

The pattern of symmetry class p6 shown in Figure 4.11 is constructed on an isometric. In this case, the connection of lines and spaces at all six sides of a hexagonal repeating unit does not produce a continuous linear network but generates sets of six-fold rotational equilateral triangles each of which contains three-fold rotational elements instead. Since two equilateral triangles have an equal area to a hexagon, the translation unit could be either a hexagonal shape consisting of six-fold rotational elements or two equilateral triangles each of which contains three-fold rotational elements.

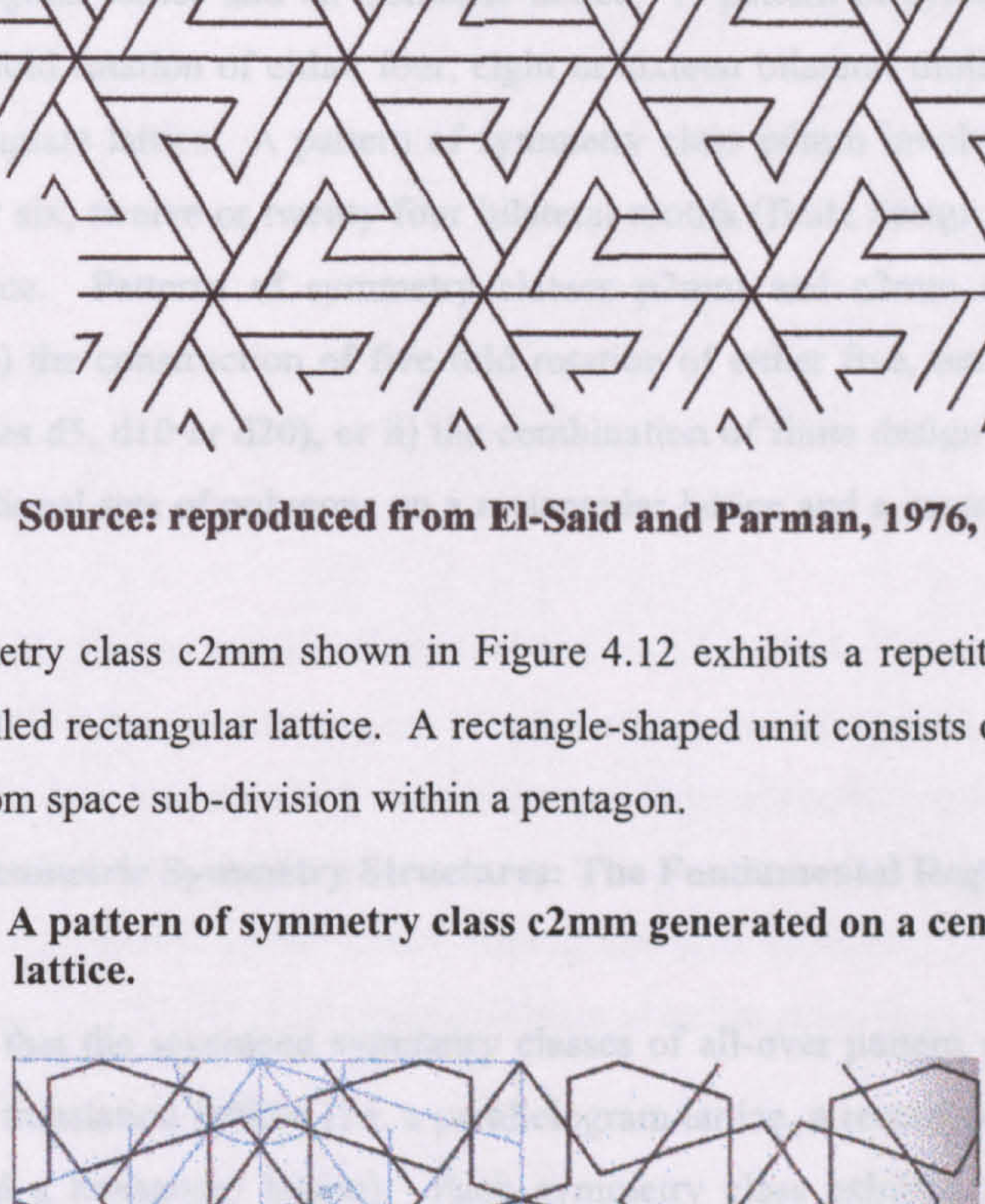
Figure 4.11 A patttern of symmetry class p6 generated on an isometric lattice



Source: reproduced from El-Said and Parman, 1976, p. 71

The pattern of symmetry class c2mm shown in Figure 4.12 exhibits a repetition of ten-point star-liked motifs on a centre-celled rectangular lattice. A rectangle-shaped unit consists of a set of multi-polygonal network generated from space sub-division within a pentagon.

Figure 4.12 A pattern of symmetry class c2mm generated on a centre-celled rectangular lattice.



Source: reproduced from Wade, 1982, illustration no. 487.

Although Islamic patterns vary from simple to complicated designs, however, they share some common symmetry characteristics. It was found by Abas and Salman that from 350 samples five of the seventeen symmetry classes, i.e., classes p2mm, c2mm, p4mm, p6 and p6mm, were employed frequently, among which classes p6mm and p4mm were the most dominant. Symmetrical star-shapes with six, eight, ten, twelve and sixteen points and constellations were found as the common features evidenced in these symmetry groups [Abas and Salman, 1995, pp.138,4].

Patterns of these symmetry classes may be generated from the translation of individual sub-divided units of four-, five- and six-fold rotations incorporated with reflections on a square lattice, a rectangular lattice, a centre-celled rectangular lattice and an isometric lattice. A pattern of symmetry class p4mm may be generated from four-fold rotation of either four, eight or sixteen bilateral motifs (finite design of classes d4, d8 or d16) on a square lattice. A pattern of symmetry class p6mm involves the construction of six-fold rotation of either six, twelve or twenty-four bilateral motifs (finite design of classes d6, d12 or d24) on an isometric lattice. Patterns of symmetry classes p2mm and c2mm may have two alternative construction means: i) the construction of five-fold rotation of either five, ten or twenty bilateral motifs (finite design of classes d5, d10 or d20), or ii) the combination of finite design of classes d4, d6, d8, d12, d16 or d24 with additional sets of polygons on a rectangular lattice and a centre-celled rectangular lattice respectively.

4.3.2 Seventeen Geometric Symmetry Structures: The Fundamental Regions and the Symmetry Groups

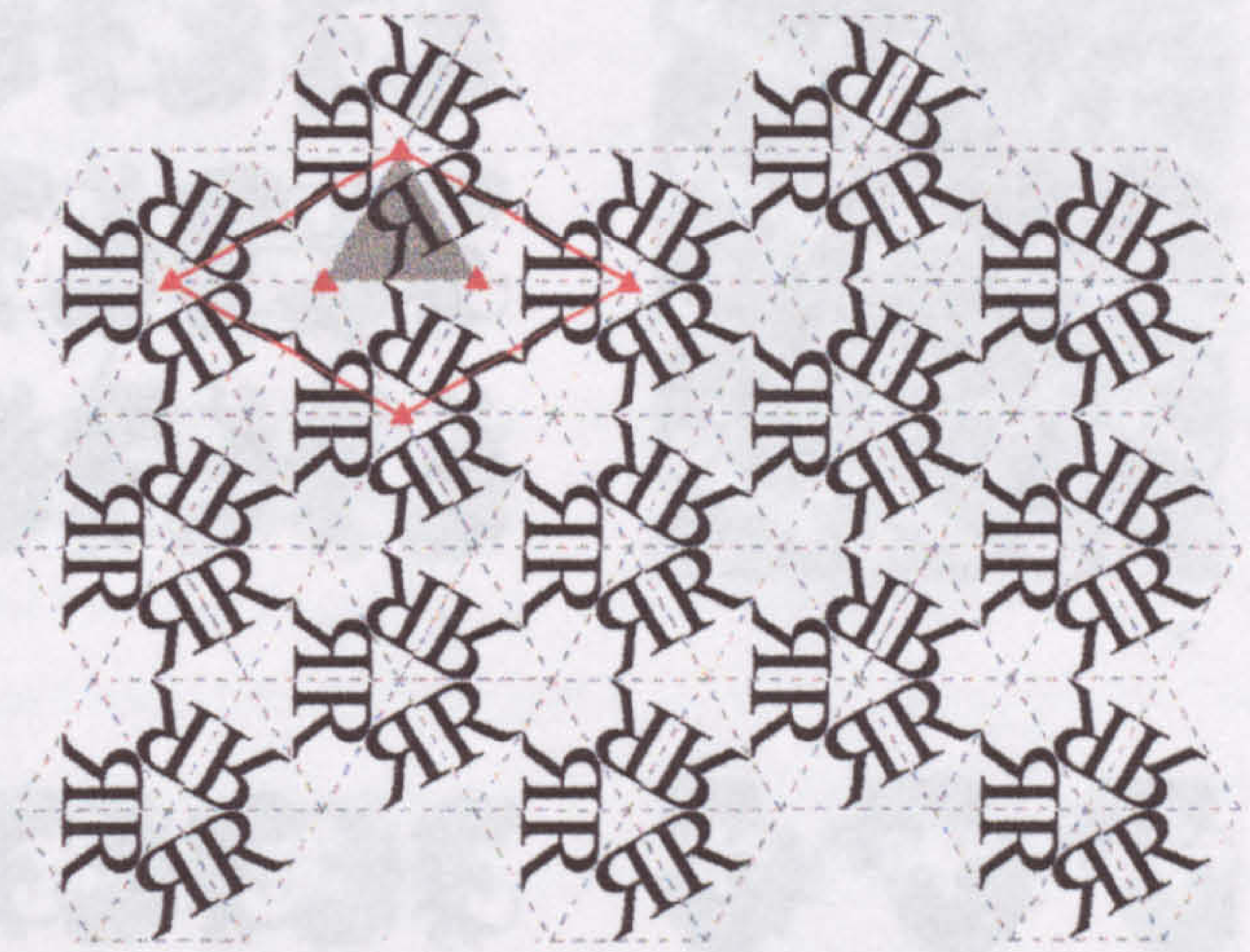
It is well established that the seventeen symmetry classes of all-over pattern may be constructed on the basis of five types of translation lattices (i.e. a parallelogram lattice, a rectangular lattice, a square lattice, a rhombic lattice and a hexagonal lattice). Each symmetry class exhibits a repetition of a unit cell containing a group of fundamental regions with a particular symmetry group. The symmetry group governs the pattern construction, not only the organisation of fundamental regions within a unit cell but also the connection between unit cells. A fundamental region is basically filled with design elements before being repeated to adjacent intervals and then mapped onto itself. Additional modification can then be applied regularly to every constituent part.

The pattern in Figure 4.13b, for example, is built up on a structure of symmetry group p3m1. A 60°-120°-60°-120° rhomboid-shaped unit cell is basically divided into six equilateral-triangle shapes of fundamental regions following the structure of intersecting reflection axes. Each equilateral triangle whose sides are bounded by the reflection axes contains an "R" letter, which is shifted consecutively to the other intervals by reflection symmetry. Three-fold rotational centres arise automatically at every intersecting point of reflection axes. A unit cell is comprised therefore of three bilateral "R" letters, in which some parts of the letters are cropped following the 60°-120°-60°-120° rhombic shape (Figure 4.13a). In fact a hexagon consisting of six entire shapes of equilateral triangles has an equal area to the

60°-120°-60°-120° rhomboid containing fragments of six equilateral triangular regions; therefore both shapes could be the alternative shapes of the unit cell.

Figure 4.13a,b A pattern of symmetry class p3m1 (b) generated from the translation of a rhomboid-shaped unit cell (a)

a



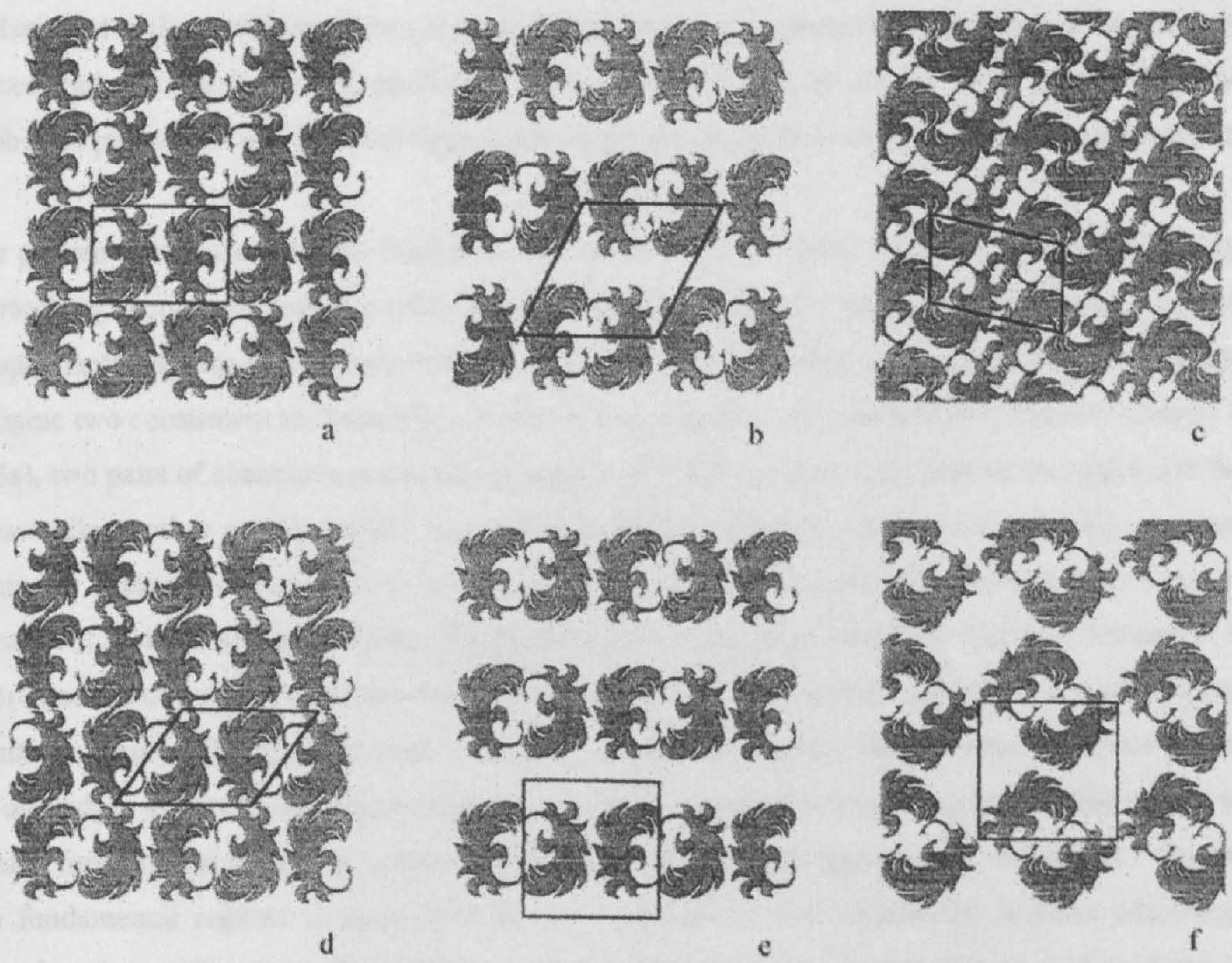
b

An extensive variety of designs can be created within each symmetry group. We can not only apply diverse kinds of motifs into the determined unit boundary, but also define different shapes of unit boundaries. Due to the fact that certain symmetry groups have alternative applications of more than one lattice type (see Table 2.2). This excludes symmetry groups of four-fold rotation which require square-shaped unit cells to be translated on the square lattices and also symmetry groups of three- and six-fold rotation which require 60º-120º-60º-120º rhomboid-shaped or hexagon-shaped unit cells to be translated on the hexagonal lattices.

Parallelogram-based symmetry groups p1 and p2 are compatible to be bounded in any shapes associated with one of five types of geometric lattices. Five rectangle-based symmetry groups, i.e., groups p1m1, p1g1, p2mm, p2gg and p2mg, have two alternatives, either a rectangle or a square. While two centre-celled symmetry groups c1m1 and c2mm are compatible to be bounded in a rhomboid, a square or a particular 60º-120º-60º-120º rhomboid.

Different shapes of the parallelogram units exhibit different alignment of symmetry operations and different organisation of fundamental regions within the same symmetry group. Moreover, a boundary of the same shape may enclose different design areas. All of these variables result in different design outcomes. As indicated in Figure 4.14a-f, and shown by Horne [1997, p.95], there are more than four varieties of patterns of symmetry class p2 generated from four types of geometric lattices. All six patterns share symmetry class p2, but exhibit different design outcomes. The patterns in Figure 4.14a,b are constructed on a rectangular lattice and a rhombic lattice respectively. Two varieties in Figure 4.14c,d are generated from two parallelogram-shaped unit cells which are different in shape and content. Two varieties in Figure 4.14e,f are generated from two square-shaped unit cells which have identical shape but contain different contents.

Figure 4.14a-f Six varieties generated using symmetry group p2



Source: reproduced from Horne, 1997, p.95

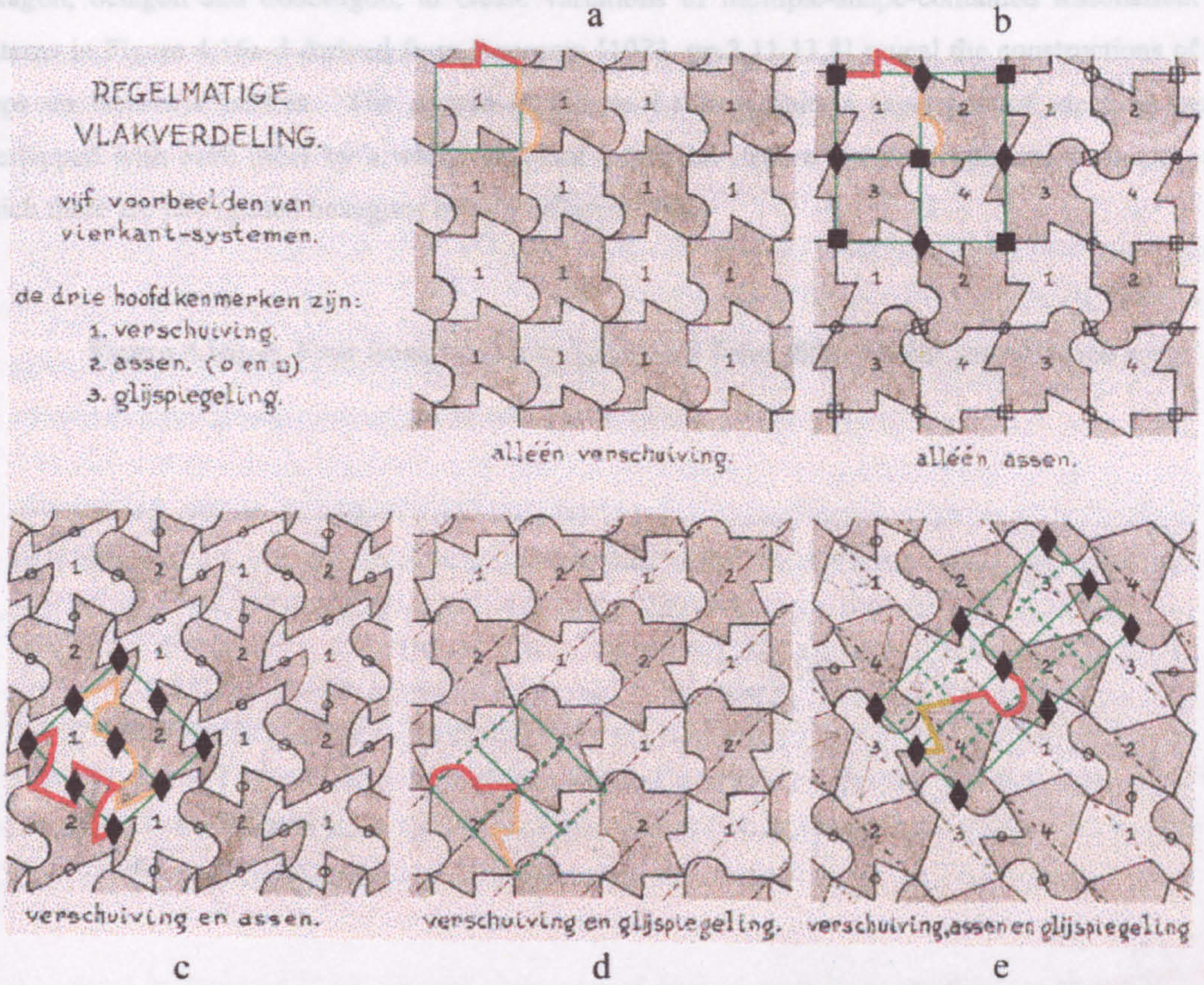
The process in which the boundary of each unit undergoes the same modification with respect to individual symmetry groups, which Escher called a *transitional system* [Schattschneider, 1990, p.59], can also be applied to create varieties of designs particularly interlocking patterns. It should be noted that the boundaries of a fundamental region and a unit cell are not necessarily straight edges.

In fact each side of both fundamental region and unit cell performs two functions: i) as the unit boundaries, and ii) in some cases, as the line of a symmetry operation (e.g. reflection or glide-reflection axes). Any sides which are on glide-reflection axes or else are lines between two rotational centres or boundaries between two units, could be modified to any interlocking sections or curves shared by two adjacent motifs. This includes all sides of the unit cell of pattern symmetry class p1 and all sides of fundamental regions of patterns symmetry classes p1g1, p2gg, p2, p3, p4 and p6. Meanwhile, all sides of fundamental regions of patterns symmetry classes p2mm, p3m1, p4mm and p6mm have to preserve straight lines of reflection axes to shift the regions from one side to another side equally. In the case of the remaining classes, i.e., classes p1m1, c1m1, c2mm, p2mg, p31m and p4gm, certain sides of fundamental regions admitting reflection symmetries have to preserve straight lines while the other sides are applicable to any interlocking sections or curves.

The modification of the polygonal sides in association with individual symmetry groups of seventeen symmetry classes can be seen in M.C. Escher's interlocking patterns and tiling patterns [Grünbaum and Shephard, 1987]. As Schattschneider remarked, fourteen of seventeen all-over symmetry classes were found being used among 144 designs by Escher. The absence of the three symmetry classes, i.e., classes p1m1, p4mm and p6mm, might suggest the self-imposed constraint of fitting figurative shapes, in which a fundamental region would need two or more straight edges as reflection axes to produce any of these three classes [Schattschneider, 1986, pp.94-95]. The preferable uses of classes p111, p1g1, p2gg and p211 exhibit the packing of non-bilateral figures admitting translation, two-fold rotation and glide-reflection.

Five pattern samples created by Escher, as shown in Figure 4.15a-e, present the possibility of applying interlocking sections or curves on four sides of the parallelogram shapes generated using five symmetry groups. Both patterns of symmetry classes p1 in Figure 4.15a and p4 in Figure 4.15b are generated using the same two constituent sections but in different organisations. In a pattern of symmetry class p1 (Figure 4.15a), two pairs of constituent sections enclosed a unit cell: one pair is located on the upper and the lower sides while another one is located on the left and the right sides. Since the unit cell of a pattern of symmetry class p4 (Figure 4.15b) contains four fundamental regions, the organisation of two pairs of constituent sections along each side of a fundamental region is governed by four-fold rotation at the unit centre and the unit corners and two-fold rotation at the mid-sides on the unit edges. In case of a pattern of symmetry class p2 (Figure 4.15c) two-fold rotation at the unit centre, the unit corners and the mid-sides of the unit edges governed the organisation of two sets of two-fold rotational sections which bound all sides of each fundamental region. A pattern of symmetry class p1g1 (Figure 4.15d) whose unit cell consists of two fundamental regions is built up from the repetition of two constituent sections admitting glide-reflections in one direction. Meanwhile, a pattern of symmetry class p2gg (Figure 4.15e) whose unit cell consists of four fundamental regions is also built up from two constituent sections but the organisation is governed by glide-reflections in perpendicular directions.

Figure 4.15a-e Five patterns created by Escher showing edge modification governed by five symmetry groups



Source: reproduced and derived from Schattschneider, 1990, p.35

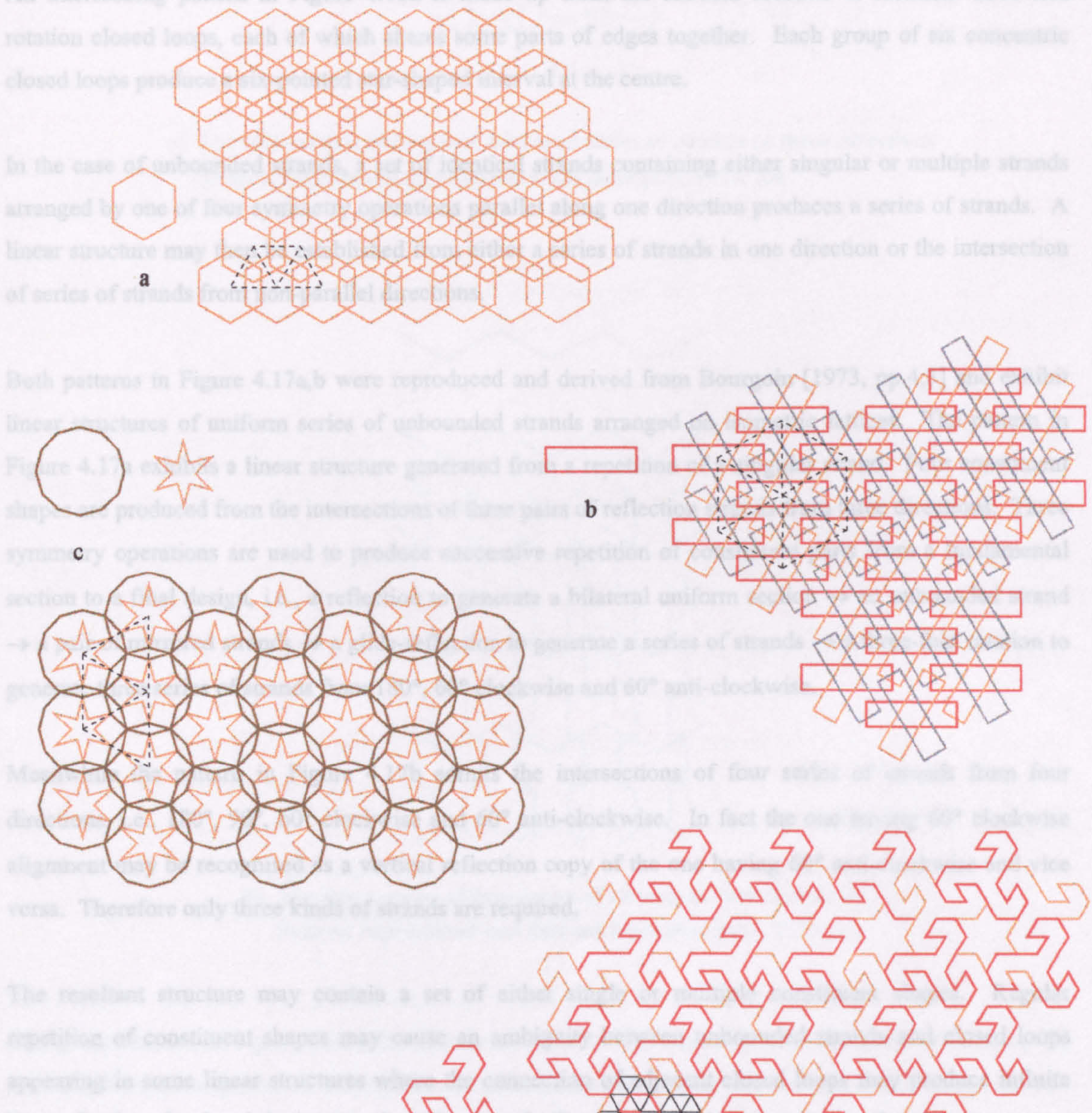
4.3.3 Linear Construction

Grünbaum and Shepard [1993] and Wade [1976] unlocked the intricate structures of Islamic interlacing patterns by discovering that most of the patterns were generated by strands containing a number of constituent sections. Excluding the complexity of crossing layers of strands, the patterns exhibit linear structures of polygonal networks. A line connected between two points is basically drawn as a constituent section. One of four symmetry operations is used to shift the section to the next pairs of points until it maps onto itself. The consecutive repetition of a series of the uniform sections forms either an unbounded strand or a closed loop [Grünbaum and Shepard, 1993, p.148] also known as an open or closed path [Wade, 1976, p.81].

An all-over structure is possibly created from a repetition of either unbounded strands or closed loops or a combination of both. The identical closed loops including either singular or a series of multiple shapes is repeated regularly corresponding with the lattice points. One of the four symmetry operations may be used to create the inter-connection between constituent shapes, i.e., edge-to-edge contact, corner-to-corner contact, overlaying, overlapping and interlocking.

Wade [1976, pp.28-29,34,39] suggested the possible arrangements of some basic polygonal shapes, e.g. hexagon, octagon and dodecagon, to create variations of multiple-shape-contained tessellation. Four patterns in Figure 4.16a-d derived from Bourgoin [1973, pp.2,11,13,8] reveal the constructions of closed loops on isometric lattices. The pattern in Figure 4.16a exhibits a repetition of identical hexagons overlapped with each other by a whole side and a quarter. There are three constituent shapes, among which there are two related hexagons having different sizes.

Figure 4.16a-d Four linear patterns generated from different shapes of closed loops



Source: reproduced and derived from Bourgoin, 1973, pp.2,11,13,8

The pattern in Figure 4.16b is also generated from a singular closed loop. But, in this case, there are two possible construction ways: i) the repetition of a group having the union of three concentric rectangular closed loops arranged on a hexagonal lattice, or ii) the intersection of series of rectangular closed loops from three directions, i.e., 180°, 60° clockwise and 60° anti-clockwise. As a result of overlapping copies of the rectangular closed loops the pattern contains five constituent shapes.

In Figure 4.16c the combination of two shapes of closed loops, i.e., a dodecagon and a six-pointed star, produce three constituent shapes. The overlapping of dodecagons by two sides establishes a repetition of six-pointed polygons with corner-to-corner contact. A 30° six-pointed star whose size is fitted properly to the six-pointed polygon is overlaid at every centre of all six-pointed polygons.

An interlocking pattern in Figure 4.16d is made up from the six-fold rotation of identical three-fold rotation closed loops, each of which shares some parts of edges together. Each group of six concentric closed loops produce a six-pointed star-shaped interval at the centre.

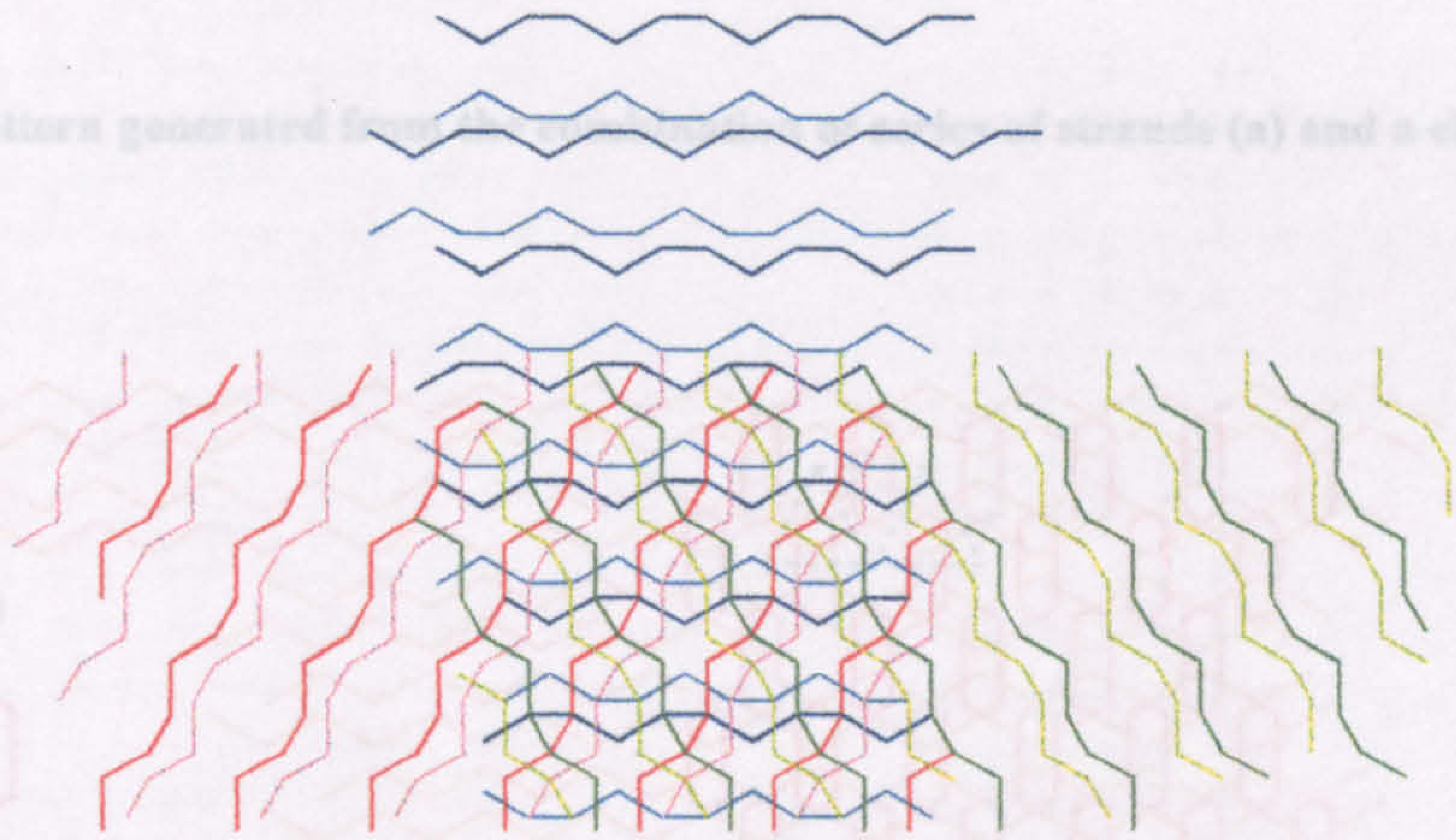
In the case of unbounded strands, a set of identical strands containing either singular or multiple strands arranged by one of four symmetry operations parallel along one direction produces a series of strands. A linear structure may then be established from either a series of strands in one direction or the intersection of series of strands from non-parallel directions.

Both patterns in Figure 4.17a,b were reproduced and derived from Bourgoin [1973, pp.4,3] and exhibit linear structures of uniform series of unbounded strands arranged on isometric lattices. The pattern in Figure 4.17a exhibits a linear structure generated from a repetition of a singular strand. Four constituent shapes are produced from the intersections of three pairs of reflection strands from three directions. Three symmetry operations are used to produce successive repetition of constituent parts from a fundamental section to a final design, i.e., a reflection to generate a bilateral uniform section → an unbounded strand → a pair of mirrored strands → a glide-reflection to generate a series of strands → a three-fold rotation to generate three series of strands from 180°, 60° clockwise and 60° anti-clockwise.

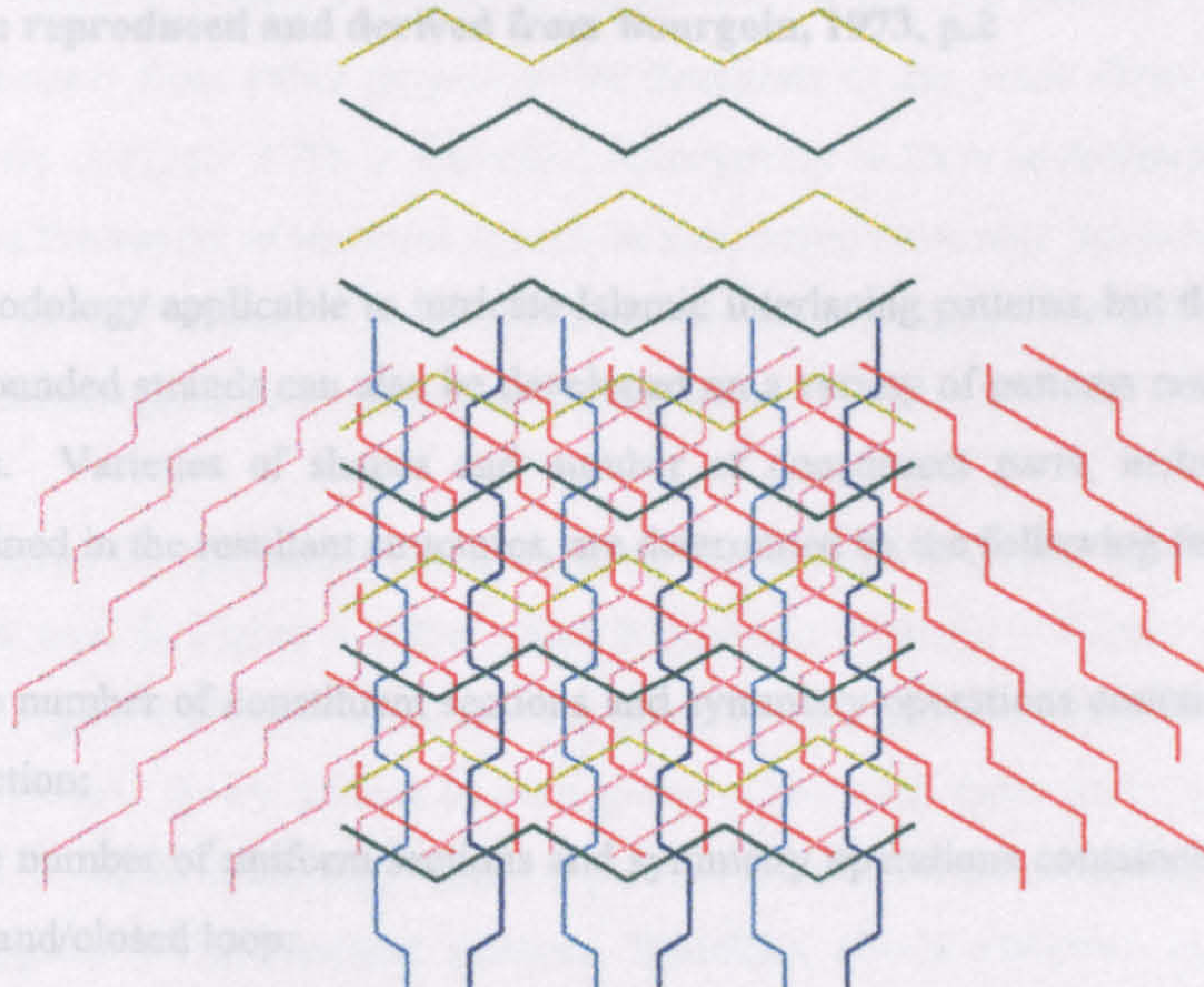
Meanwhile the pattern in Figure 4.17b admits the intersections of four series of strands from four directions, i.e., 180°, 90°, 60° clockwise and 60° anti-clockwise. In fact the one having 60° clockwise alignment may be recognised as a vertical reflection copy of the one having 60° anti-clockwise and vice versa. Therefore only three kinds of strands are required.

The resultant structure may contain a set of either single or multiple constituent shapes. Regular repetition of constituent shapes may cause an ambiguity between unbounded strands and closed loops appearing in some linear structures where the connection of adjacent closed loops may produce infinite lines. On the other hand the intersection of series of unbounded strands may create enclosed spaces.

Figure 4.17a,b Two linear patterns generated from series of strands



a) The intersection of identical mirrored series of strands in three directions
Source: reproduced and derived from Bourgoin, 1973, p.4

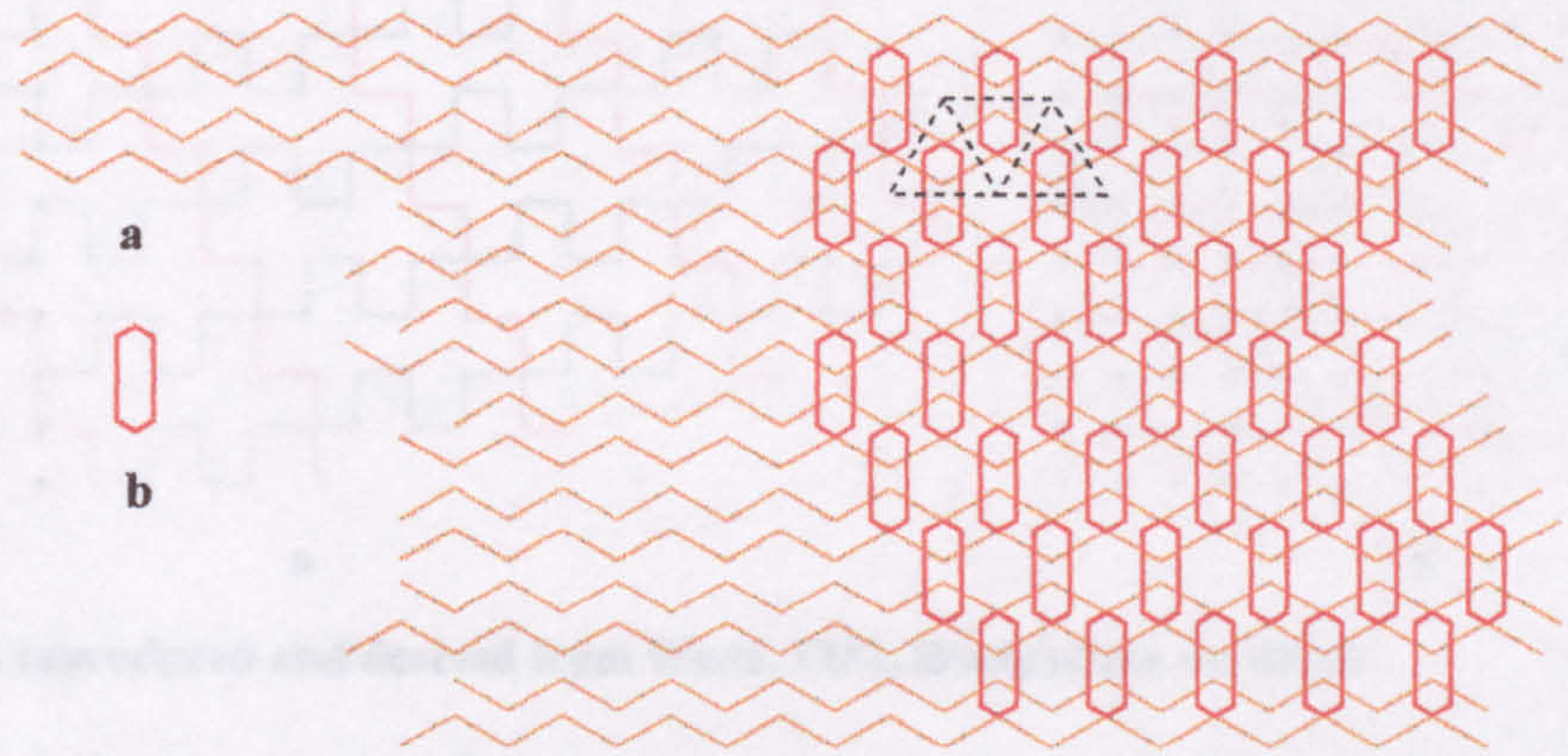


b) The intersection of four series of strands in four directions
Source: reproduced and derived from Bourgoin, 1973, p.3

The pattern shown in Figure 4.18 (also shown in Figure 4.16a) is generated not only by overlapping the hexagonal closed loops, but also by the combination of the closed loop and a series of unbounded strands. Based on a hexagonal lattice, a series of 120° zigzag lines consists of pairs of parallel strands, each of which is arranged alternately with the mirrored copy of itself (Figure 4.18a). The identical closed loops, one is shown in Figure 4.18b, are repeated regularly to complete the vertical sides of the hexagonal lattice.

Moreover, instead of the hexagonal shape, a closed loop in Figure 4.18b can also be used as a construction unit to generate the same pattern but in association with six-fold rotation along all sides of the hexagonal lattice, or otherwise three-fold rotation about every vertex of the hexagonal lattice.

Figure 4.18 A pattern generated from the combination of series of strands (a) and a closed loop (b)



Source: reproduced and derived from Bourgoin, 1973, p.2

Not only is the methodology applicable to intricate Islamic interlacing patterns, but the principle of linear construction by unbounded strands can also be developed on a variety of patterns ranging from simple to complicated designs. Varieties of shapes and number of constituent parts, including the symmetry characteristics contained in the resultant structures, are determined by the following factors:

i) the number of constituent sections and symmetry operations contained in each uniform section:

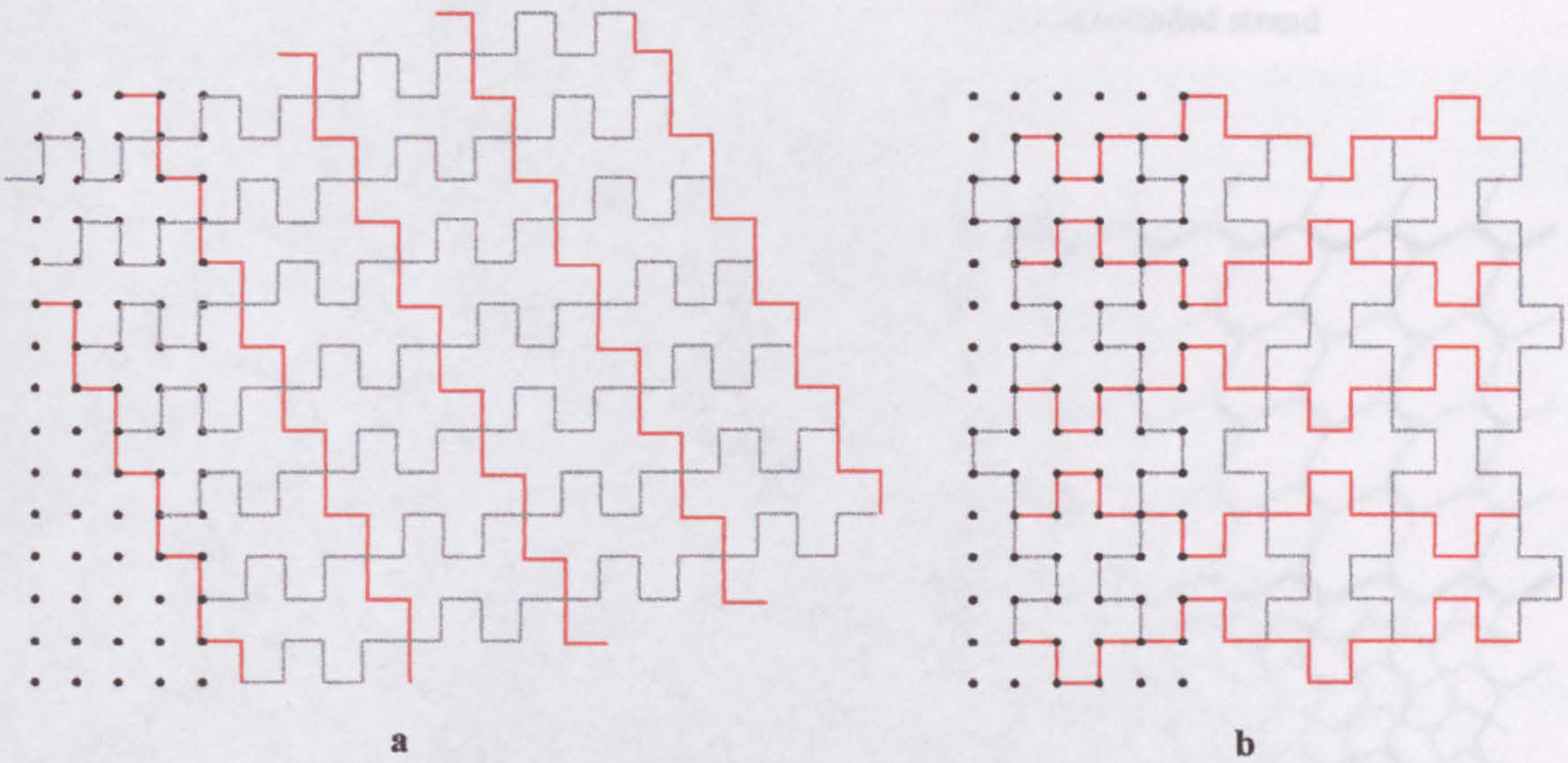
ii) the number of uniform sections and symmetry operations contained in each unbounded strand/closed loop:

iii) the number of unbounded strand/closed loop and symmetry operations contained in each series of lines:

iv) the number of series of lines and symmetry operations arranged in different directions.

In the case of the patterns in Figure 4.19a-b, both have the same shape of constituent parts, but have different orientations. A pattern of symmetry class p2 (shown in Figure 4.19a) is built up from the intersections of two series of unbounded strands: one admits horizontal translation while the other one admits vertical translation. A pattern of symmetry class p4gm (shown in Figure 4.19b) is also generated from the intersections of two series of unbounded strands, but in this case both series consist of pairs of identical mirrored strands arranged perpendicularly to each other.

Figure 4.19a,b Two patterns having identical constituent shapes but generated from different series of strands



Source: reproduced and derived from Wade, 1982, illustrations no. 41,42

A number of square-based varieties, as shown in Figure 4.20, reveal the intersections of series of 120° zigzag unbounded strands from either perpendicular directions or the same direction. A 120° zigzag uniform section shown in Figure 4.20a is translated successively to form an unbounded strand shown in Figure 4.20b. In fact the copies of identical strands in successive rows may not necessarily preserve the same orientation as the next to one. The resultant patterns shown in Figure 4.20c-m illustrate varieties created from different arrangements of the unbounded strands within each series.

Three groups of patterns are generated based on three intersecting points of perpendicular series, i.e., point A covering patterns in Figure 4.20c-e, point B covering patterns in Figure 4.20f-g and point C covering patterns in Figure 4.20h-k. While another group reveals the intersections of series in the same direction (Figure 4.20l-m). Every pattern in each group obtains the same primary structure generated from the intersection of series containing regularly parallel strands, on which additional series of strands are subsequently applied. The resultant patterns, therefore, obtain different symmetry groups and constituent shapes from their primary structures. Table 4.1 summarises the features of construction series and resultant designs generated from 120° zigzag strands, as illustrated in Figure 4.20c-m.

It is found that different arrangements of strands offer a variety of linear patterns. As evidenced in the three primary structures, the three different intersecting points produce three different polygonal networks as classified as the pattern symmetry classes p4, c1m1 and c2mm. But, on the other hand, the patterns classified as the same symmetry class may not necessarily be generated from the same combination of series. As seen in Figure 4.20e and Figure 4.20i, both patterns share the same symmetry class p4gm but they are generated on different intersecting points, from different strand arrangements and different shapes of constituent parts.

Figure 4.20a-m Square-based varieties generated from series of 120° zigzag lines in different arrangements

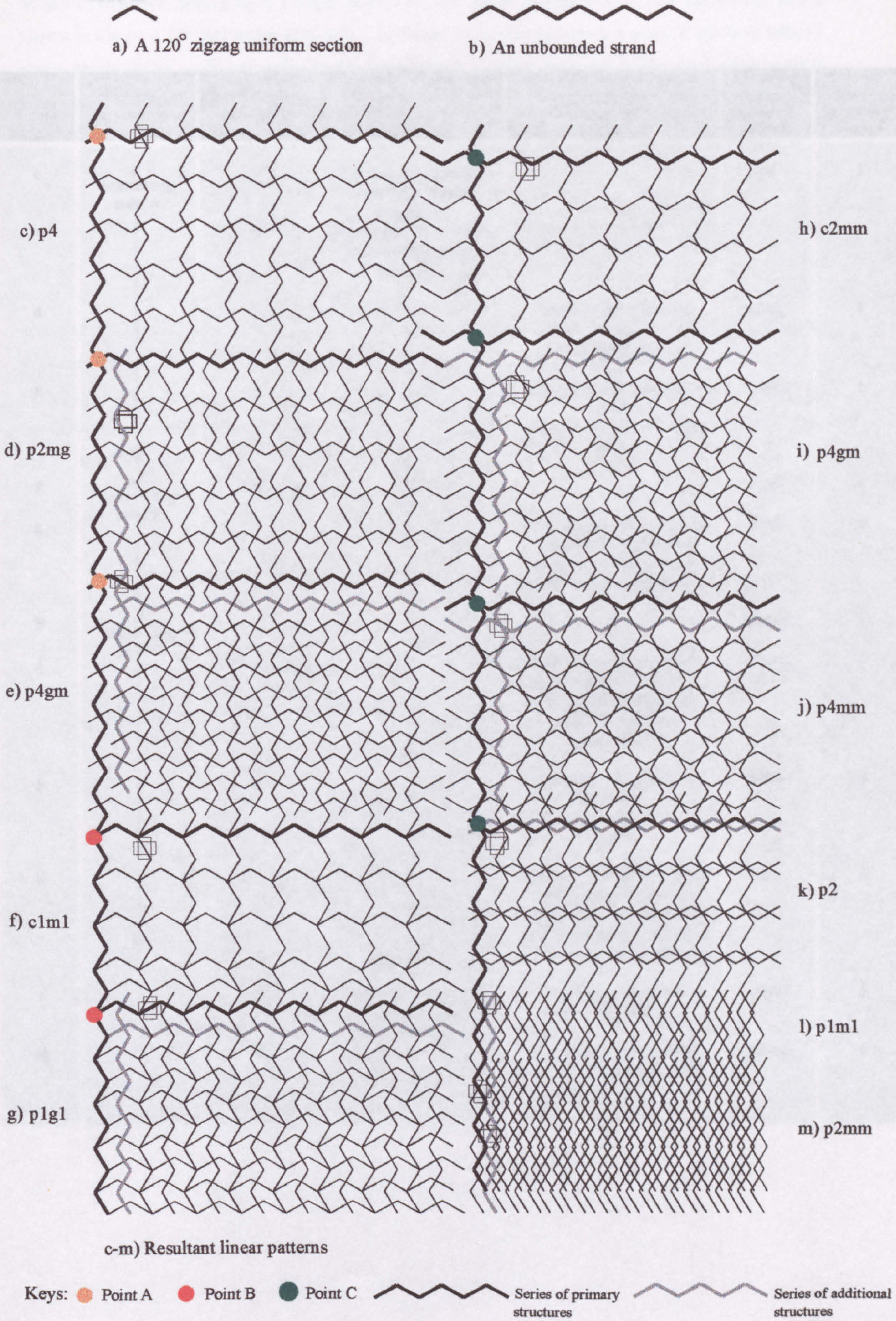


Table 4.1 Summary of square-based patterns generated from series of 120° zigzag strands as shown in Figure 4.20c-m

Figure	The intersection of perpendicular series	The intersection of series in the same direction	The number of series and symmetry operations contained in primary structures	The number of series and symmetry operations of additional strands associated with the primary structures	Symmetry classes of resultant patterns	Number of constituent shapes
c	✓ Intersecting point A	-	Vertical: one parallel series with regular translation Horizontal: one parallel series with regular translation	-	**p4**	1
d		-		Vertical: one mirrored series Horizontal: none	**p2mg**	1
e		-		Vertical: one mirrored series Horizontal: one mirrored series	**p4gm**	1
f	✓ Intersecting point B	-		-	**c1m1**	1
g		-		Vertical: one parallel series with regular translation Horizontal: one parallel series with half-side sliding	**p1g1**	2
h	✓ Intersecting point C	-		-	**c2mm**	1
i		-		Vertical: one parallel series with regular translation Horizontal: one parallel series with regular translation	**p4gm**	1
j		-		Vertical: one mirrored series Horizontal: one mirrored series	**p4mm**	2
k		-		Vertical: one parallel series with regular translation Horizontal: one mirrored series arranged in the same row as the primary structure	**p2**	2
l	-	✓	Vertical: one parallel series with regular translation	Vertical: a set of two mirrored strands	**p1m1**	5
m	-	✓		Vertical: a set of two mirrored and one parallel series	**p2mm**	6

In the case of curvilinear varieties, several examples shown in Figure 4.21 are built up from the repetitions of half-circled arcs ranging from a single half-circled arc shown in Figure 4.21a to a half-circled strand shown in Figure 4.21b and series of strands. A variety of regular patterns are constructed from either a single series shown in Figure 4.21c or the intersections of more than one series arranged in vertical direction shown in Figure 4.21d-h or perpendicular directions shown in Figure 4.21i-l. Table 4.2 summarises the features of construction series and resultant designs of the curvilinear patterns illustrated in Figure 4.21c-l.

A symmetry group of a resultant pattern associates with the combination of symmetry operations along a process of construction from uniform sections, continuous strands, series of strands to an all-over structure. Symmetry operations of the uniform sections are not necessarily presented in the symmetry group of the all-over structure. Since both the 120° zigzag section and the half-circled arc are bilateral elements, they tend to produce the patterns whose symmetry groups contain reflection symmetries as evidenced by most of the samples presented in Figure 4.20 and 4.21. But this excludes the patterns with rotation and glide-reflection symmetries in Figure 4.20c, g, k and Figure 4.21d where the combinations of strands disturb the continuity of the reflection axes in any direction.

Figure 4.21a-l Curvilinear varieties generated from series of half-circled arcs in diffferent arrangements

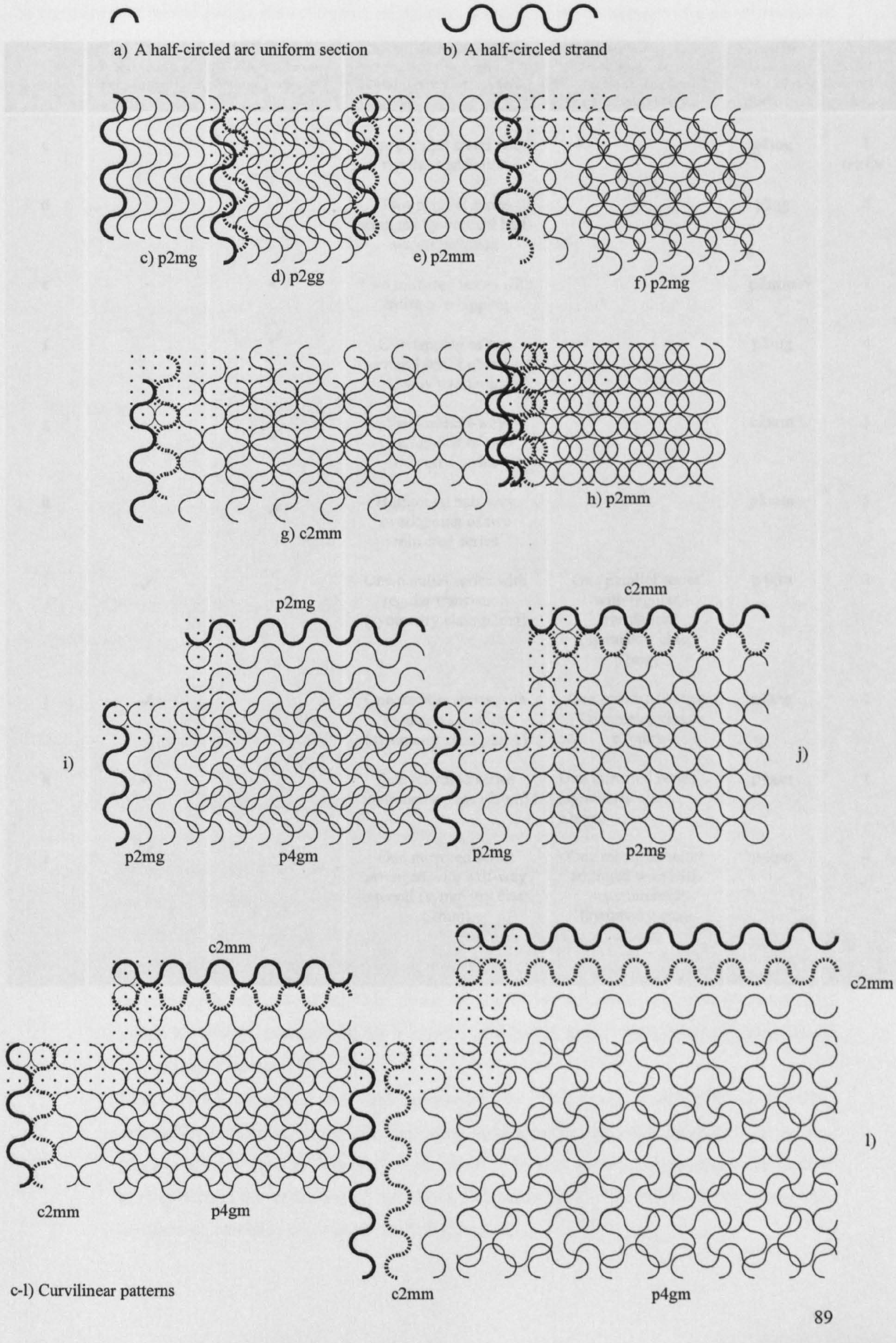


Table 4.2 Summary of curvilinear patterns generated from series of half-circled strands as shown in Figure 4.21c-l

Figure	The intersection of perpendicular series	The intersection of series in the same direction	The number of series and symmetry operations contained in vertical series	The number of series and symmetry operations contained in horizontal series	Symmetry classes of resultant patterns	Number of constituent shapes
c	-	-	One parallel series with regular translation	-	**p2mg**	1 (stripe)
d	-	✓	Two parallel series arranged by vertical half-way translation	-	**p2gg**	2
e	-	✓	Two mirrored series with entire overlapping	-	**p2mm**	1
f	-	✓	Overlapping of two mirrored series obtaining half-way overlapping	-	**p2mg**	4
g	-	✓	Vertical half-way overlapping of two mirrored series	-	**c2mm**	3
h	-	✓	Horizontal half-way overlapping of two mirrored series	-	**p2mm**	5
i	✓	-	One parallel series with regular translation (symmetry class p2mg)	One parallel series with regular translation (symmetry class p2mg)	**p4gm**	2
j	✓	-	One parallel series with regular translation (symmetry class p2mg)	One mirrored series (symmetry class c2mm)	**p2mg**	1
k	✓	-	One mirrored series (symmetry class c2mm)	One mirrored series (symmetry class c2mm)	**p4gm**	1
l	✓	-	One mirrored series arranged with half-way interval (symmetry class c2mm)	One mirrored series arranged with half-way interval (symmetry class c2mm)	**p4gm**	4

4.4 Designers' Construction Means: The Hybrid Approach

In the context of textile design, the emergence of patterns is based on the recurrence of a set of designs at regular intervals to accommodate fabric length in the longitudinal direction and width in the transverse direction. The size and a number of repeats are determined in association with the fabric width, which is imposed by the fabric end use and mechanical means of production.

The rectangle and square are regularly employed as the basic shapes for the repeating units. As Watson pointed out one complete repeat of woven figurative designs must be capable of being enclosed within a rectangle whose boundary edges correspond vertically and horizontally with warp and weft directions [Watson, 1954, 6th ed. 1996, p.266]. This could be applicable to the interlooping structure of courses and wales of knitting and also the rectangular-blocked layout of screen printing.

In this section, attention is focused on the construction of rectangle- and square-shaped repeating units to generate the all-over structure and the distribution of motifs within each unit.

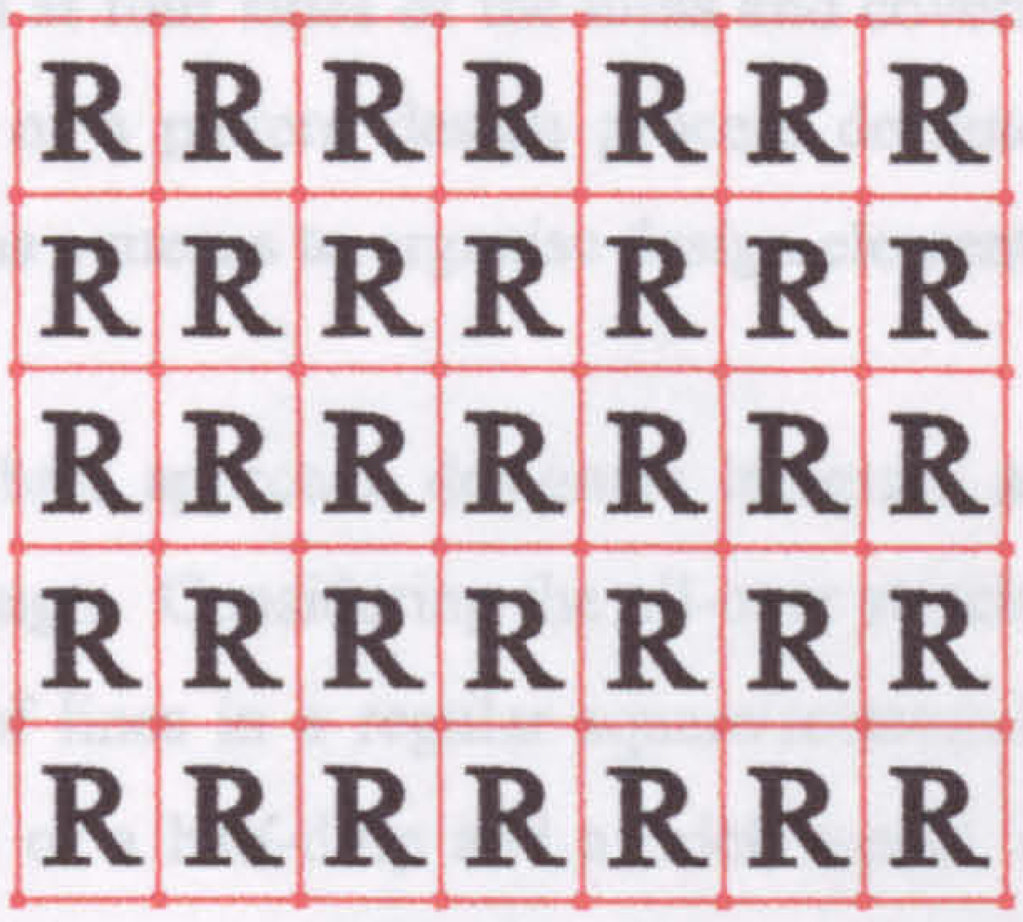
4.4.1 Textile Repeating Formats: The Construction of Square/ Rectangular Units.

When a repeating unit is translated longitudinally and transversely design elements at all sides of each unit must join perfectly with each other. The connection at four sides of rectangle- and square-based units determines some possible repeating formats for the unit distribution.

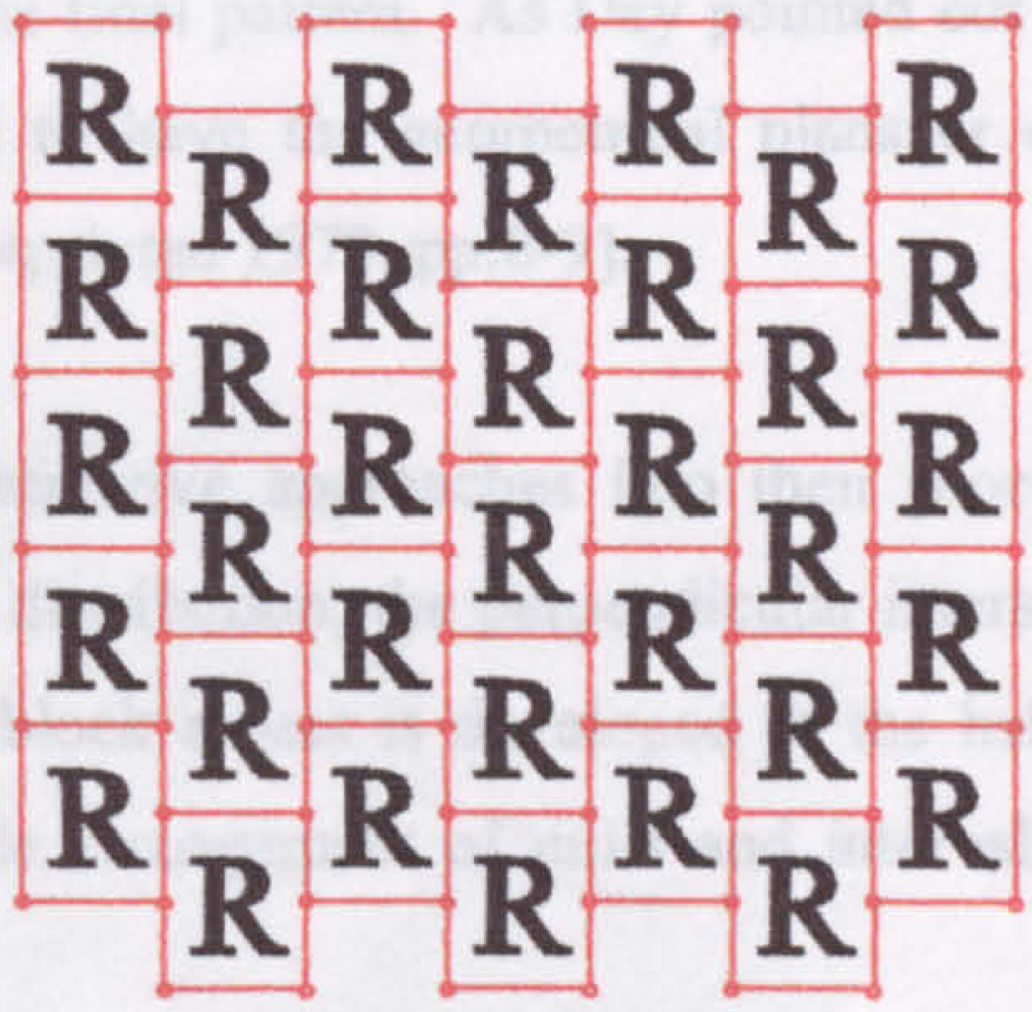
As mentioned by Day [1903, reprinted 1979], Watson [1954, 6th ed. 1996], Phillips and Bunce [1993] and McNamara and Snelling [1995], designers usually employ block, half-drop and brick repeats as their customised formats.

- **Block repeat**: A block repeat, as shown in Figure 4.22a, is the most fundamental format in which a repeating unit is translated successively through two non-parallel directions. Each corresponding point of the unit is the corresponding point of one other unit and each side of the unit is also the side of precisely one other unit.
- **Half-drop repeat**: A half-drop repeat, as shown in Figure 4.22b, is used to vary the linear repetition of a block repeat in a vertical direction. Repeating units arranged in a vertical-column layout are placed half-way down the successive columns. The unit corresponding points are located vertically at the mid-sides of adjacent units. As a result it causes visual increasing of a unit-width.
- **Brick repeat**: In contrast to the half-drop repeat, the brick repeat, as shown in Figure 4.22c, establishes horizontal emphasis. Repeating units arranged in a horizontal-row layout are placed half-way right or left of the upper row. The unit corresponding points are located horizontally at the mid-sides of the above and below units. Therefore the repeating unit seems to be extended twice in the vertical direction.

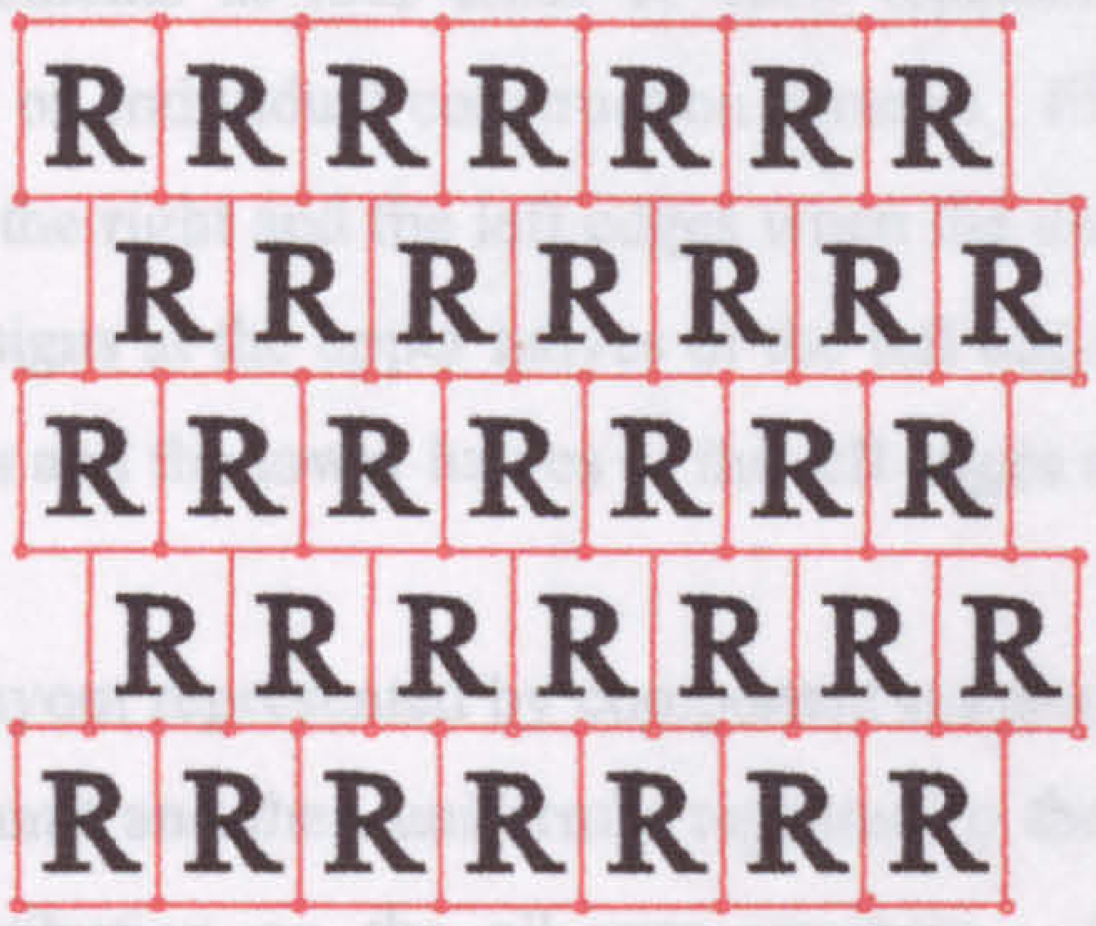
Figure 4.22a-d Four types of repeating formats



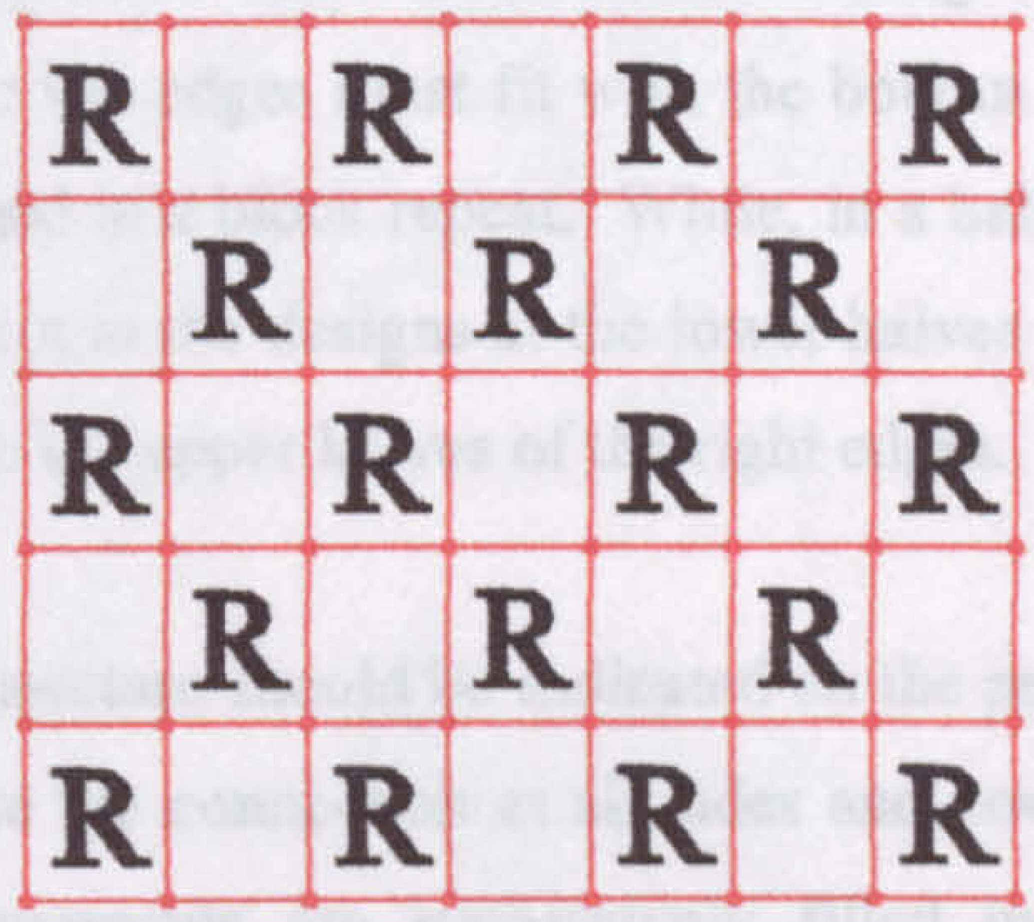
a) A block repeat



b) A half-drop repeat



c) A brick repeat



d) A diaper repeat

It should be noted that both half-drop and brick repeats exhibit half-way translation but in different directions. In the case of using non-directional elements the half-drop format with a quarter turn provides the same result as the brick format and vice versa. When bilateral reflection symmetry is developed on a pair of repeating units packed on either half-drop or brick format, the resultant design exhibits glide-reflection symmetry. Moreover, it is possible to apply ⅓, ¼ or other fractional ratios to drop and brick repeats to produce a variety of stepped arrangements [Phillips and Bunce, 1993, p.57].

Based on these three repeating formats a range of varieties can be derived by applying reflection and rotation symmetries, also known as turnover and turn-round, to each repeating unit or adding spaces between units or unit-columns/rows [see examples in Phillips and Bunce, 1993]. The **diaper repeat**, as shown in Figure 4.22d, could be the special case of a block repeat which produces a pattern in which each rectangle-or square-shaped unit is placed alternately with a rectangle or a square shape of blank spaces. Thus there are connections between units only at the corners of the repeating units.

The rectangle- or square-shaped repeating unit may contain a group of elements having random or regular arrangement. Balance of element distribution is subjected to designers' experiences and skills. Some patterns exhibit gaps, optical stripes or strong directional designs resulted from many causes, e.g., an un-balance combination of figures and backgrounds or inappropriate element organisation.

To achieve a perfect outcome, a rough design layout should be developed primarily concerning the connection at four sides of the units and coverage design of the final pattern. As Day pointed out at the beginning of a pattern design process designers are advised to have the geometrical plans or design skeletons as a means to organise design elements [Day, 1903, reprinted 1979, pp.8-9].

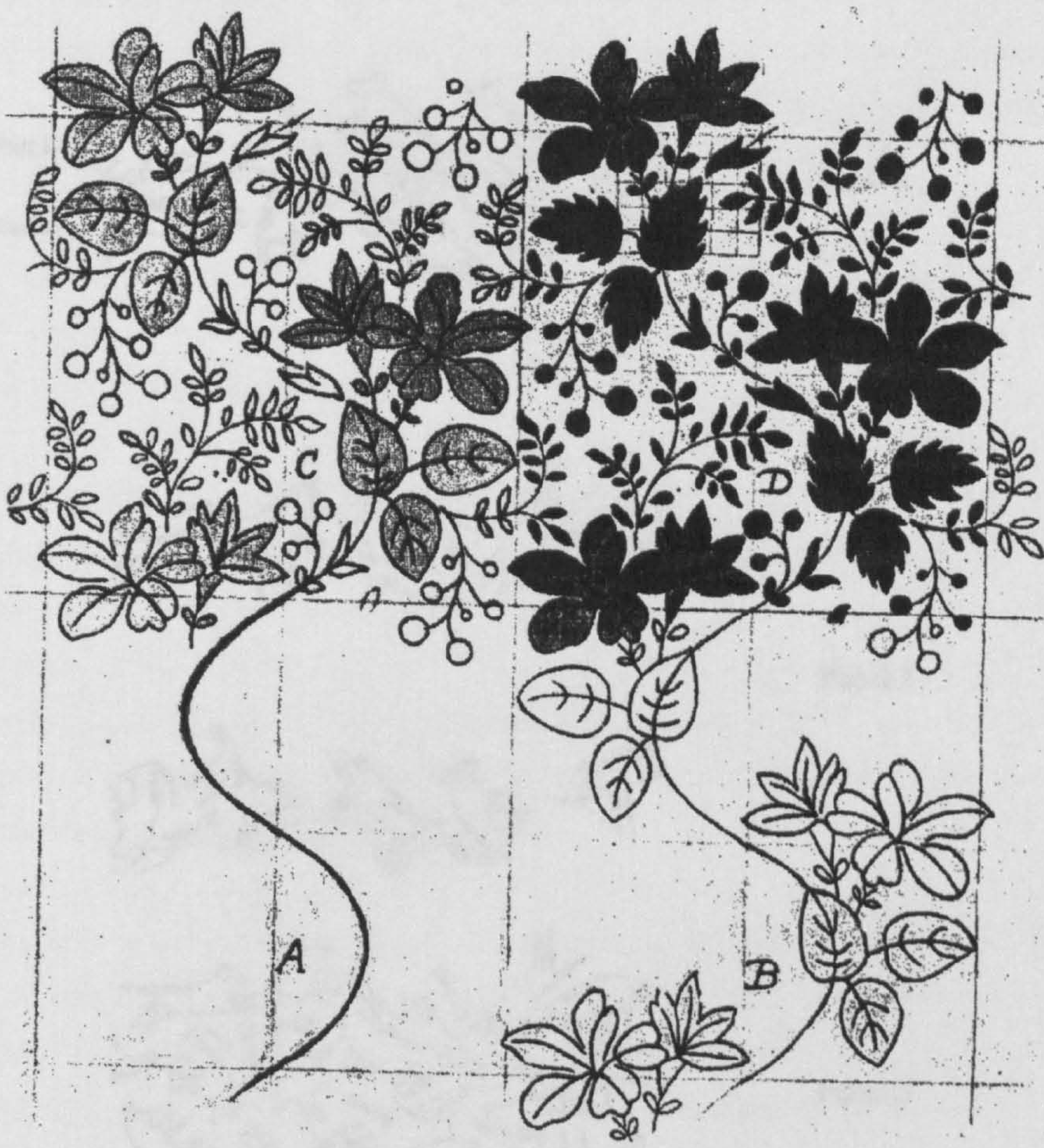
As the hybrid approach designers integrate additive and subtractive approaches into their process of pattern design. Considering the all-over structure for the unit distribution, the perpendicular intersection of series of lines in a regular square/rectangular lattice or a block repeat is developed to the half-way translation of a half-drop and a brick repeat, and the alternate arrangement of units and intervals of a diaper repeat.

Design elements at four sides of each repeating unit must join to the others of succeeding repeats dependent on individual construction formats. Elements at the top edges must fit with the bottom edges and so do the right and the left edges when the units are arranged in a block repeat. While, in a half-drop repeat, designs at the upper halves of the left edges must connect to the designs at the lower halves of the right edges and the lower halves of the left edges must join with the upper halves of the right edges.

A rough layout represented by component masses or a linear structure should be indicated on the primary repeating unit and then uniformly repeated to the others to see the connection at all sides and coverage motif distribution on the all-over structure. Additional ornaments are successively filled in each component mass or along a linear structure and then repeated to the other parts following the underlying layout. One or more of four symmetry operations may be developed on every stage of motif distribution and construction planning. An investigation is made on two case studies, i.e., a continuous pattern and an isolated pattern, as mentioned below.

Figure 4.23, a drawing reproduced from Watson [1954, 6th ed. 1996, p.271], exhibits four steps to create design layout of a foliage pattern in four units of block repeat. A two-fold vertical curved line is firstly drawn in the repeating area A and repeated to the others as the axial guidelines, each of which bisects every rectangular unit into two halves. A pair of reflection motifs containing flowers and leaves are then introduced as the major elements fitted symmetrically to the inverse- and outverse-curved lines representing the plant stems as shown in area B. Small details of foliage designs are filled in the intervals with respect to the orientations of curved stems and two major figures, as shown in area C. A colour distribution is the final touch developed on a complete design as shown in area D.

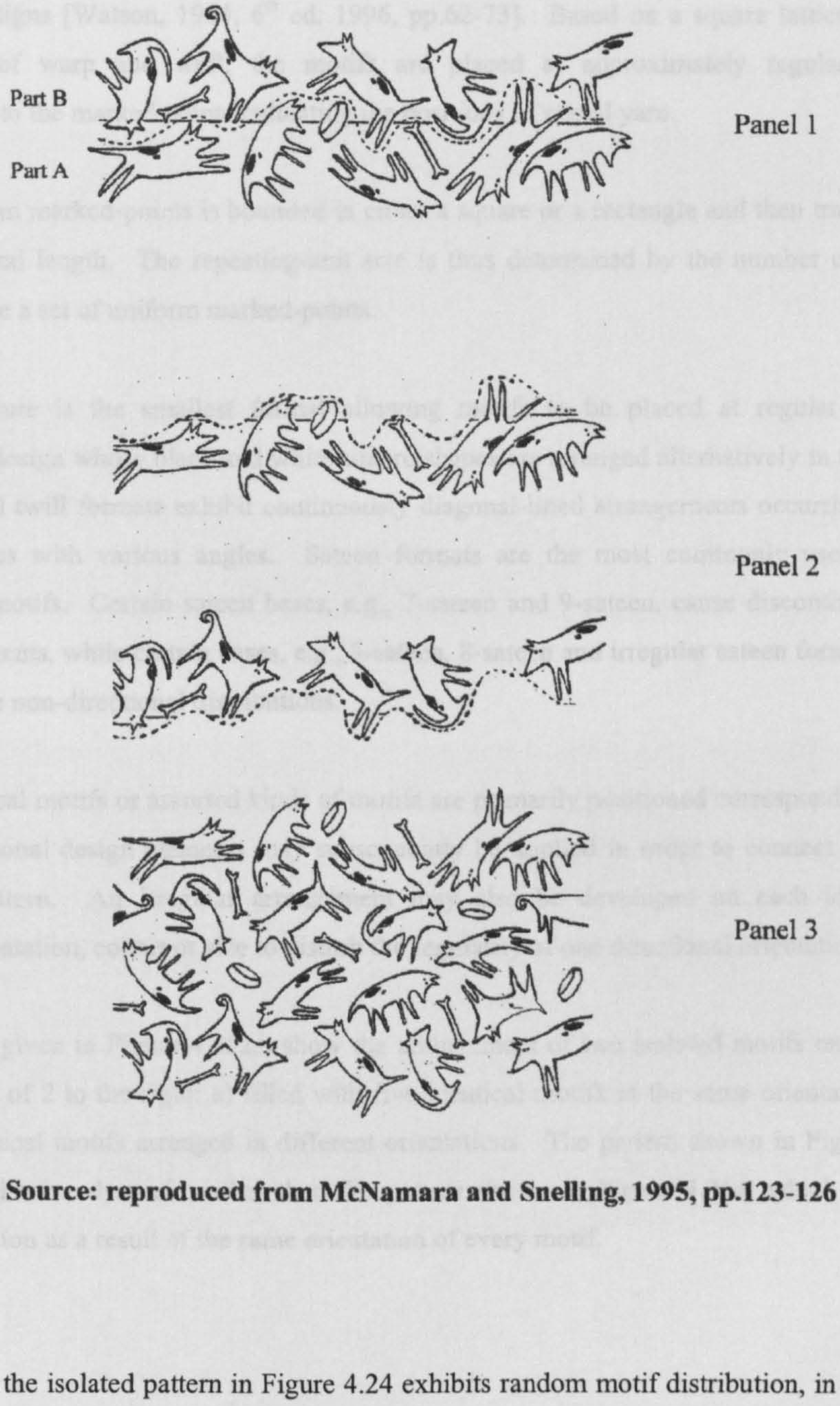
Figure 4.23 Illustration showing four steps contained in the design plan of a foliage pattern



Source: reproduced from Watson, 1954, sixth edition 1996, p.271

McNamara and Snelling [1995, pp.123-126] suggested a technique to produce an isolated pattern with a perfect motif distribution in Figure 4.24. A number of assorted dog figures are arranged randomly along the connecting edge of the top and the bottom repeat. Then the connecting edge is modified from a straight line to a cut-through line running around the figures: not going through any of them. As a result there are two separated parts, i.e., part A and B, as shown in panel 1. All figures in part A are shifted down to a lower edge of the bottom repeat where a cut-through line is located precisely on the same position as the upper repeat. Existing figures at the top and the bottom edges, as shown in panel 2, guide the distribution of additional figures to be filled in the interval between them, as shown in panel 3. The cut-through line between these two parts thus acts as an interlocking boundary underlying the translation of the repeat upward and downward. It is noted that the process can also be applied to the left- and right-sided edges of the repeat in the same manner, if required.

Figure 4.24 Three panels of repeating manipulation to generate an isolated pattern



Source: reproduced from McNamara and Snelling, 1995, pp.123-126

It is found that the isolated pattern in Figure 4.24 exhibits random motif distribution, in which no figures are identical or repeated in the same manner. Meanwhile, the continuous pattern in Figure 4.23 presents some degree of uniform motif organisation along inverse and outverse of the axial curve. The process of regular space sub-division may be further developed within a rectangle- and square-shaped repeating unit as a uniform guideline for the motif distribution. Each unit may contain a number of sub-divided rectangles or squares in which not every interval must be filled with design elements.

4.4.2 Weave Structures: Sub-divided Formats in Square/Rectangular Units

It is well established that weave structures provide the formula for motif distribution as in for example spot repeats [Day, 1903, reprinted 1979, pp.128-138 and McNamara and Snelling, 1995, pp.134-136] or spot figure designs [Watson, 1954, 6th ed. 1996, pp.62-73]. Based on a square lattice representing the interweaving of warp and weft, the motifs are placed at approximately regular distances apart corresponding to the marked points indicating the positions of raised yarn.

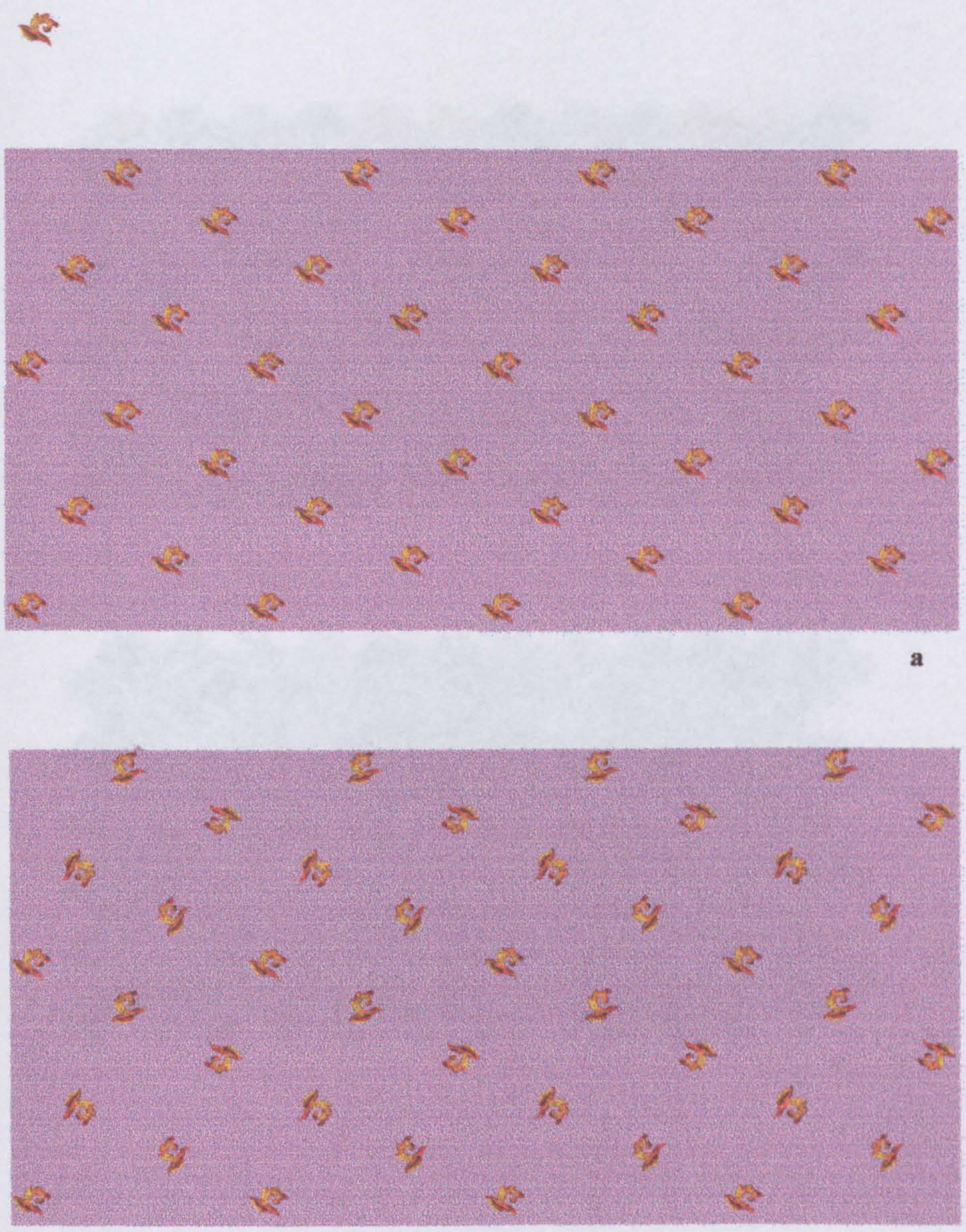
A set of uniform marked-points is bounded in either a square or a rectangle and then translated across the fabric width and length. The repeating-unit size is thus determined by the number of warp and weft, which complete a set of uniform marked-points.

A plain structure is the smallest format allowing motifs to be placed at regular intervals, e.g. a checkerboard design whose black and white square shapes are arranged alternatively in two perpendicular directions. All twill formats exhibit continuously diagonal-lined arrangements occurring in the cloth as diagonal stripes with various angles. Sateen formats are the most commonly used for isolated or disconnected motifs. Certain sateen bases, e.g., 7-sateen and 9-sateen, cause discontinuously diagonal-lined arrangements, while certain bases, e.g., 5-sateen, 8-sateen and irregular sateen formats (e.g. 4-and 6-sateen) provide non-directional distributions.

A set of identical motifs or assorted kinds of motifs are primarily positioned corresponding to the marked points. Additional design elements may subsequently be applied in order to connect these motifs as a continuous pattern. An irregular arrangement may also be developed on each identical motif by modifying orientation, colour or size to disturb the regularity of one directional orientation.

The examples given in Figure 4.25a,b show the arrangement of two isolated motifs on 5-sateen formats with the count of 2 to the right: a) filled with five identical motifs in the same orientation, and b) filled with five identical motifs arranged in different orientations. The pattern shown in Figure 4.25b is non-directional and more dynamic rather than the pattern shown in Figure 4.25a, which exhibits a visual diagonal direction as a result of the same orientation of every motif.

Figure 4.25a,b Two isolated patterns arranged on 5-sateen formats: a) filled with identical motifs of the same orientation, and b) filled with identical motifs in different directions.

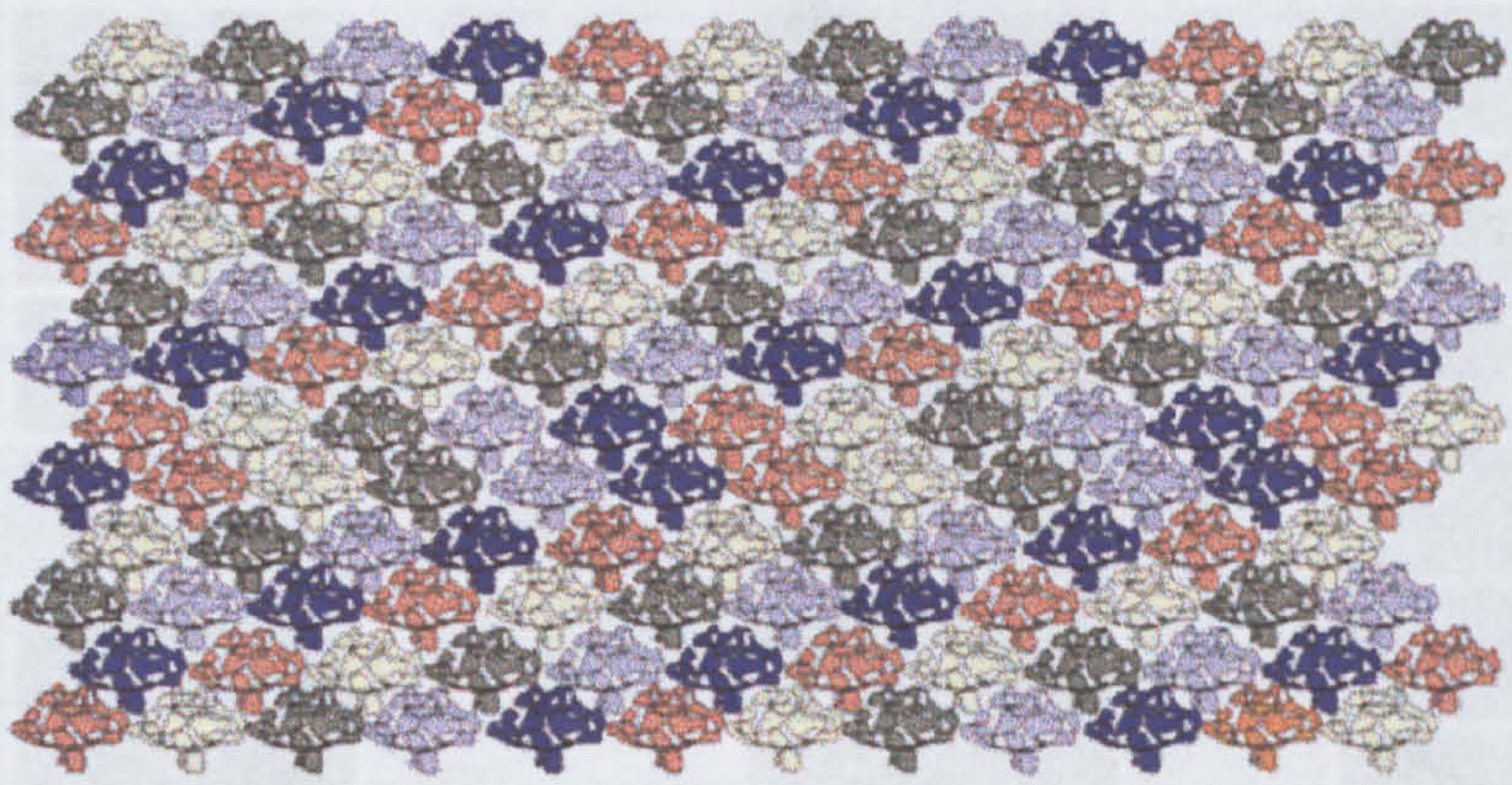


a

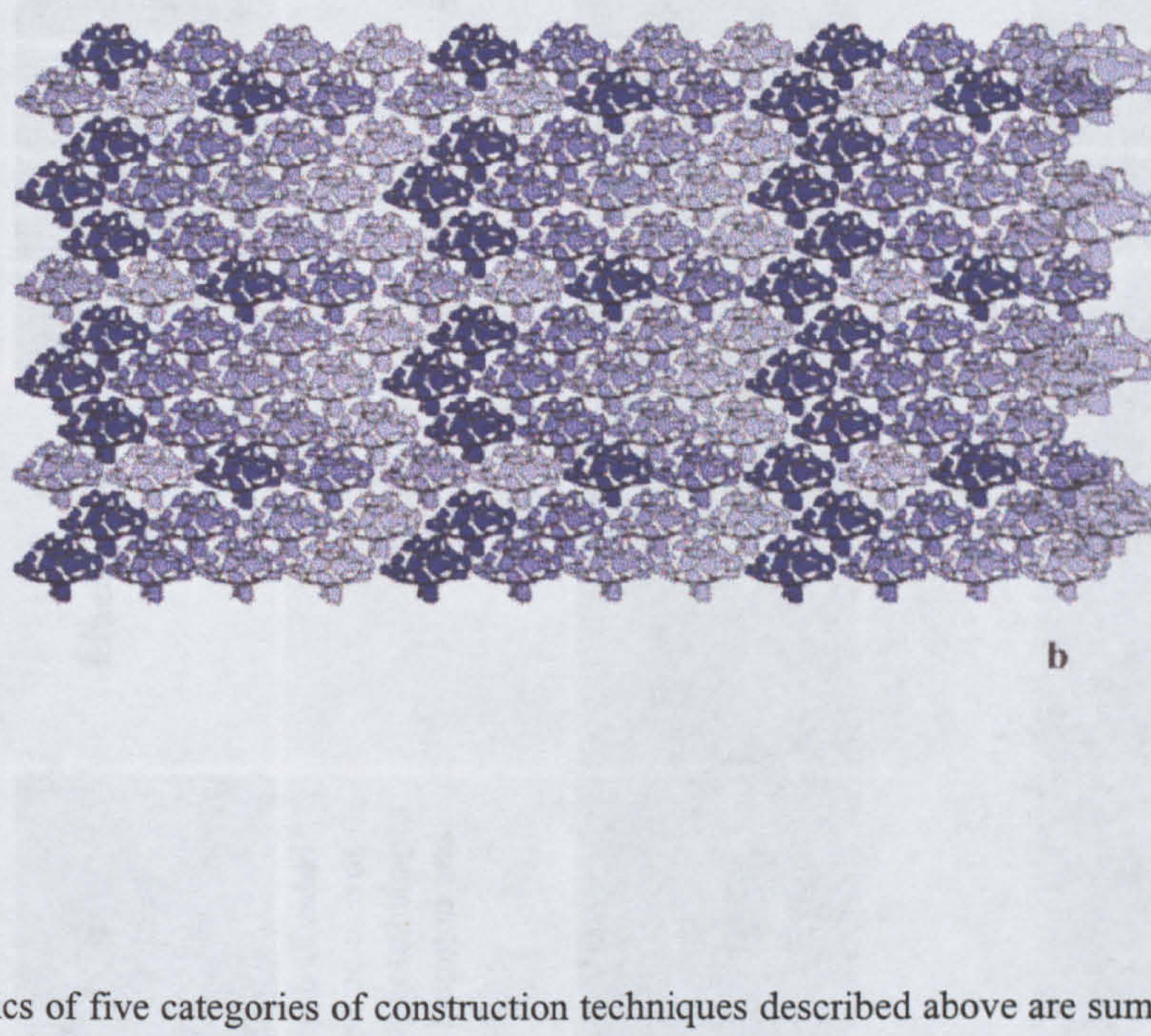
b

A range of varieties may also be created from the combination of a weave structure format with other design features or techniques. The patterns shown in Figure 4.26a-b, for example, exhibit two scale patterns in which series of five- and four- colour distributions have been applied based on the 5- and 4-sateen formats.

Figure 4.26a,b Two scale patterns obtained two series of colour distributions: a) based on 5-sateen format, and b) based on 4-sateen format.



a



b

The characteristics of five categories of construction techniques described above are summarised in Table 4.3.

Table 4.3 Summary of five categories of construction techniques

Construction techniques	Construction units			Unit contents						Design outcomes					
	Repeating units			Number of elements		Symmetry properties		Element organisations			Continuous patterns		Isolated patterns		
	Fundamental regions	Unit cells	Linear units	Singular	Multiple	Asymmetry	Symmetry-obtained	Determined	Un-determined	Interlocking patterns	Regular distribution	Random distribution	Regular distribution	Random distribution	Special Features
Islamic pattern constructions		A space sub-divided unit based on one of three polygonal shapes (square, hexagon and pentagon)			✓		✓		✓	✓	✓				- Geometric appearance - Economical use of elements
Seventeen geometric symmetry structures	✓	One or a group of fundamental regions generated using one of seventeen symmetry groups		✓	✓	✓	✓	✓		✓	✓		✓		- Rigorously symmetry-performed designs - Economical use of elements
Linear construction			Sections→ lines→ series of lines	✓	✓	✓	✓		✓	✓	✓		✓		- All-over structure concern
Textile repeating formats		Rectangle-/sqaure-/paralellogram-shaped units			✓	✓	✓		✓			✓		✓	- Randomly distributed designs - Diversity of elements
Weave structure formats		A group of marked points bounded in either rectangle or square			✓	✓	✓		✓		✓	✓	✓	✓	- Systematically random distributions

4.5 Development of the Repeating Pattern Concepts to Designs

Since art, science and technology are the fundamental elements of designs, the mathematical principles of geometric symmetry have been applied to the domain of regular repeating pattern design. Four categories of designs containing a range of traditional Islamic patterns to computer-generated images given below reveal the development of the repeating pattern concept in which geometric structures have been transformed to varieties of designs serving individual aspects.

4.5.1 Two-dimensional Graphics

Graphic artists who are concerned with the properties of harmony and order transform rigid geometrical skeletons to a wide variety of designs as the matters to communicate particular ideas to their audiences. Degree of design transformation varies from a straightforward use of pure geometrical structures to complicated curvilinear contour derived from a wide range of inspirations. Three case studies were examined, i.e., Islamic patterns, Vasarely's illusion art and Escher's interlocking figures. Illustrations in Figure 4.27a-d show three varieties generated on an isometric lattice within a hexagonal shape: a) an Islamic interlacing pattern, b) *Hat* by Vasarely, and c) *Verbum* by Escher.

Islamic Patterns: Pure Geometric Designs

Since Greek mathematical knowledge was transferred into the Arabic world, the Muslims have fostered the concept of plane geometry hand in hand through their art and cultural items by employing a wide range of media and patterning techniques, e.g., tilings, brickwork, stucco, stone and wood carving. In Morocco, traditional mosaic patterns exhibit the utilisation of sets of uniform shapes in which identical constituent parts are used regularly to create diverse combinations [Abas and Salman, 1995, p.24].

Islamic patterns reveal underlying geometric structures leaving space to be filled with different kinds of ornamental elements. In fact they are non-figurative designs arisen from religious proscription [Abas and Salman, 1995, p.9]. The equilateral triangle, the square and the hexagon are three fundamental shapes associated with the Islamic cultural context. The square symbolises the earth or materiality while the equilateral triangle and the hexagon are regarded as the human consciousness and the heaven, respectively [Critchlow, 1976, p.24]. Stylised motifs originally derived from nature are mainly used to embellish rigid geometric structures [Albarn, 1974, pp.34-35]. Commenting on Islamic geometrical designs Kappraff stated that:

> *" A sacred art is not necessarily made of images... Ornamentation with abstract forms enhances contemplation through its unbroken rhythm and endless interweaving... Continuity of interlacement invites the eye to follow it, and vision is transformed into rhythmic experience accompanied by the intellectual satisfaction given by the geometric regularity of the whole... "*
>
> [Kappraff, 1991, p.202]

Designs are basically developed on the polygonal networks which involve the symmetrical and proportional space sub-division within each repeating unit and the relationship of all units on the associated lattice, as mentioned previously in section 4.3.1. Certain patterning techniques have been employed to transform these polygonal networks to a variety of traditional Islamic designs.

To obtain an openwork pattern every structure line is embolden equally to distinguish a net-liked design at a foreground from polygonal intervals at a background. Complex layered-design can be created by a interlacing technique where strands from different directions pass over and under each other, as shown in Figure 4.27b. Both openwork and interlacing patterns exhibit continuous linear structures, while mosaic patterns reveal a modular filling technique in which assorted colours and decorative elements are applied individually unit by unit.

Vasarely: Geometric Abstraction

By using a repetition of identical or related geometric shapes, the painter, Victor Vasarely, took a further step to synthesise Impressionism and Cubism through the geometric abstraction designs [Spies, 1971, p.5]. Regarding his two-dimensional graphic studies (1933-1938), Vasarely remarked that:

> " *The two dimensions are far from being exhausted. Does not optics, even in the form of illusion, belong to kinetics? Does not aggressing the retina in fact make it vibrate?* "
>
> [cited by Joray, 1965, pp. 68,112]

His constructive geometric designs whether in black and white or multi-coloured illustrations portray the re-arrangement of regular repeating elements. Transformation in terms of unit orientation, gradation of shapes, scales and colours, as well as the optical illusion of black and white contrast, is the technique intentionally introduced to substitute static sense of a regular repetition to visual dynamic movement. The effects of these techniques, however, interrupt the regularity of the initial geometric frameworks. Each transformed unit seems to be an individual assembly packed together to form a configuration shape or design.

Hat 1 (1971) shown in Figure 4.27c, for instance, consists of a number of subsidiary rhombic units which are repeated concentrically on the isometric grid to form the three-dimensional effect of a cube at the centre and a hexagon as the whole figure. Perfect colour combination especially in term of value is also another significant factor strengthening three-dimensional visual illusion of this design.

M.C. Escher: Chaotic Figures on Rigid Geometric Lattices

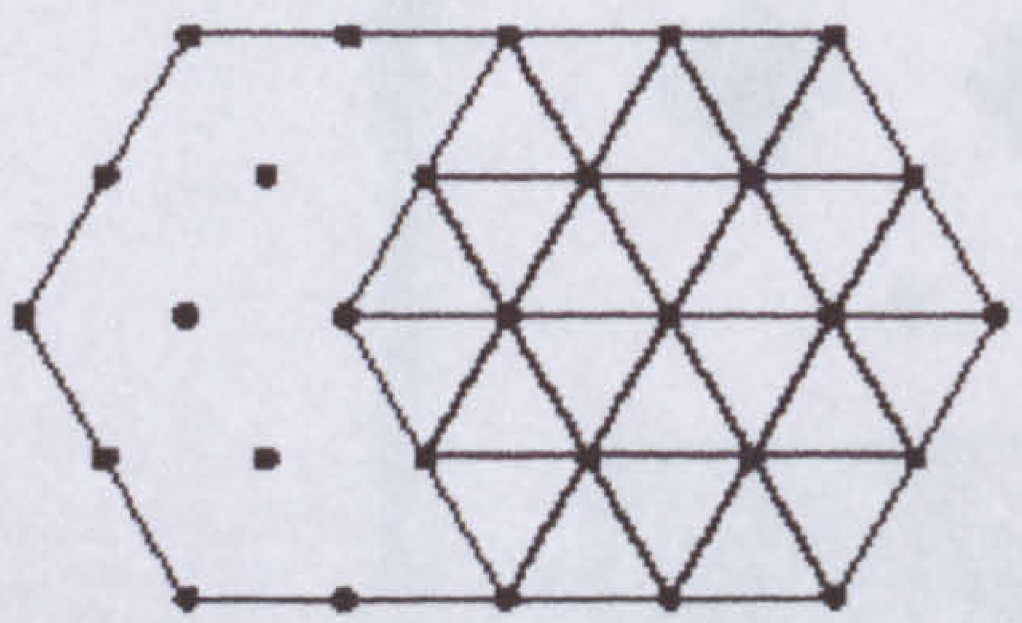
The Dutch graphic artist, M.C. Escher, was the pioneer who merged visual art with logical mathematical principles. Beginning with a fascination in the Moorish decorative tilings of the Alhambra in Spain, he turned rigid geometrical networks of those patterns into a variety of vivacious patterns of the fantasy world. As Schattschneider commented:

> *" Escher's use of symmetry was unusual in many respects: he did not merely fit congruent shapes together to form decorative patterns, but made recognisable shapes in contrasting colours whose interpretation challenge the viewer's sense of perception. "*
>
> [Schattschneider, 1990, p. ix]

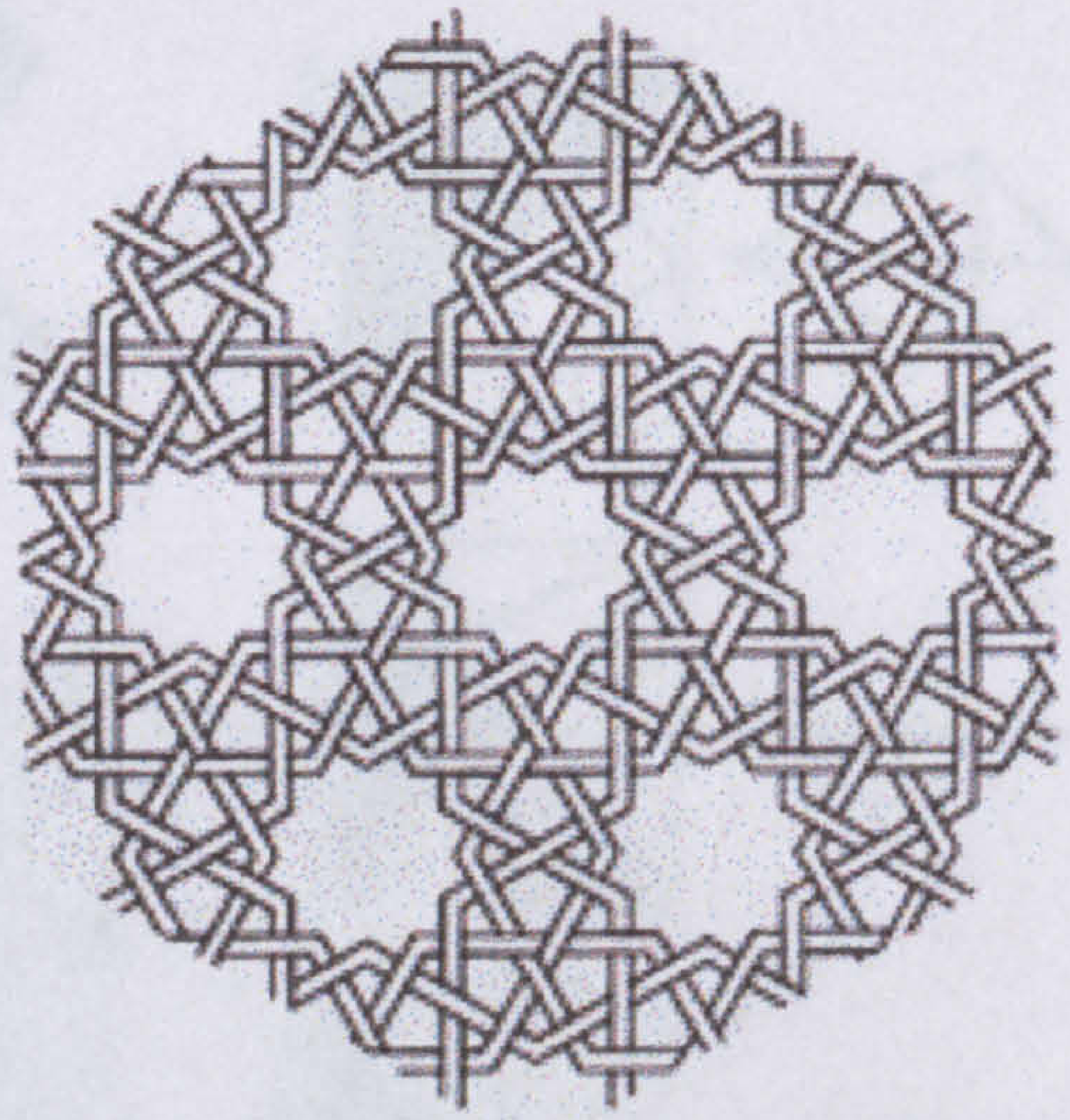
Escher established his own symmetry classification system under a title *The Regular Division of the Plane* to explore how the plane can be filled with congruent figures in a regular manner. An experiment on numerous patterns was carried out with the geometric symmetry concern. Certain factors, e.g. symmetry characteristics in terms of symmetry operations and orientation, repeating units in terms of a number of constituent motifs and organisation, and a number of colours for colour-counterchange, were focused.

As Ernst pointed out Escher's investigation on the regular division of flat surface structure would be categorised into three design themes: i) *Metamorphoses*, which exhibits successive transformation of geometrical shapes to figurative motifs, e.g., *Verbum*, 1942, as shown in Figure 4.27d, ii) *Cycles*, by which an identical motif or a set of motifs is repeated regularly on the infinite plane, e.g., *Reptiles*, 1943, and iii) *Approaches to Infinity*, by which a series of related motifs in different scales and proportions is packed together to generate a configuration design with three-dimensional effect, e.g., *Circle Limit II*, 1959 [Ernst, 1994, pp.20-23]

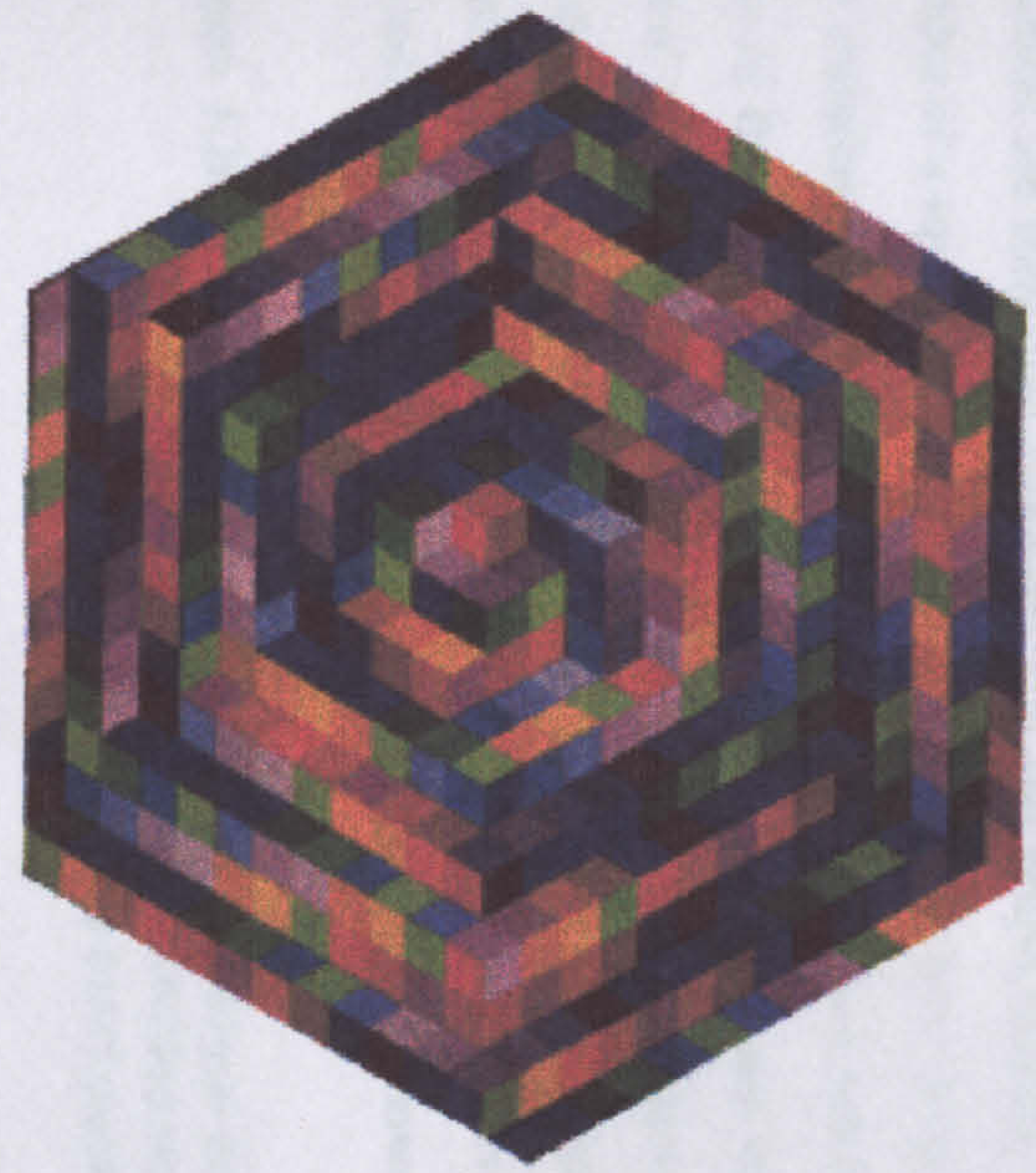
Fantasy and imagination are the essential matters involving three main aspects, i.e., the recognisability of the motifs, the use of contrast colours, and the dynamic evolution and transition of forms [Schattschneider, 1990, p.29]. Polygonal shapes of constituent units within geometric lattices are thus replaced by interlocking shapes of recognisable creatures or objects. Visual dynamic balance is created upon the patterns without backgrounds or the compositions in which figures and backgrounds change functions alternatively at every stage [Bool, 1986, p.19]. Sharp contrast colours especially black and white are used to ensure individuality and equal value of adjacent motifs and emphasise successive sequences of motif transition changes (e.g. from birds to fishes).



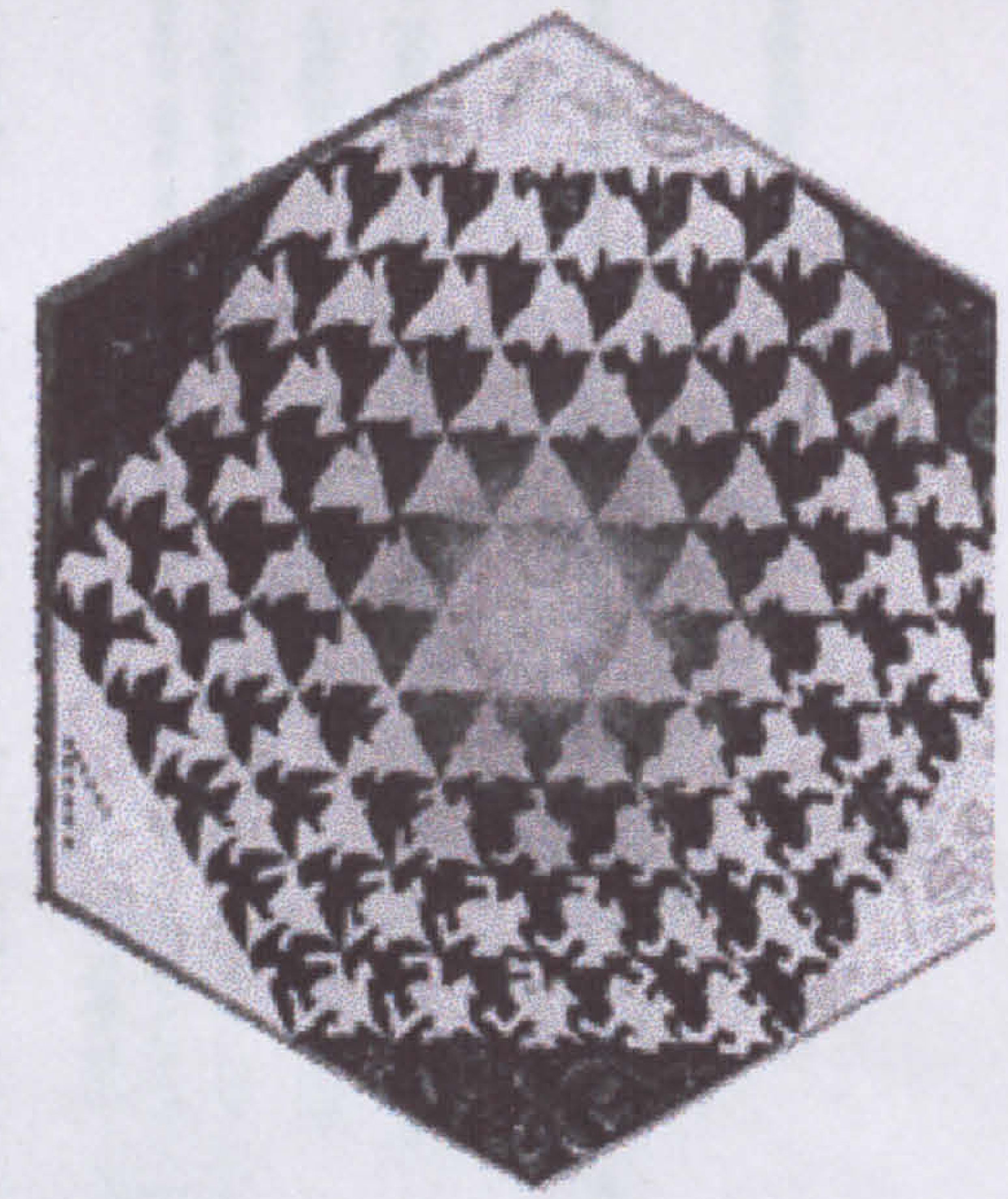
a) A hexagonal shape containing an isometric lattice



b) An Islamic interlacing pattern depicting the construction of seven twelve-pointed stars in a hexagonal shape.
Source: reproduced from Wade, 1976, p.85



c) *Hat* by Vasarely:
three-dimensional visual illusion derived from a concentric series of rhombic shapes.
Source: reproduced from Spies, 1971, p.172



d) *Verbum* by Escher:
transformation of an isometric lattice to an interlocking pattern of recognisable figures.
Source: reproduced from Schattschneider, 1990, p.259

Figure 4.27a-d Three design varieties generated on isometric lattices within the hexagonal shapes

4.5.2 Three-dimensional Objects

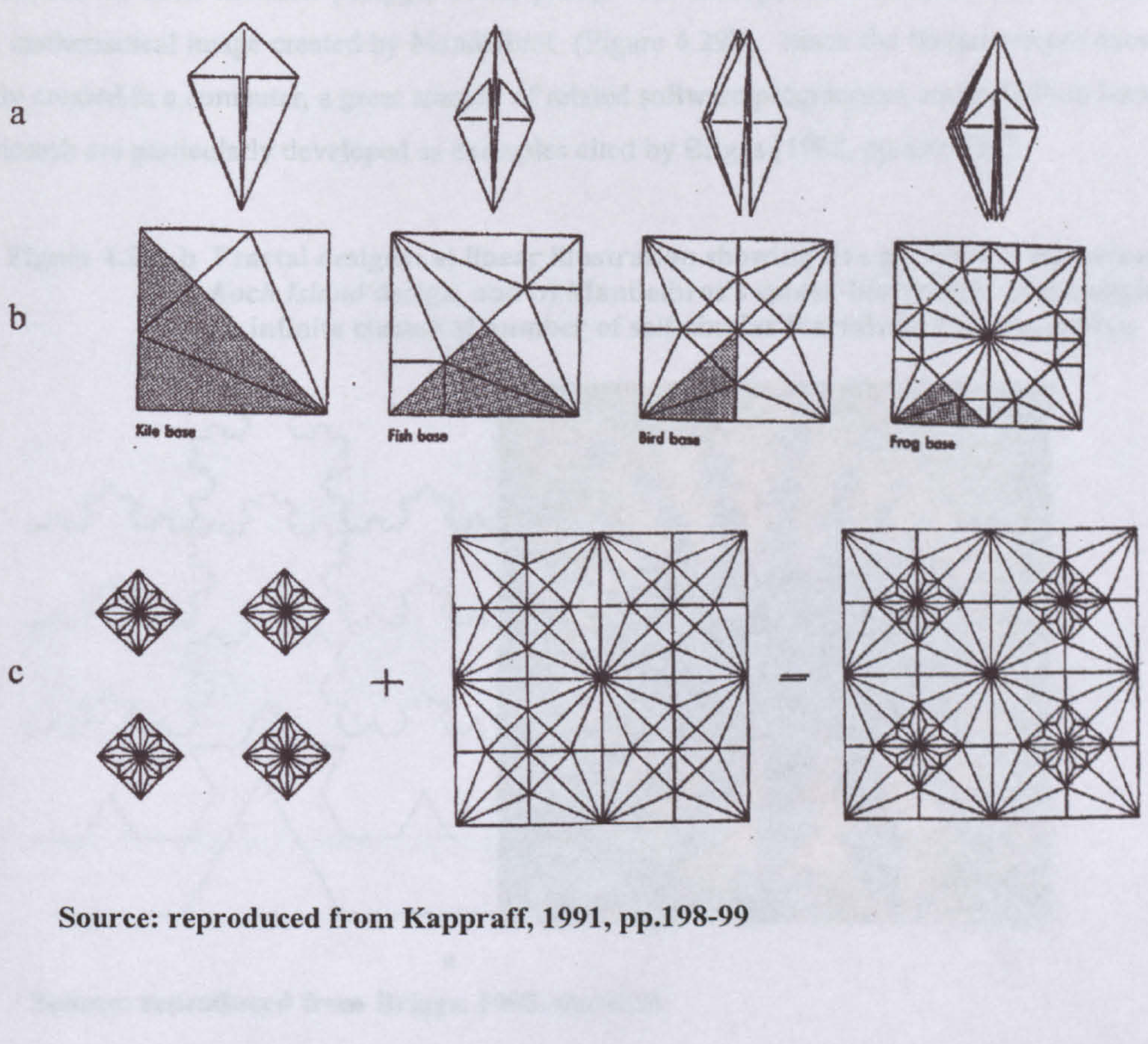
Origami: From Two-dimensional Sheets to Three-dimensional Figures

Origami is a paper folding art originally invented by the Chinese. A three dimensional figure is created by folding a single square sheet of paper. As stated by Kappraff:

> *" Traditional origami uses four basic folded bases: the kite, fish, bird and frog. ...when these bases are unfolded, they reveal a sequence of geometric patterns based on a single module."*
> [Kappraff, 1991, p.198]

The folding guidelines on the unfolded paper sheet exhibit space sub-division by a series of straight lines. Folded structures are formatted symmetrically and proportionately with respect to the right angles and four equal sides of a square shape. Considering the four basic shapes (Figure 4.28 row a-b), a kite as the prime shape contains two identical 30°-60°-90° triangles as a result of space sub-division along the 45° diagonal line of the square shape. Both triangular modules are mirrored images of each other. Many more identical modules are required to construct the other three designs in the same manner. Four modules complete a fish while eight and sixteen modules fulfil the bird and frog respectively. Combinations of more than one of four fundamental bases arise a variety of designs, e.g., a combination of four frog bases onto four bird bases produces an octopus, as shown in Figure 4.28 row c.

Figure 4.28a-c Four fundamental bases of traditional origami: kite, fish, bird and frog in row a) folded figures, row b) unfolded sheets, and row c) a combination of four frog bases onto four bird bases produces an octopus.



Source: reproduced from Kappraff, 1991, pp.198-99

4.5.3 Computer-generated Images

During the last few decades of the late 20th century there was much fascination in computer-generated images containing repetitions of uniform or related elements in certain regular arrangements. Four innovations, i.e., Fractal designs, Stereogram, Photomosaics and Maeda's computer-coded graphics, as mentioned below, reveal the development of designs along side the technological achievement of matching software programmes.

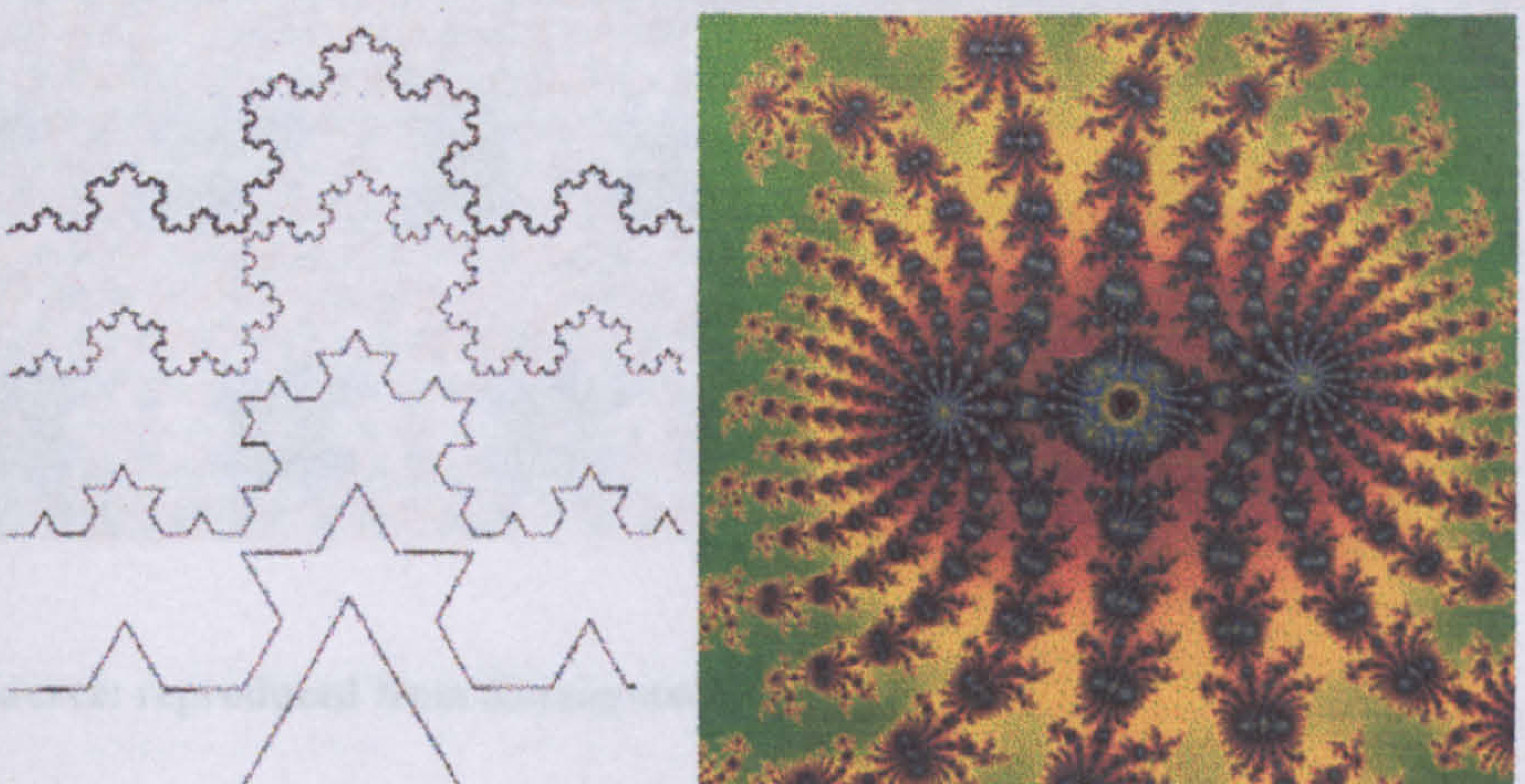
Fractal Designs

Traced back to the 1960s, Benoit Mandelbrot, an IBM researcher, discovered the new geometrical non-linear phenomenon which he called fractal geometry.

> *" Fractals are geometric shapes that are equally complex in their details as in their overall form. That is, if a piece of a fractal is suitably magnified to become the same size as the whole, it should look like the whole, either exactly, or perhaps only after a slight limited deformation."*
>
> [Mandelbrot, 1993, p.12]

The concept covers a great deal of substance ranging from visual forgeries of nature, e.g., snowflakes, cauliflower, tree leaves and branches, and landscape surfaces, to invisible microscopic cross sections of DNA molecules or turbulent strands of gas and fluid. The significant matter co-existing in all of them is the repetition of self-similar shapes at a variety of scales, which are orderly packed. The orderness, as a result, characterises the artistic chaos of the final images. A series of lines in Figure 4.29a demonstrate the generating sequence of *Koch island* fractal by consecutively adding a triangle to the middle of every straight section at each iteration [Briggs, 1992, p.66]. An example also includes the world's most complex mathematical image created by Mandelbrot (Figure 4.29b). Since the fractal images have been practically created in a computer, a great amount of related software programmes compatible to both IBM and Macintosh are particularly developed as examples cited by Briggs [1992, pp.182-183].

Figure 4.29a,b Fractal designs: a) linear illustration showing five generating sequences of *Koch Island* design, and b) Mandelbrot's spider-like fractal image depicting an infinite cluster of number of self-similar fractals in different scales.



a b

Source: reproduced from Briggs, 1992, pp.66,26

Stereogram

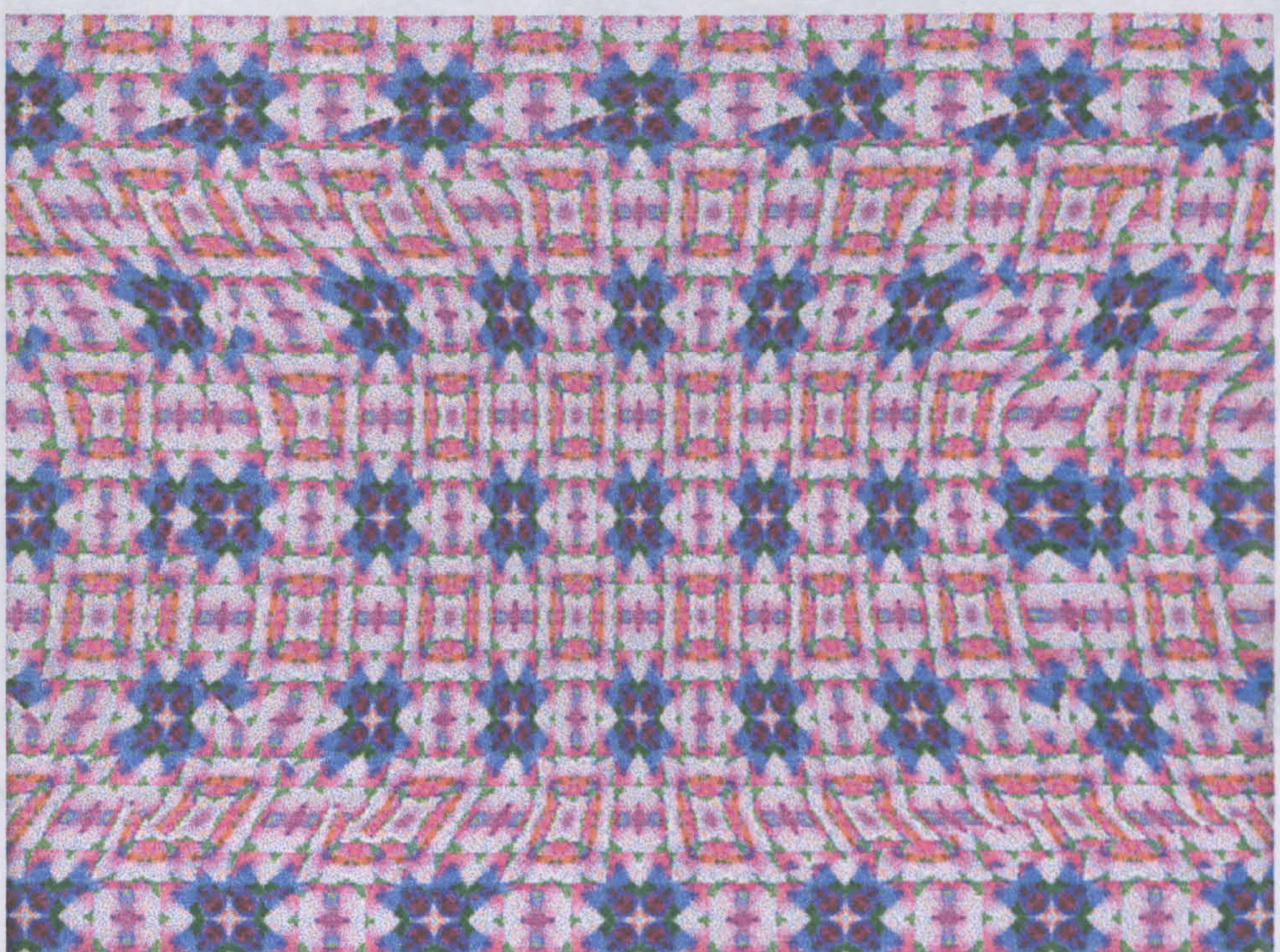
The stereogram reveals the technological invention of perceiving three-dimensional images concealed in two-dimensional graphics. The principle of stereovision was established by Charles Wheatstone in the 1830s as the fact that:

> *" The small differences between the images projected to the two eyes supply a vivid sense of the depth of 3D space."*
>
> [cited by Tyler, 1994, p.83]

Basically the stereogram images contain a number of repeating units of either recognisable or un-recognisable motifs, each of which, however, have a slight difference in perspective.

By using either parallel or cross-eyed viewing techniques, the human brain is able to convert the visual perceptions from two-dimensional graphics into illusory three-dimensional images [Rheingold, 1994, p.6]. The concept of stereovision intrigues a great deal of psychologists, graphic designers and computer programmers to develop further advances in this field. The random-dot stereograms, for example, was invented by Bela Julesz in 1960 [Tyler, 1994, p.83]. An example includes *Poly 2000*, a three-dimensional graphic of polyhedrons consisting of regular triangles, created by Shiro Nakayama shown in Figure 4.30.

Figure 4.30 *Poly 2000*, stereogram created by Shiro Nakayama



Source: reproduced from *Stereogram*, 1994, p.16

Photomosaic

Robert Silvers has developed a software programme to generate the photomosaic, an image made up of thousands of tiny pictures [Hawley, 1997, pp.10-11 and http://www.photomosaic.com]. A digitised original image is initially divided into a regular geometric grid, usually being as a rectangle, in which a number of selected photographs from the database are filled: one cell contains one image.

Similar to the principle of image pixel-construction, assembled images are arranged according to their colours, textures and element orientations to match with each individual cell of the original image. The assembled images usually have graphic meanings associated with the original image as seen from a portrait of Marilyn Monroe on the cover of Life magazine comprising of hundreds of former Life covers, or a portrait of the Liberty Statue containing hundreds of people from different genres as shown in Figure 4.31.

Figure 4.31 ***Liberty*****, photomosaic created by Robert Silvers**

Source: reproduced from Hawley, 1997, p.10

Maeda's Computer-code Graphics

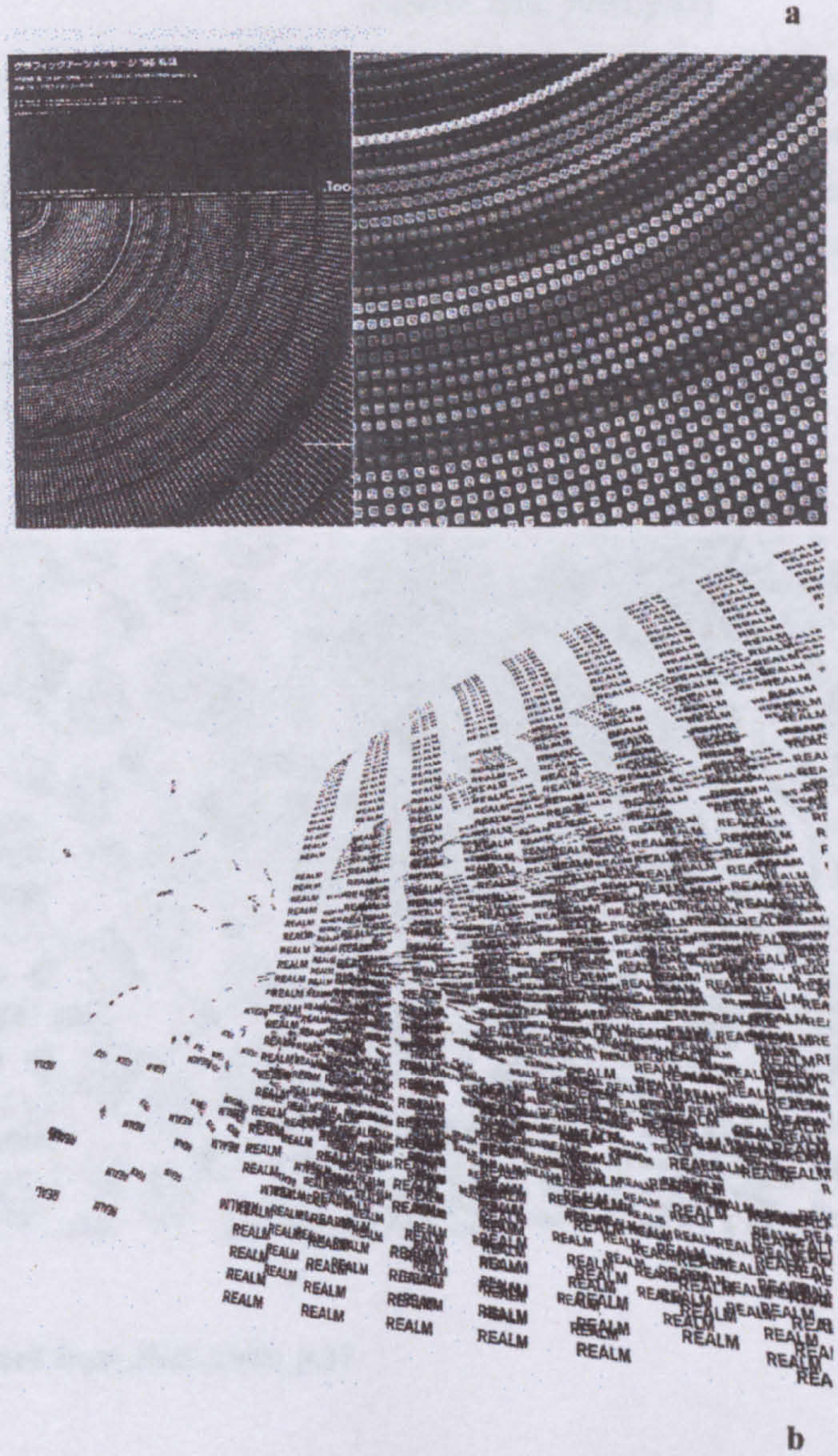
The marriage between innovative graphics and generating software is apparent from Maeda's designs [http://www.maedastudio.com]. In this case the image-pixels are substituted by a variety of computer codes. As Jacobs noted:

> *" Maeda's absorption with the minutiae of code-writing is similar to a painter's obsession with the gradations of brush strokes. He loves patterns formed by repetition and often fills a page or screen with thousands of words, numbers of lines."*
>
> [Jacobs, 1998, p.62]

Series of digitised codes are possibly arranged and manipulated on either regular grids or any structures, e.g., curves, gradation formats or object outlines, to create configuration designs which have three-dimensional effects. For example, the image in the poster produced for a Japanese marker maker (Figure 4.32a) depicts an immense complexity of a concentric relief generated from series of concentric rings, each of which contains repetition of tops of pen-cabs of the same colour. Meanwhile, an illustration for *Gilbert Paper* (Figure 4.32b) establishes a sense of loss of dimensionality by a multi-directional series of a single word "REALM".

Figure 4.32a,b Maeda's computer-code graphics: a) a poster depicting a concentric configuration design made up of series of tops of marker cabs, and b) illustration for Gilbert Paper depicting repetition of a word "REALM" in multiple directions

a



b

Source: reproduced from Jacobs, 1998, pp.60,61,64

4.5.4 Contemporary Household Products

A great deal of CAD programmes have been developed particularly to complete individual types of designs. Nonetheless, common computer-graphic programmes provide the open-ended results in which designers can apply to fit a diversity of designs in a multi-disciplinary area. Two interior products mentioned below are examples revealing two possibilities to apply simple symmetry operations to common repeating elements.

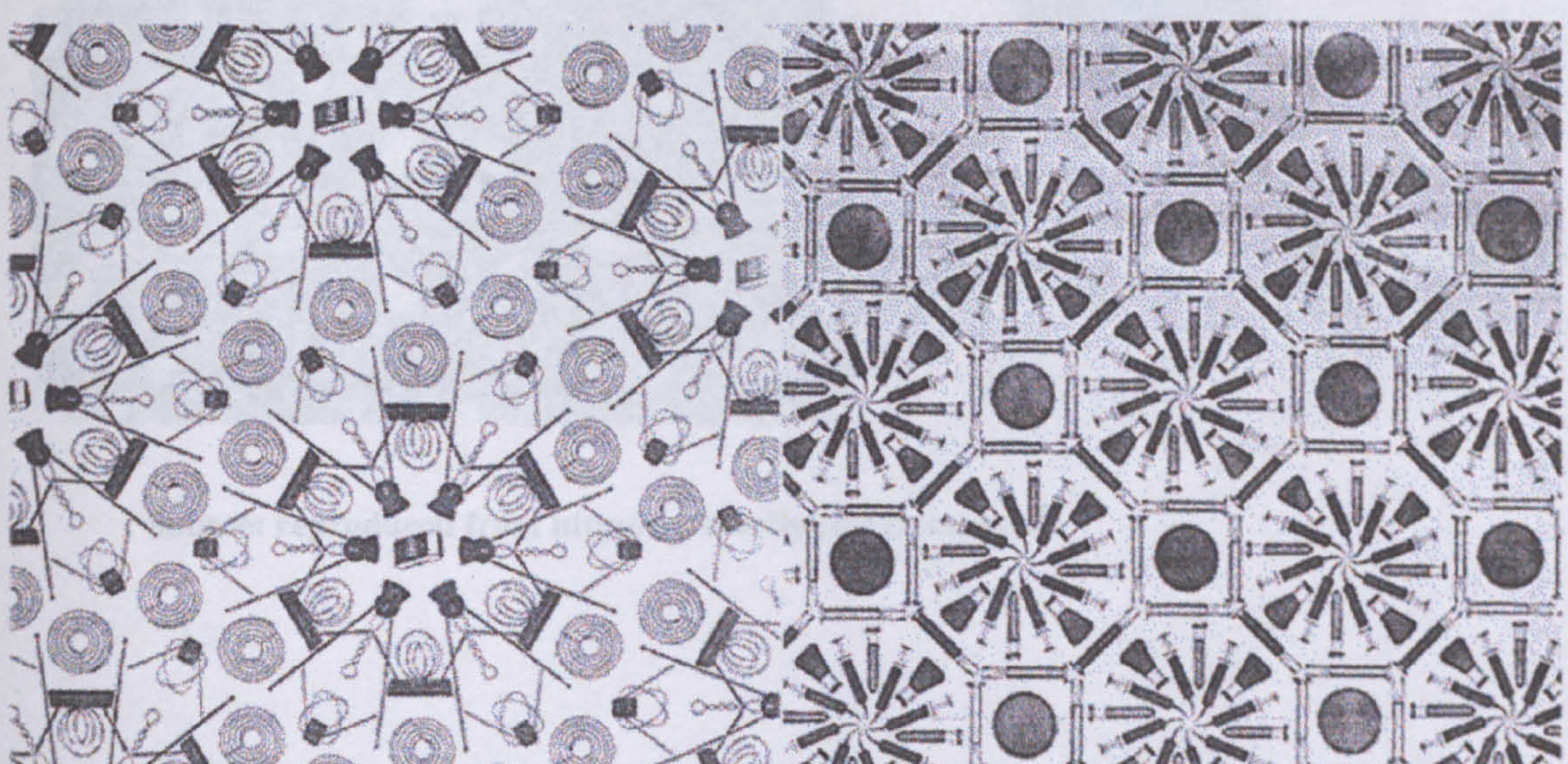
Davies + Starr's Wallpaper

The idea of this collection is generated from the difference of visual perception between all-over patterns and repeating elements. Repetition of one thing may create a pattern, which is recognised as the other thing. As Chalkie Davies, the photographer, described one design:

> *"It looked beautiful from a distance, like a heart but when you got closer you could see it was an incredible vicious knife."*
>
> [cited by Hall, 1999, p.27]

Unusual objects, e.g., chain saws, spiked heels, TV antennae, burning matches, Concorde or space rockets are introduced as repeating elements to place on the wall. Straightforward use of symmetry operations is applied to build up configuration forms and at the same time create dynamic visual perception within the patterns, as evidenced as six- and four-fold rotation symmetries in two designs in Figure 4.33a,b.

Figure 4.33a,b Davies+Starr's wallpaper patterns: a) a six-fold rotation of TV antennae, and b) a four-fold rotation of laboratory equipment



a b

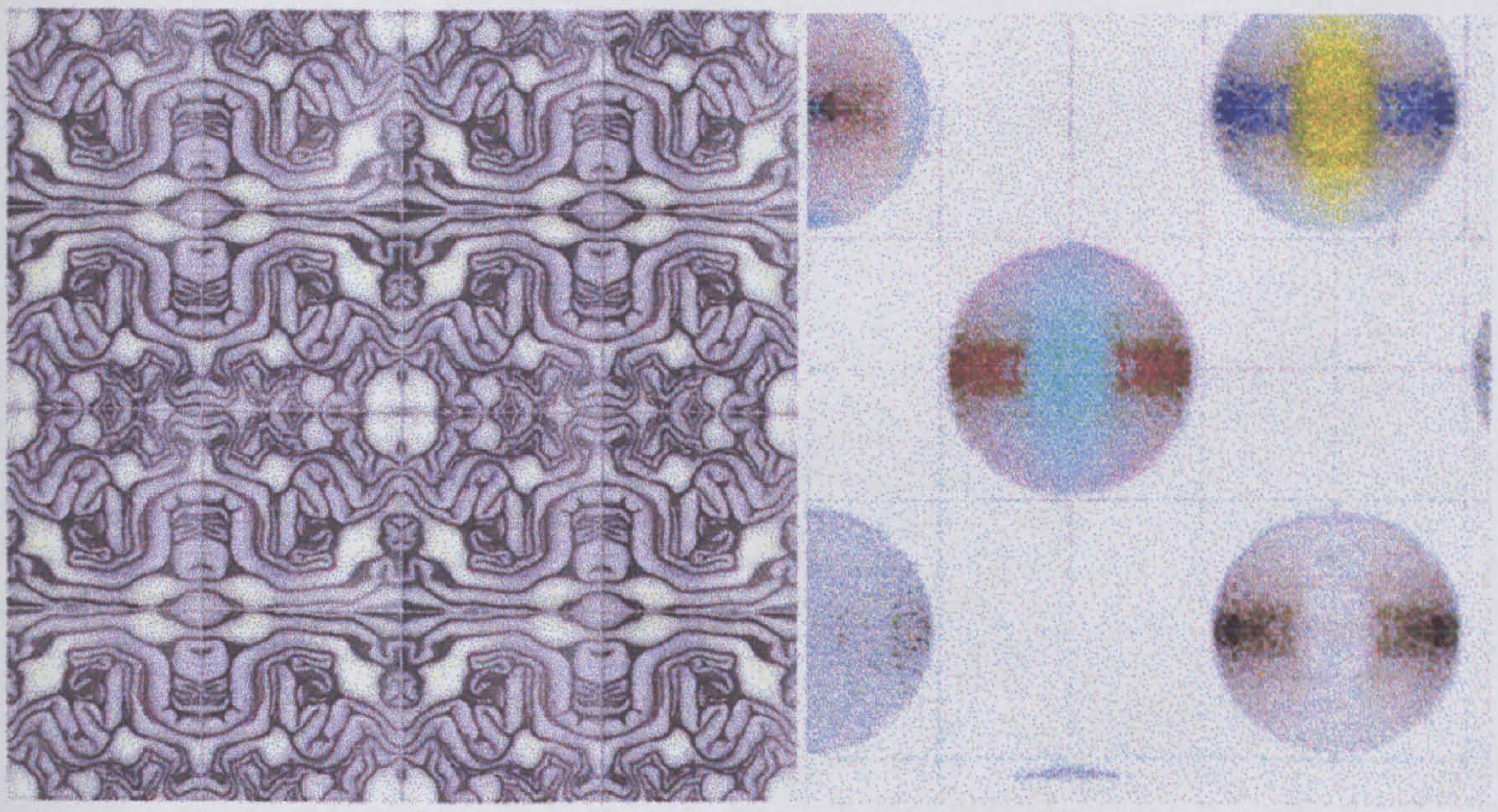
Source: reproduced from Hall, 1999, p.27

Crinson's Digitiles

A collection of ceramic wall tiles designed by Dominic Crinson reveals the integration of computer-generated patterns and conventional ceramic printing [http://www.digitile.ac.uk]. Visual materials are introduced in terms of their textures and closed-up surfaces, e.g. bubble surface, glass surface and cabbage section. They are scanned, modified, cropped in the square boundaries (standard sizes: 10, 15 and 20 cm.) and then arranged. Due to the nature of the modular system, a set of uniform units is reused regularly in different arrangements to create a variety of designs. Each pattern may consist of either a repetition of identical elements having reflection or rotation symmetries or a combination of assorted elements.

A cabbage pattern in Figure 4.34a, for example, obtains reflection symmetry at all sides of each square unit as a means to repeat itself to adjacent regions. Reflection symmetry around the unit edges of cropped elements produces interesting configuration designs. An isolated spot pattern in Figure 4.34b is made up of a set of four circle-quarters admitting perpendicular reflections.

Figure 4.34a,b Crinson's digital ceramic tiles: a) a cabbage pattern and b) a spot pattern



a **b**

Source: reproduced from http://www.crinson.demon.ac.uk

"Geometry the basis of all patterns – Breaks in the simple stripe give cross-lines – Hence the lattice and the chequer, on which a vast variety of patterns is built."

[Day, 1903, reprinted 1979, p.10]

Chapter 5 Design Variations Generated from the Seventeen All-over Pattern Symmetry Groups

5.1 Introduction

Symmetry and regularity in patterns involve the repetition of uniform designs underlaid by the physical laws of two-dimensional geometric construction. There are a total of seventeen distinct classes of all-over patterns generated by combinations of one or more of four symmetry operations. Although the number of symmetry groups is limited, there is an extensive variety of designs possible within each symmetry group due to various features. For example, two-colour counterchange may be developed onto the seventeen classes, and this leads to a total of forty-six classes of counterchange patterns.

This chapter explores design variations based on the seventeen symmetry groups of all-over patterns. The primary focus is on the combination of symmetry groups within unit cells and symmetry operations between units produced by individual repeating formats. Combinations of two/three features of unit cells and six repeating formats applied to all seventeen symmetry groups produce a total of 124 representative patterns for analysis. These and related concepts are explored further in this chapter.

5.2 Sources of Data

It is noted that a band pattern is generated from a linear repetition of uniform units in one direction. An all-over pattern has two alternative constructions, either a repetition of uniform units in two non-parallel directions or a construction from a series of band patterns. Symmetry connection between units and bands identify symmetry groups to which they belong. Each of the seventeen symmetry classes of all-over patterns obtains an individual symmetry group, which underlies the organisation of determined numbers of fundamental regions. Design elements and a symmetry group contained in a unit cell are the critical elements that may affect a variety of designs within the same symmetry group.

Based on the investigation of variations of finite designs of classes cn and dn, seven symmetry groups of band patterns and seventeen symmetry groups of all-over patterns generated using varieties of finite designs of classes cn and dn (Chapter three), Horne remarked:

> *" The classification by symmetry group of design structure and design unit provides a new approach to design analysis. Some of the designs constructed from this classification system seem to exhibit a more "chaotic" or "random" appearance depending on the symmetry group and subgroup."*
>
> [Horne, 1997, p.133]

The concept initiated basically from the fact that the fundamental region is not necessarily an asymmetrical motif in every case. It may contain multiple motifs admitting n-fold rotation or reflection, known as a finite design of class cn (n≥2) and a finite design of class dn (n≥1).

Different numbers of n-fold rotation and reflection within a unit produce different design features within the same symmetry group, e.g., varieties of finite designs of class d2 may be generated from finite designs of classes c2, c3, c4,..., etc., or varieties of band pattern class p2 may be generated from finite designs of classes d1, d2, d3,..., etc.

The interrelation between certain symmetry groups of finite designs, seven symmetry classes of band patterns and seventeen symmetry classes of all-over patterns indicates that one design may be the special case of another one, which may require a particular symmetry orientation. For example, a band pattern class p111 generated using finite designs of class d2 may be identical to a band pattern class pmm2 if the perpendicular reflection axes within units correspond to the perpendicular axes of a band. In the same manner, an all-over pattern class p1 generated using finite design of class c3 may be identical to an all-over pattern class p3, if the unit cell undergoes repetition on a hexagonal lattice.

However, varieties of fundamental regions admitting other symmetry operations apart from n-fold rotation and reflection of finite designs of classes cn and dn have not been discussed. Also, varieties of repeating formats have not been taken into account as the variant that may affect the symmetry position changes between units.

5.2.1 Unit Content

Accepting that a repeating unit is a uniform region undergone by translation in two non-parallel directions to cover the plane all units must have the same shape and content. Nonetheless, there is an extensive variety of content possible within a unit. The examination of the unit content is focused, firstly, on the relationship of positive and negative spaces within and between units (the positive space is recognised as a figure whereas the negative space is considered as a background) and, secondly, on possible symmetry groups that the unit may contain.

The connection between positive and negative spaces along the unit edges indicates two categories of designs:

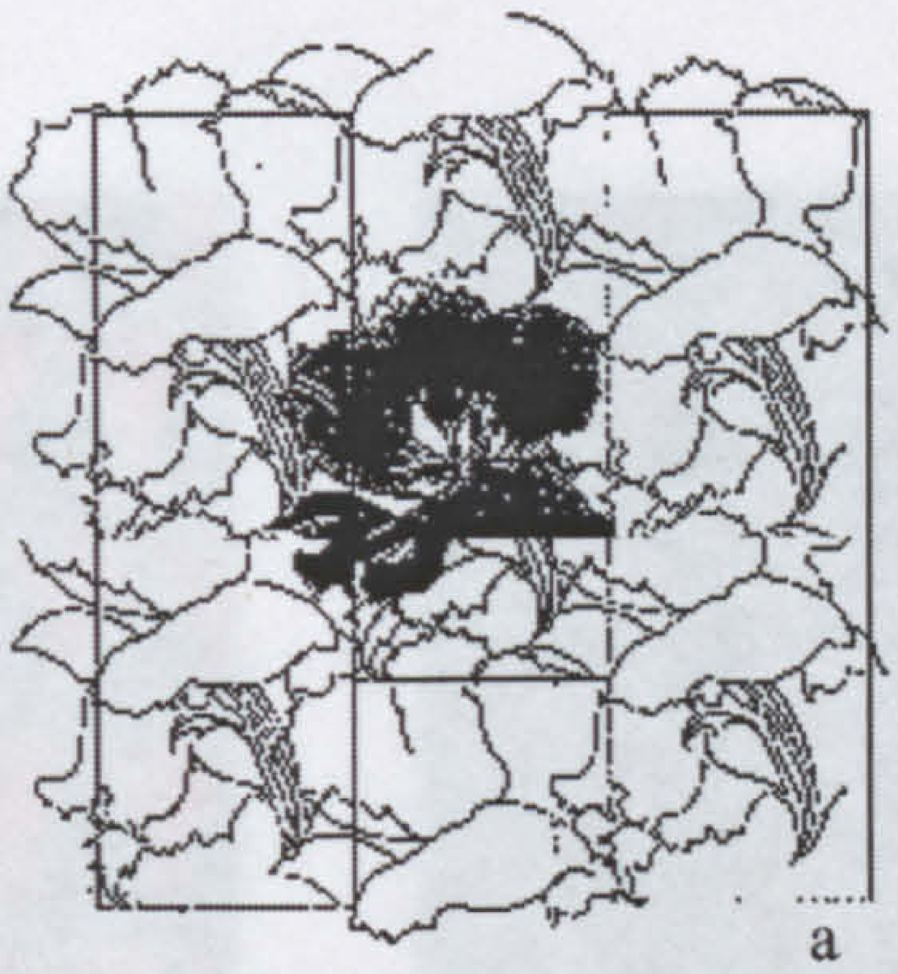
i) Disconnected design in which a pattern contains a repetition of individual units where there are no connections of figures between units. Examples include an isolated pattern or floating pattern containing detached motifs floating on a background space which flows continuously (Figure 5.1a), a patchwork-like pattern (Figure 5.1b) and an interlocking pattern (Figure 5.1c), in which each unit is filled entirely with design elements but there are no connections of elements between units.

Figure 5.1a-c Disconnected designs: a) an isolated pattern containing a set of identical motifs, b) a patchwork-like pattern containing three identical units and c) an interlocking pattern containing two-fold rotational motifs with two-colour counterchange

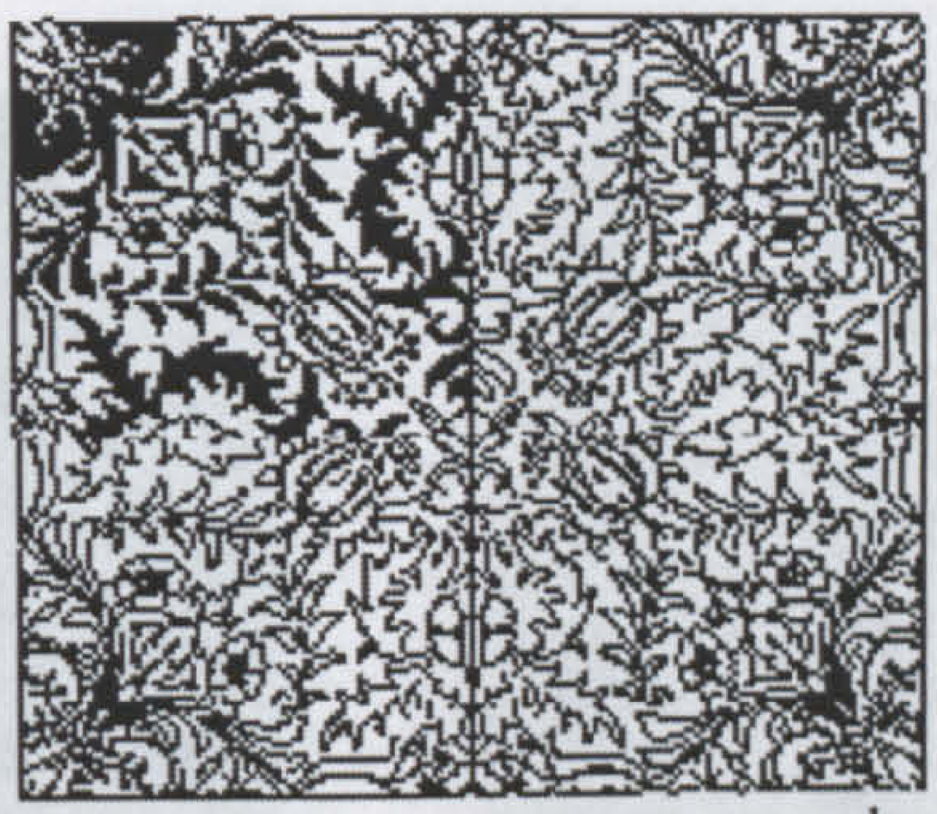
a b c

ii) Connected design in which a pattern exhibits the connection of figures and backgrounds along the unit edges by which close-shaped spaces and configuration designs may be produced. Examples include a continuous pattern (Figure 5.2a), a mirrored pattern (Figure 5.2b) and an interlocking pattern (Figure 5.2c).

Figure 5.2 a-c Connected designs: a) a continuous pattern plan in a half-drop repeat, b) a mirrored pattern and c) Escher's interlocking pattern containing two glide-reflection figures with two-colour counterchange



a



b

c

Sources: (a) and (b) are reproduced from Phillips and Bunce, 1993, pp.13,7, (c) is reproduced from Schattschneider, 1990, p.195

Connected features of design elements between units are significant matters that may restrict varieties of unit arrangement. Individual units of three disconnected designs (Figure 5.1a-c) may be rearranged in different repeating formats or by different symmetry groups to create a variety of isolated, patchwork-like and interlocking patterns. However, repeating units of three connected designs (Figure 5.2a-c) require particular structures to produce perfect connection at every pair of the unit edges. Patterns of the same symmetry group may look different due to the design elements contained in the fundamental regions. This is evidenced in three sets of seventeen symmetry classes of all-over patterns generated from three kinds of design elements, i.e., isolated motifs (Figure 5.3), series of lines (Figure 5.4) and groups of tiny ornaments (Figure 5.5). All illustrations were produced from Terrazzo programme [see Appendix A2].

Figure 5.3 Nine of seventeen symmetry classes of all-over patterns generated from isolated motifs

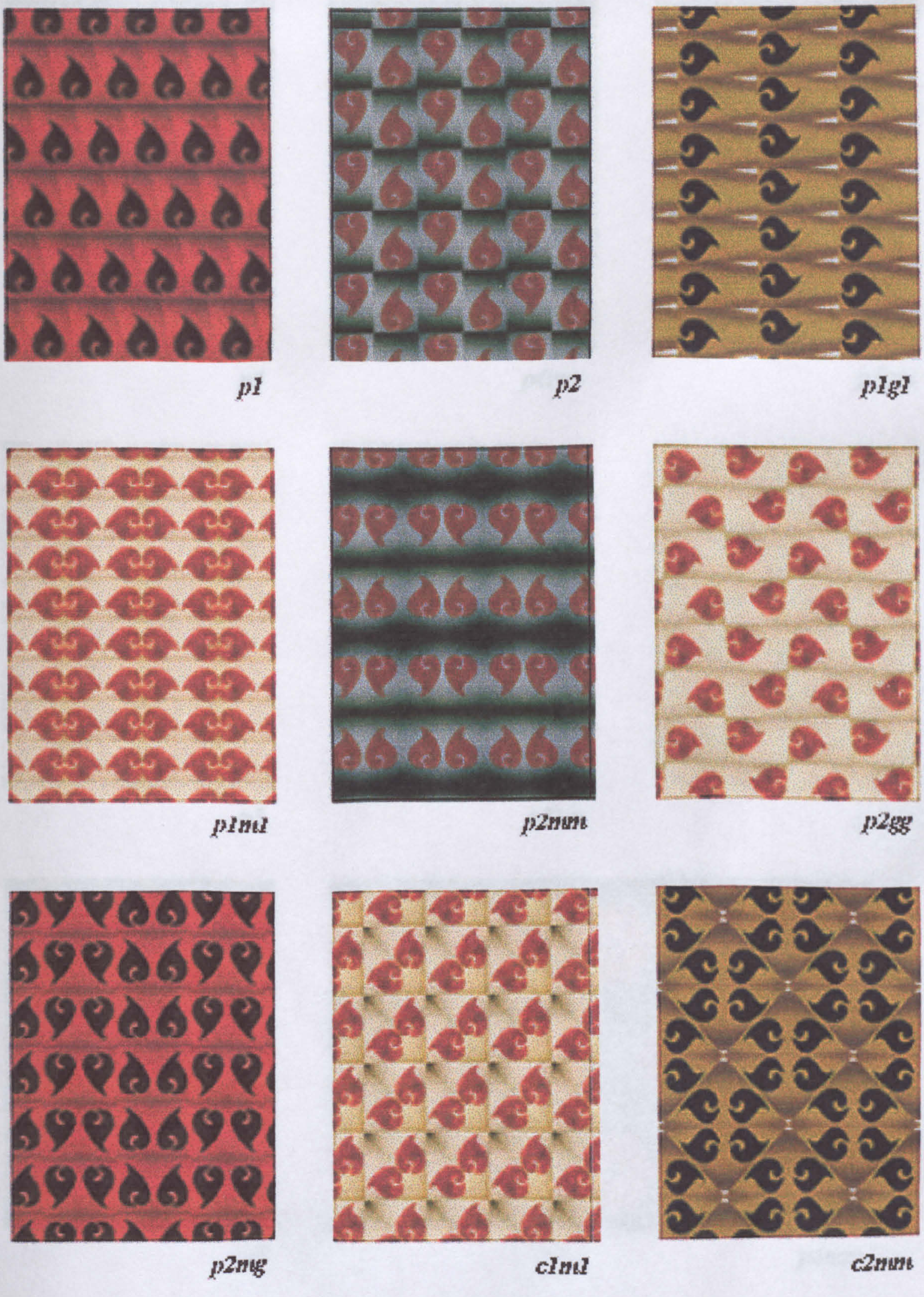


Figure 5.3 (continued) Eight of seventeen symmetry classes of all-over patterns generated from isolated motifs

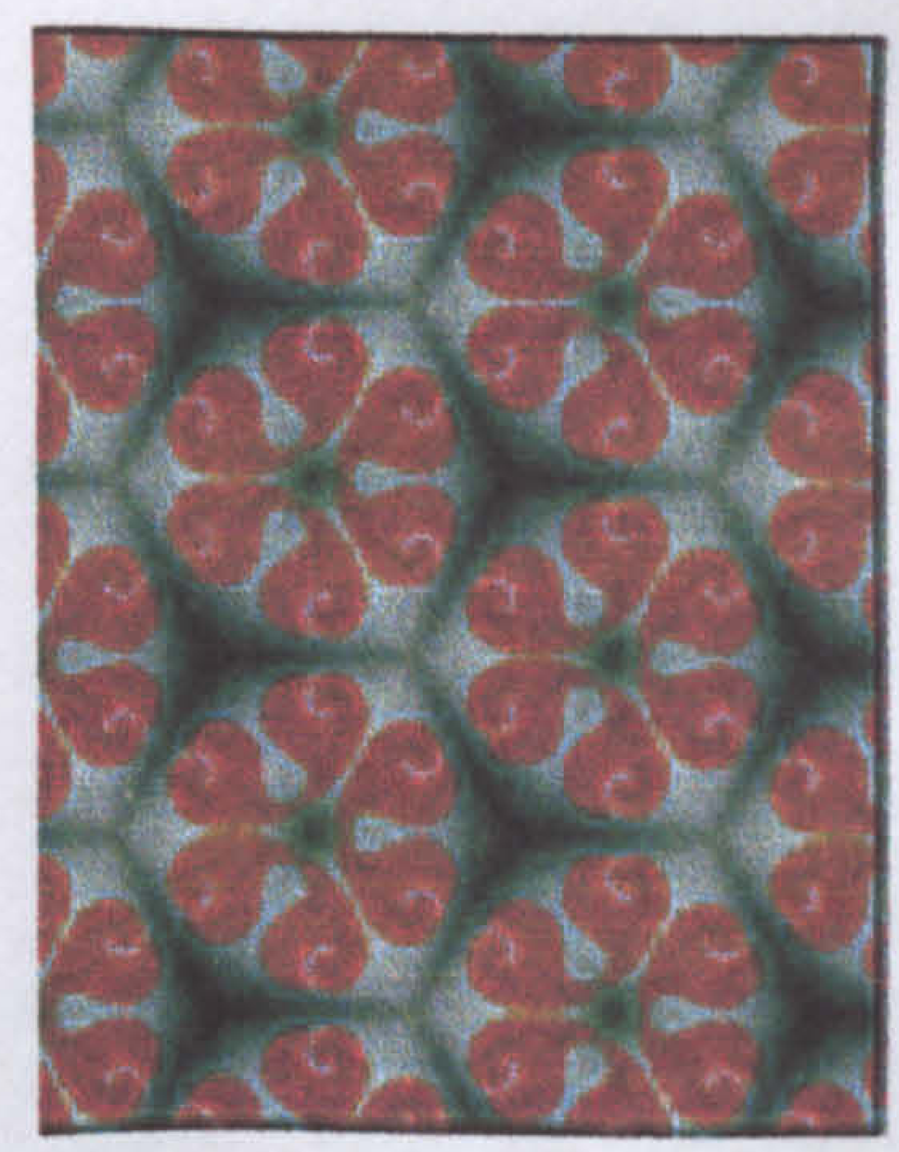
p4

p4mm

p4gm

p3

p31m



p3m1

p6

p6mm

Figure 5.4 Nine of seventeen symmetry classes of all-over patterns generated from series of lines

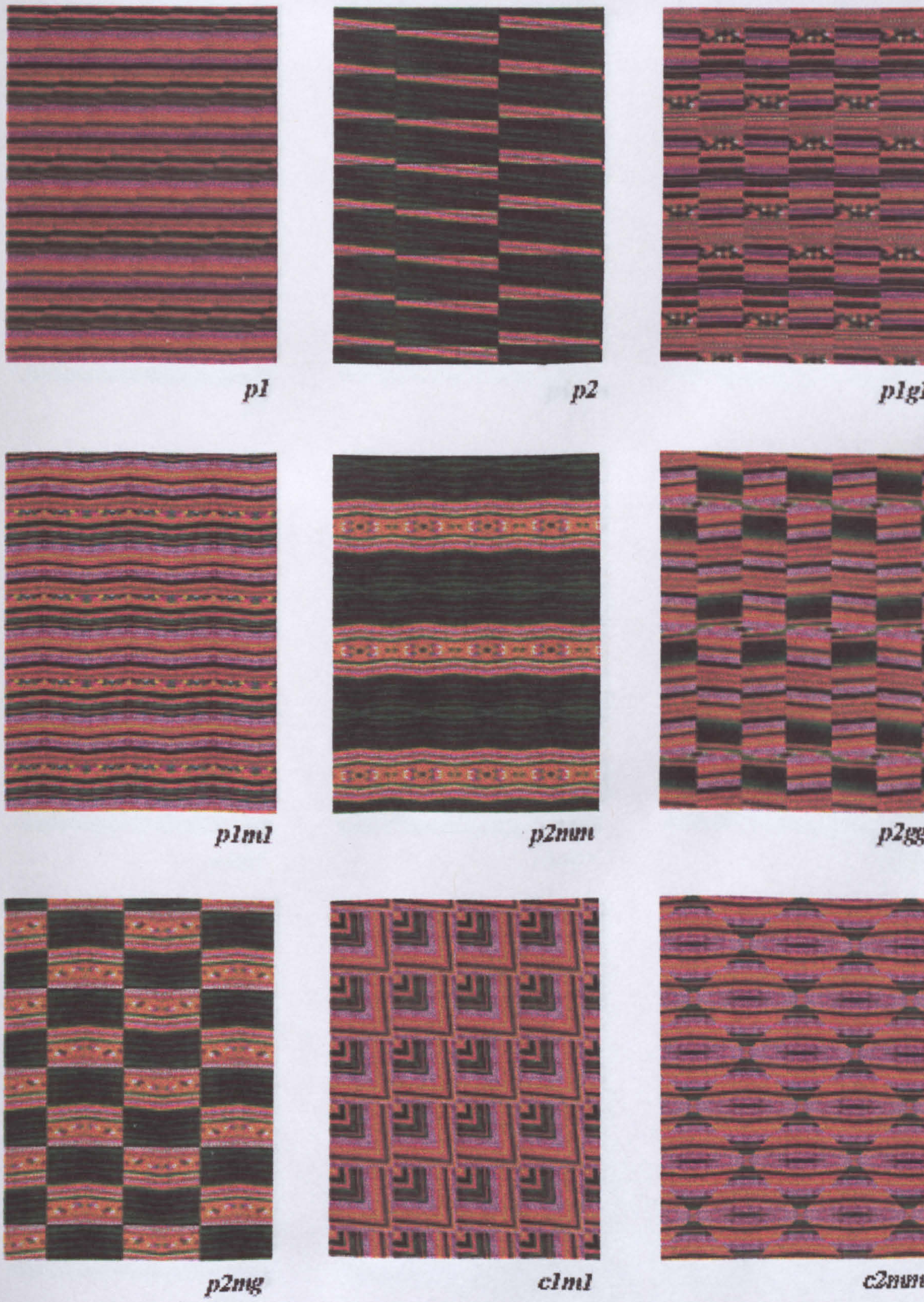


Figure 5.4 (continued) Eight of seventeen symmetry classes of all-over patterns generated from series of lines

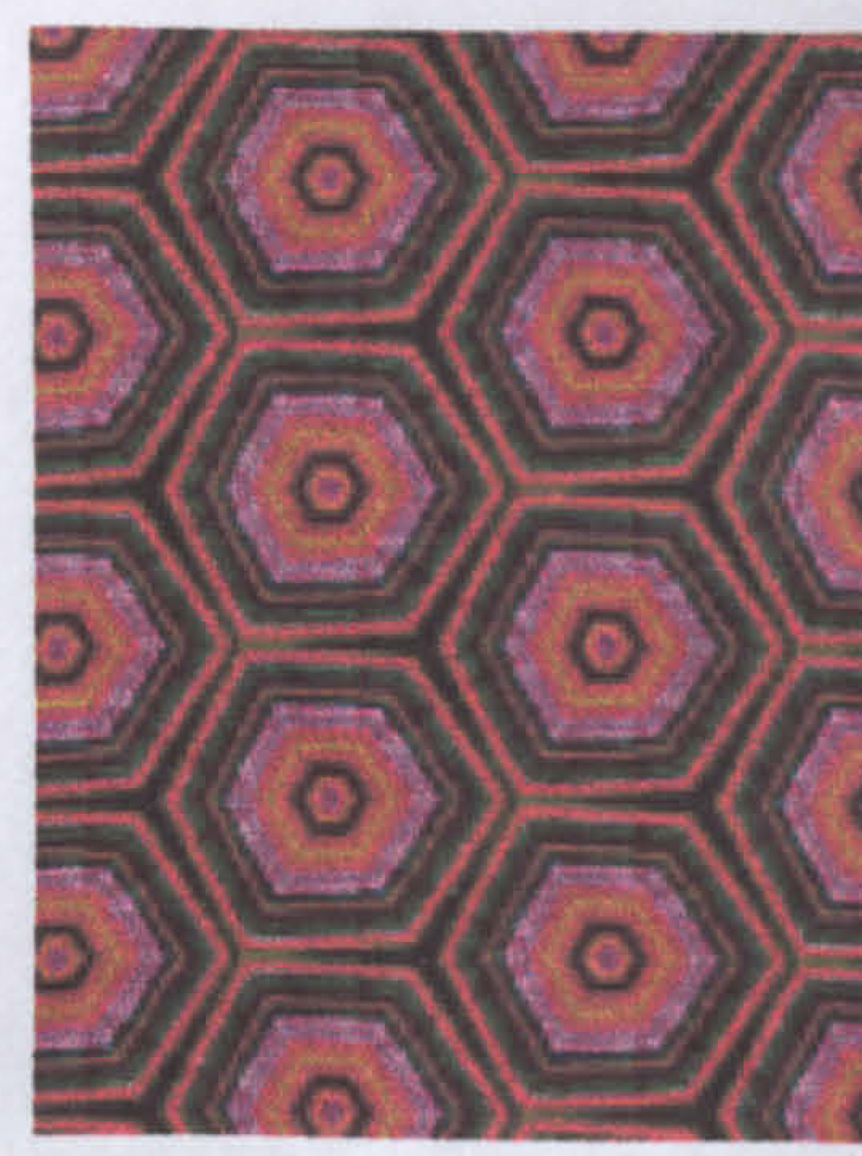
p4

p4mm

p4gm

p3

p31m



p3m1

p6

p6mm

Figure 5.5 Nine of seventeen symmetry classes of all-over patterns generated from groups of tiny ornaments

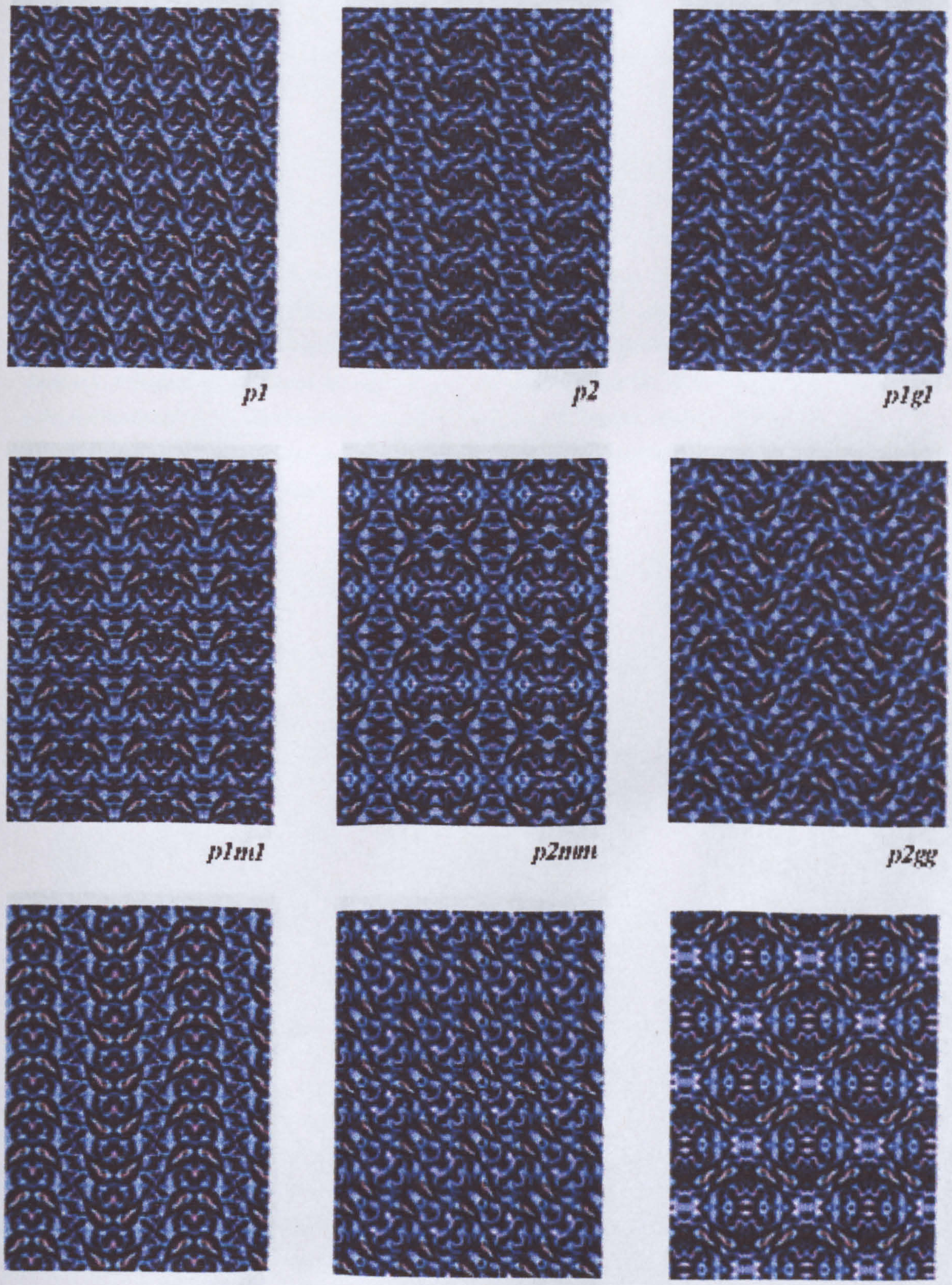
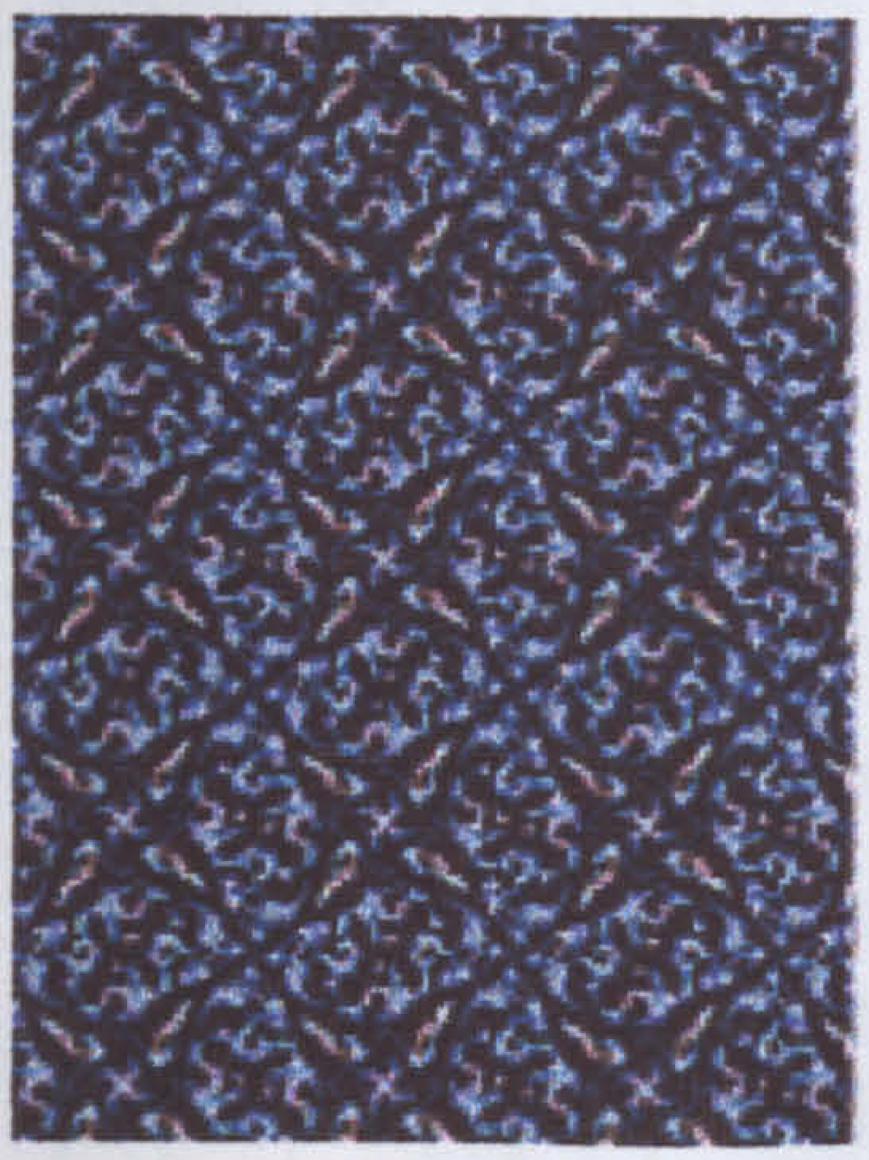
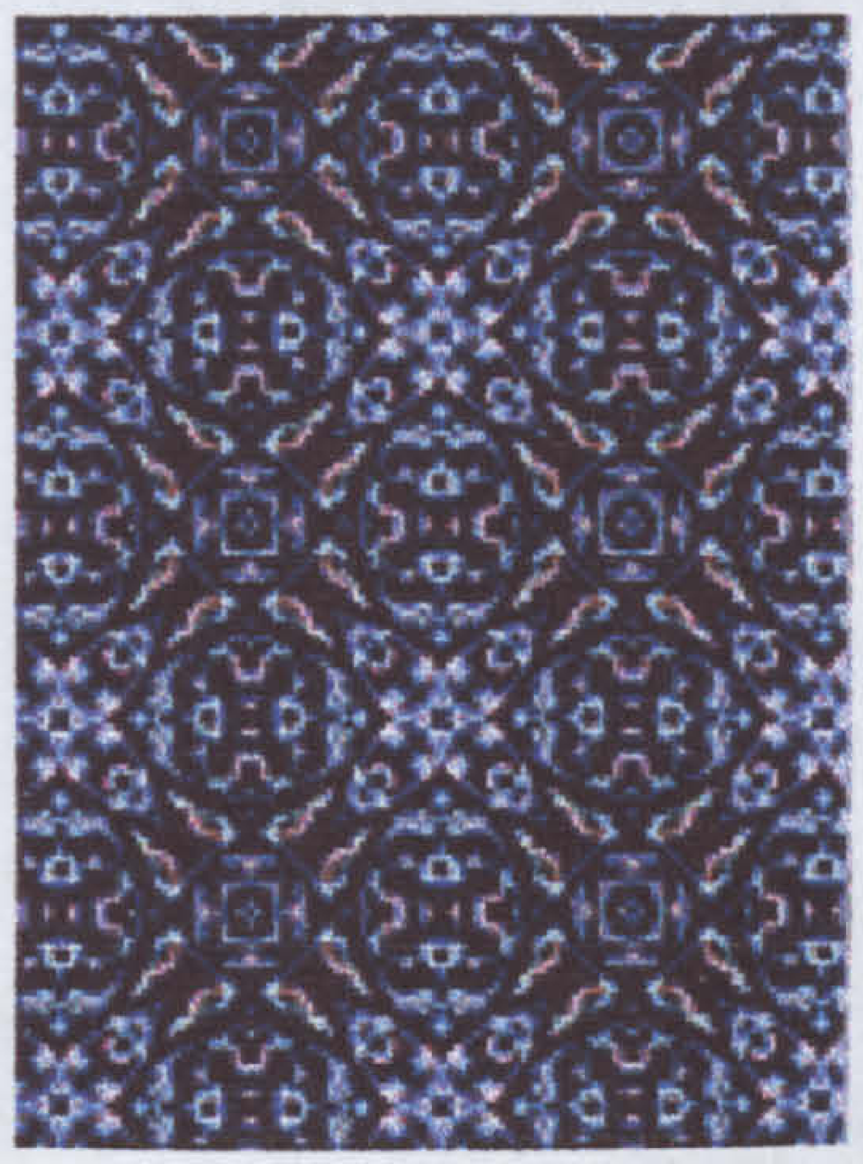


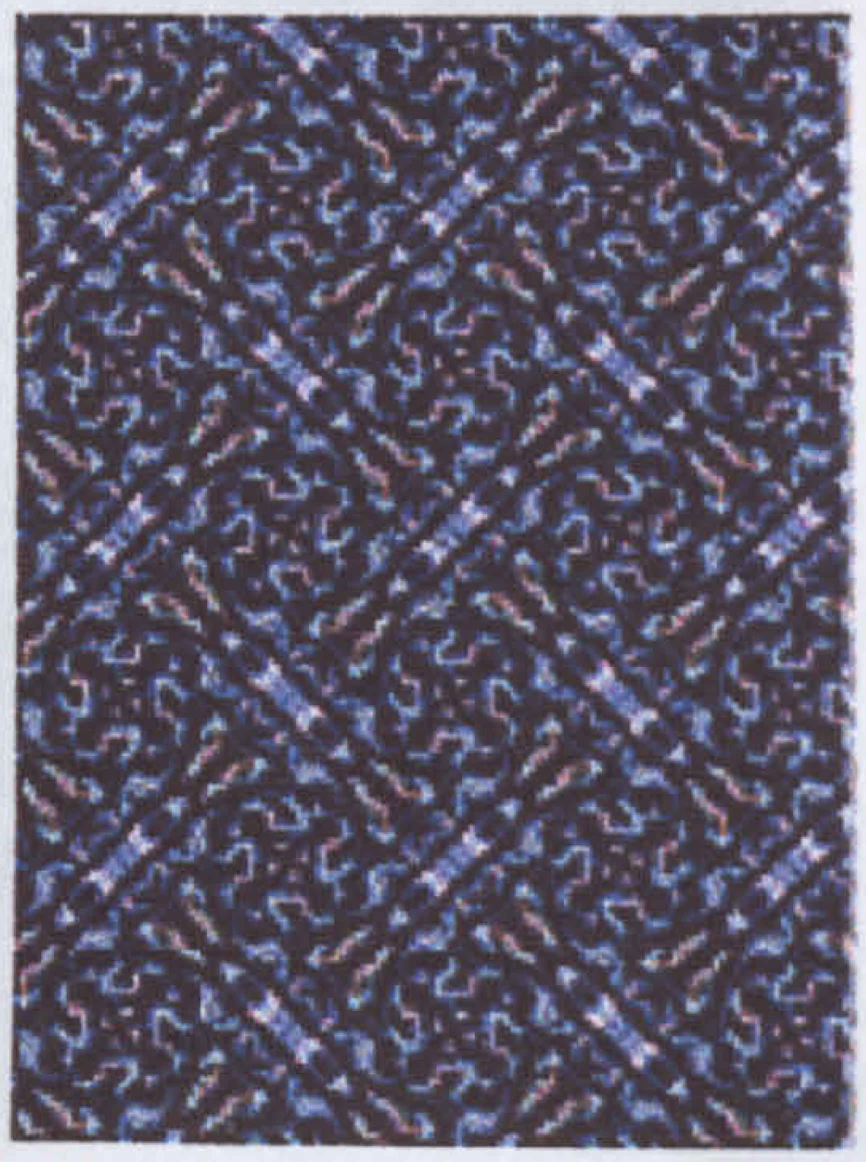
Figure 5.5 (continued) Eight of seventeen symmetry classes of all-over patterns generated from groups of tiny ornaments



p4

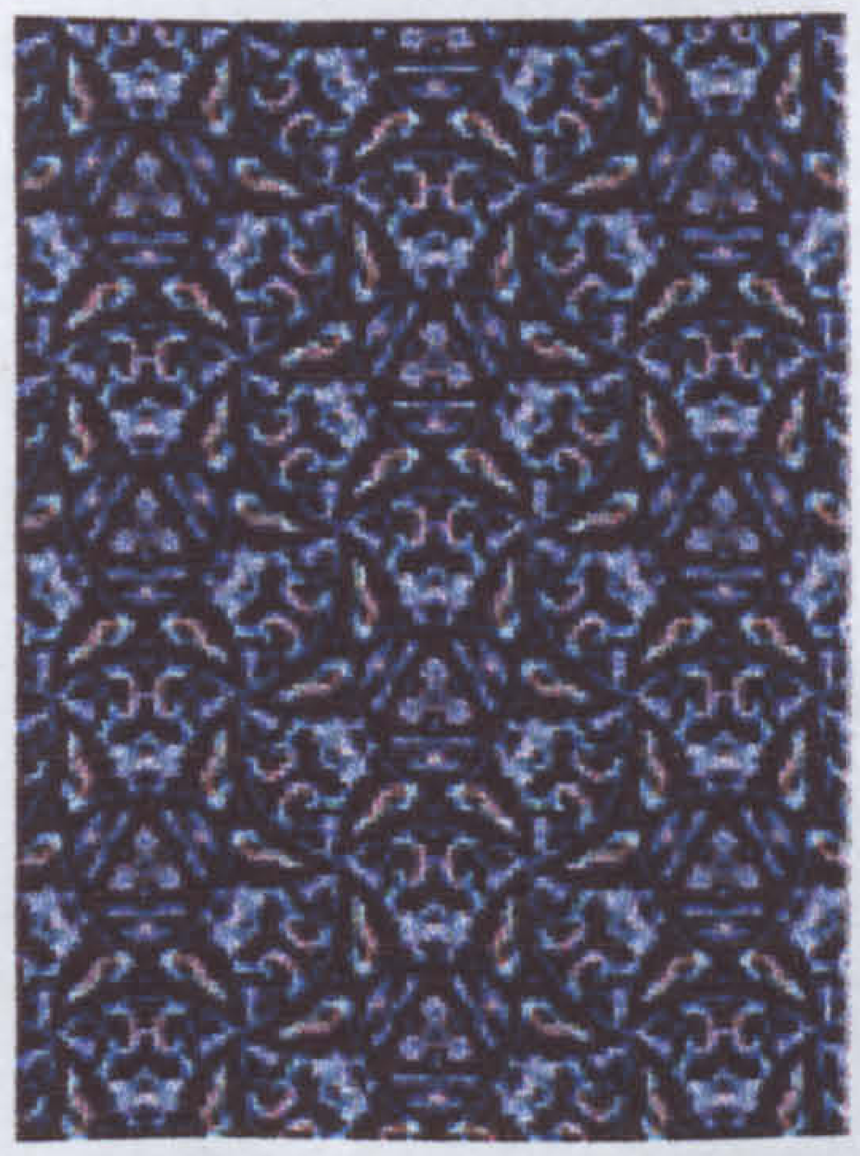


p4mm

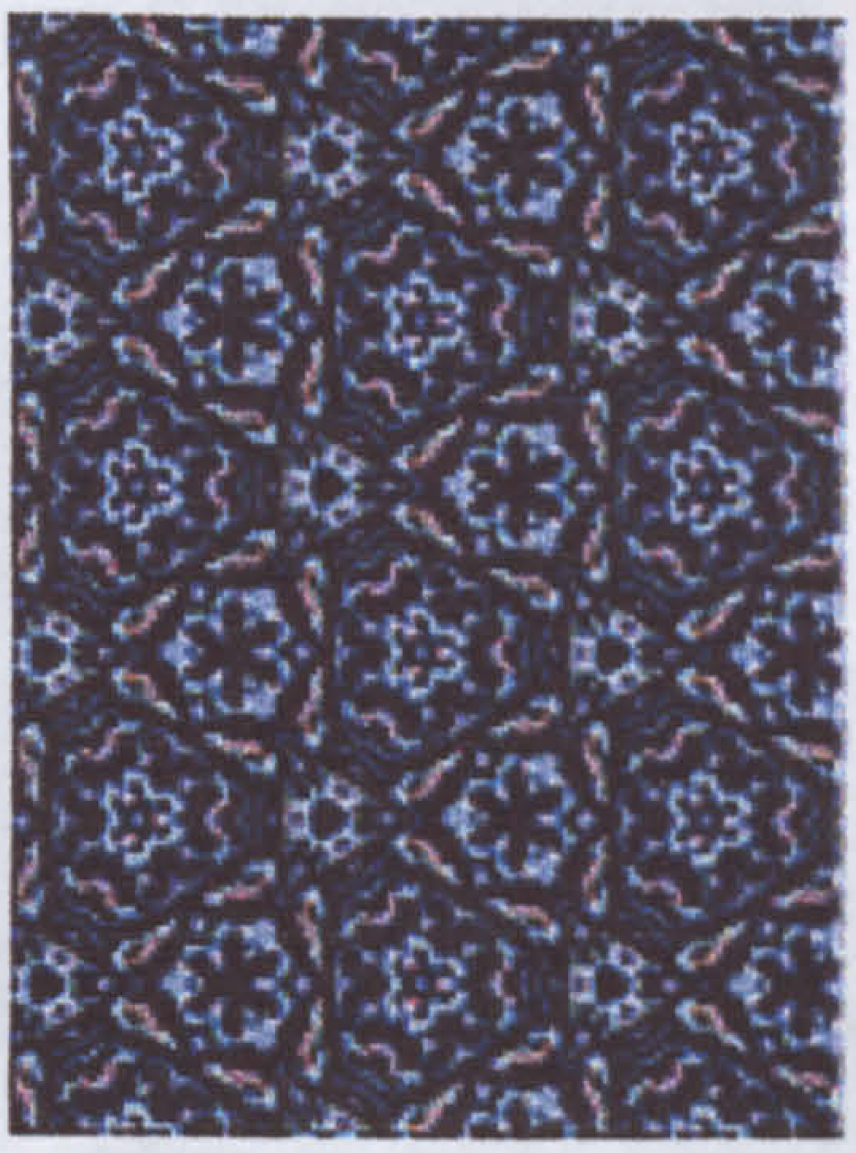


p4gm

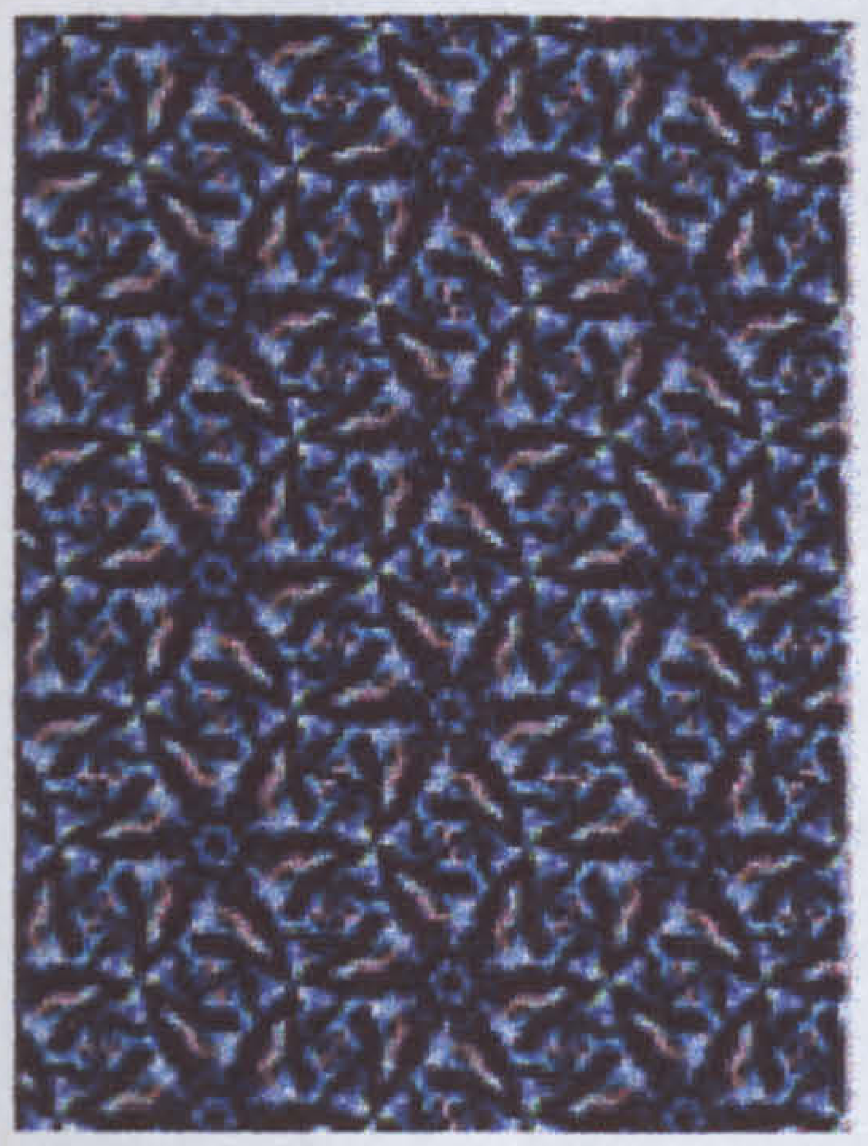
p3



p31m



p3m1



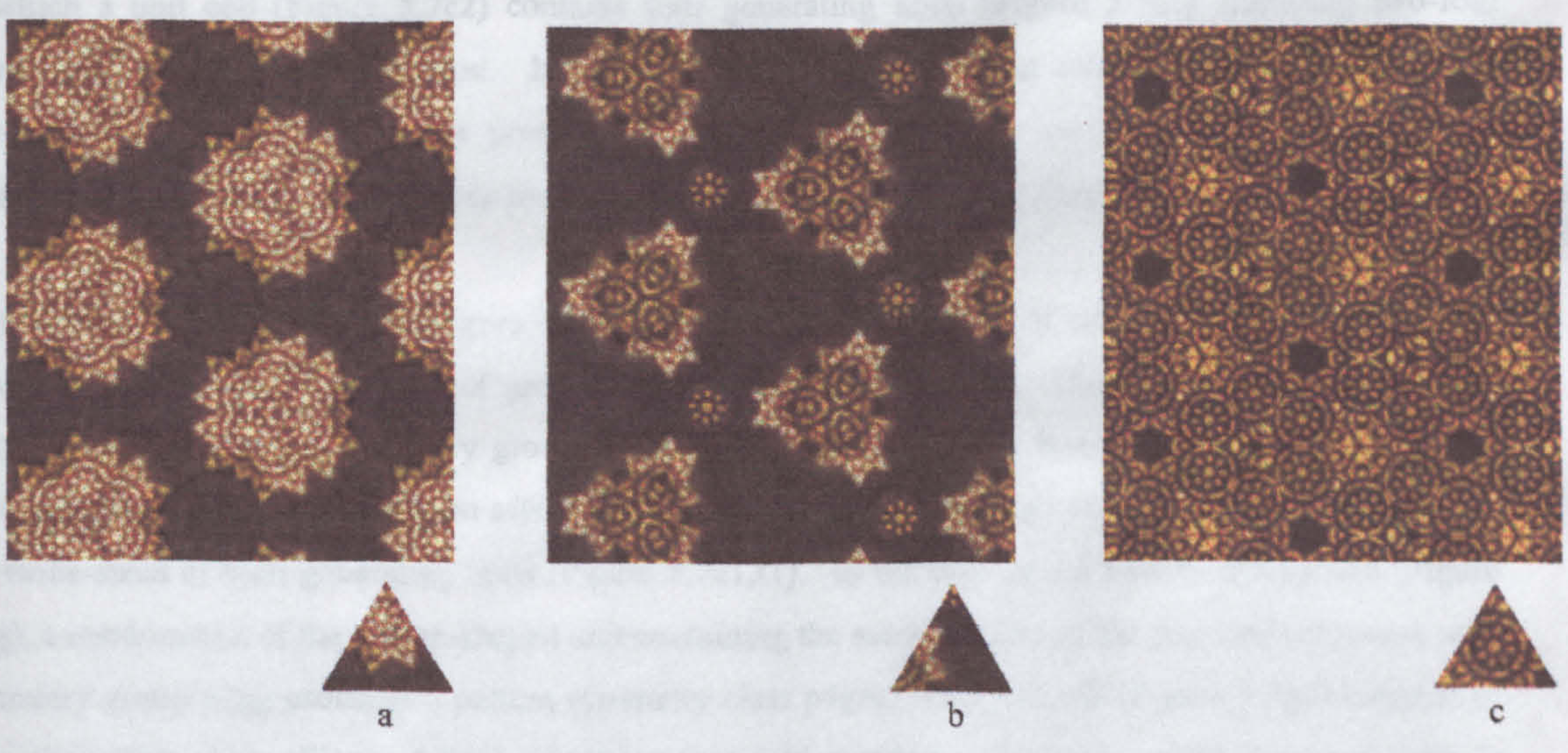
p6

p6mm

Unit boundaries and symmetry groups of patterns generated from isolated motifs are more explicitly noticeable than the ones generated from the other two elements. Units containing series of lines and groups of tiny ornaments tend to produce configuration designs due to the connection between figures and backgrounds along the unit edges. This is seen particularly in the patterns with large numbers of n-fold rotations and reflection axes. Some patterns generated from series of lines exhibit directional designs depending upon the directions of lines and symmetry groups of the unit arrangements. However, the further understanding of the relationship between figures and backgrounds involves the theories of visual recognition and perception which is beyond the scope of this thesis.

Position of elements associated with symmetry groups and boundary of the unit is also the factor that may vary designs. Three patterns shown in Figure 5.6, for example, present three different designs. Though their structures are governed by the same symmetry group p3m1 and generated using the same kind of motifs, their fundamental regions enclose different areas of figures and backgrounds. Based on a hexagonal lattice, an equilateral-triangle-shaped fundamental region containing a motif at one corner produces a repetition of an isolated motif admitting three-fold rotation shown in Figure 5.6a. While the other two fundamental regions of the same shapes and sizes produce the other two patterns. In Figure 5.6b, the fundamental region containing motifs at two corners presents a repetition of two isolated motifs, each of which admits three-fold rotation. In Figure 5.6c, the fundamental region whose entire area covered by motifs exhibits a pattern with three noticeable three-fold rotations.

Figure 5.6a-c Three variations of patterns symmetry classes p3m1 generated from different unit contents



Apart from the design elements, the second concern is on the symmetry group that the unit may contain. As the smallest area of the pattern, a fundamental region is assumed to be an asymmetrical design, but it is not necessarily the case [Horne, 1997, p.96]. It may contain a collection of motifs, which are governed by an underlying symmetry group; this is referred to as a symmetry-obtained unit.

In fact the finite designs of classes cn and dn are symmetrical in all directions due to the n-fold rotations about the fixed points. Nonetheless, in the case of a unit admitting other symmetry operations, e.g., glide-reflection or multiple translation, the direction of element arrangements seems to be critical.

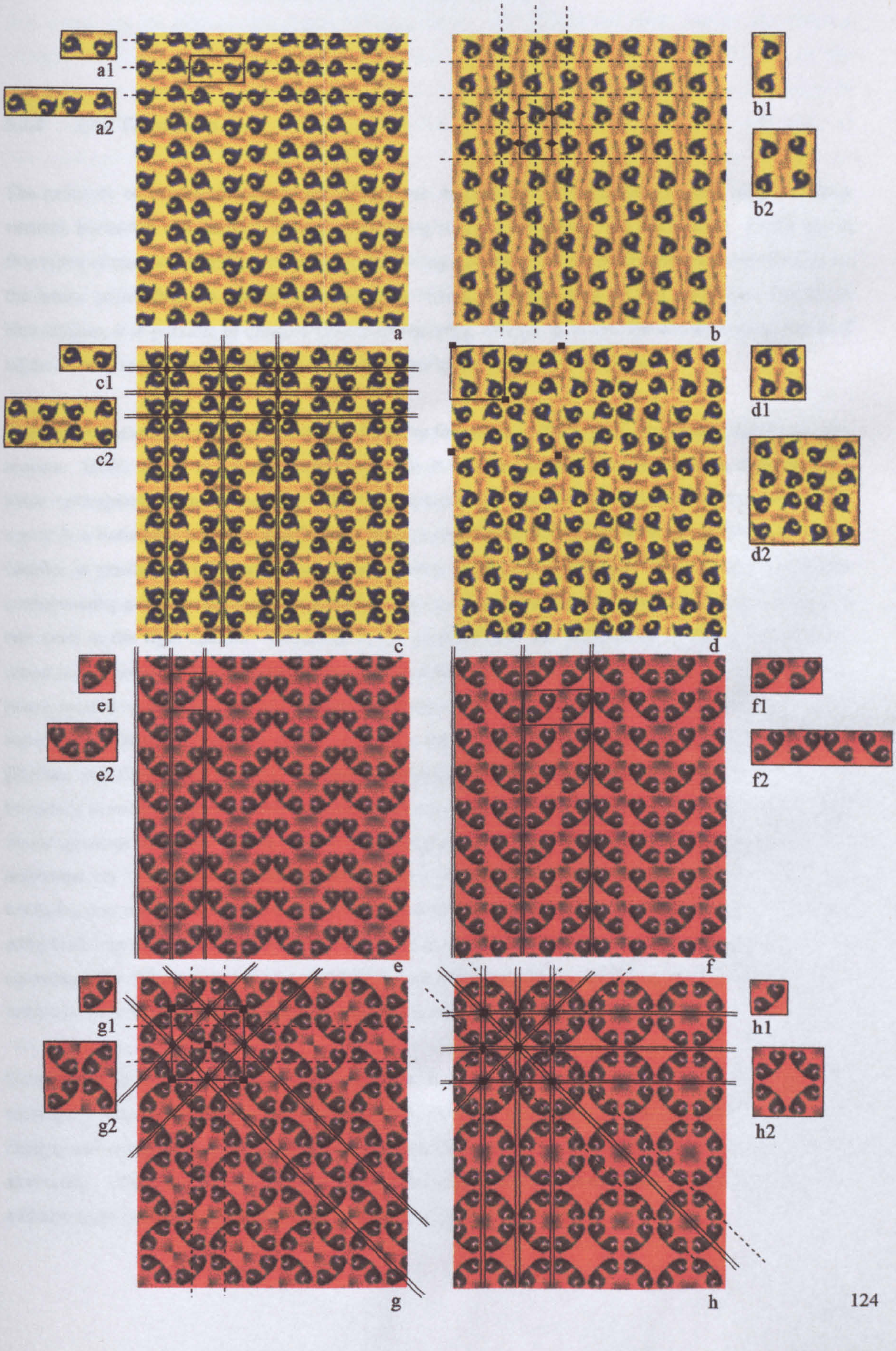
As shown in Figure 5.7, two sets of patterns are generated from two fundamental regions. The unit containing glide-reflection in one direction is used to generate the top four patterns: two by symmetry groups p1g1 (Figure 5.7a,b) and the other two by symmetry group p2mm (Figure 5.7c) and symmetry group p4 (Figure 5.7d). The centre-celled unit containing a bilateral motif is used to generate the bottom four patterns: two by symmetry groups p1m1 (Figure 5.7e,f) and the other two by symmetry group p2gg (Figure 5.7g) and symmetry group p2mm or p4 (Figure 5.7h).

Although the top two patterns (Figure 5.7a,b) are constructed by the same symmetry group p1g1, their resultant patterns exhibit different symmetry classes. The left pattern (Figure 5.7a) preserves symmetry class p1g1 due to parallel glide-reflection axes in a horizontal direction. The right pattern (Figure 5.7b) admits symmetry class p2gg as a result of a perpendicular intersection of vertical glide-reflection axes within units and horizontal glide-reflection axes of the unit arrangement. The unit cells of both patterns (Figure 5.7a2,b2) are horizontally extended twice of their generating units (Figure 5.7a1,b1) following the direction of glide-reflection of generating symmetry groups.

Both patterns in the second row preserve the same symmetry classes as their generating symmetry groups, i.e., class p2mm (Figure 5.7c) and class p4 (Figure 5.7d). A combination of glide-reflection-obtained unit and symmetry groups p2mm produces the repetition of two sets of finite designs of class d2 (Figure 5.7c), in which a unit cell (Figure 5.7c2) contains four generating units (Figure 5.7c1) admitting two-fold rotation and perpendicular reflection. In the right pattern, the generating unit (Figure 5.7d1) containing two glide-reflection-obtained units produces a pattern symmetry class p4 (Figure 5.7d), in which a square-shaped repeating unit includes four generating units admitting four-fold rotation (Figure 5.7d2).

Both patterns in the third row (Figure 5.7e,f) present the combination of units obtaining 45° diagonal reflection with vertical reflections of generating symmetry groups p1m1. They preserve symmetry class p1m1 as their generating symmetry groups, but exhibit different design features due to the numbers of generating units. Vertical reflection affects horizontal extension of the unit cells (Figure 5.7e2,f2) which are twice-sized of their generating units (Figure 5.7e1,f1). In the case of the bottom left pattern (Figure 5.7g), a combination of the square-shaped unit containing the same content as the previous two cases with symmetry group p2gg produces a pattern symmetry class p4gm. The unit cell (Figure 5.7g2) consists of four generating units (Figure 5.7g1) admitting four-fold rotation. However, when the same unit is developed on the perpendicular structure of reflection axes of symmetry group p2mm, or on the other hand four-fold rotation of symmetry group p4, the pattern obtains symmetry class p4mm (Figure 5.7h). The square-shaped unit cell (Figure 5.7h2) also contains four generating units (Figure 5.7h1) with four-fold rotation, but the constructions of reflection axes are different.

Figure 5.7a-h **The top four patterns generated from fundamental regions admitting glide-reflections in one direction are arranged: a, b) by symmetry groups p1g1, c) by symmetry group p2mm and d) by symmetry group p4. The bottom four patterns generated from square-shaped fundamental regions admitting reflections diagonally are arranged: e, f) by symmetry groups p1m1, g) by symmetry group p2gg and h) by symmetry group p2mm or p4.**
a1-h1 Fundamental regions (generating units)
a2-h2 Unit cells (resultant repeating units)



It should be noted that both design elements and symmetry groups contained in a fundamental region are the influential variants governing a variety of designs. Further investigation on different features of unit contents developed on each of seventeen symmetry classes of all-over patterns in association with different formats of unit translation are presented in section 5.3-5.5.

5.2.2 Unit Translation

The principle of brickwork reveals the regular use of uniform rectangle-shaped units repeated along vertical, horizontal and diagonal lines corresponding to the co-ordinates of x and y axes. Using one of five types of translation lattices, repeating units having any shape or content are arranged side by side on the lattice points that are parallel and consistent within the same lines of two translation directions. Nonetheless, it is possible to create a variety of repeating formats in which the corresponding points of adjacent units are not necessarily located on the same levels as the normal lattices.

Attention is focused on five types of textile repeating formats, i.e., block, half-drop, brick, diaper and step repeats. Block repeat is a symmetrical translation in two non-parallel directions where adjacent units share corresponding points together. Half-drop repeat is a vertical half-way translation, while brick repeat is a horizontal half-way translation. In the case of half-drop repeat, every repeating unit in one column is positioned half-way down adjacent units in the next columns, which means it has two corresponding points located on the mid-sides of two units in the left column and two on the mid-sides of two units in the right column. Occurring in the perpendicular direction, every repeating unit of brick repeat is positioned half-way sliding to the units in adjacent rows, which means it has two corresponding points located on the mid-sides of two units in the upper row and two on the mid-sides of two units in the lower row. Diaper repeat is a special case of a block repeat by introducing spacing between units [Phillips and Bunce, 1993, p.26]. Alternate arrangement of repeating units and intervals of the same boundary in each row and column exhibits the connection between units only at the unit corners. Step repeat presents half-side sliding of the parallelogram units in every direction, which produces an additional set of intervals whose shapes and sizes are associated with the parallelogram unit. For example, step repeats of square, rectangular and parallelogram units produce sets of square-, rectangle- and parallelogram-shaped intervals respectively, each of which has a boundary equal to one-fourth of its repeating unit. Whereas, a step repeat of the hexagonal units produces a set of equilateral-triangle-shaped intervals, each of which has a boundary equal to one-twenty-fourth of the hexagonal unit.

Considering five types of parallelogram lattices, it is found that three lattices, i.e., parallelogram, rectangular and square lattices, exhibit block repeats when every unit cell is filled with an identical design, and represent diaper repeats when the unit cells are filled with design elements and intervals alternately. They can also admit half-drop and brick repeats as the half-way translation in one direction, and step repeats as the half-side sliding on all sides of parallelogram units.

Both rhombic (centre-celled) and hexagonal lattices present half-drop repeats simultaneously as brick repeats depending upon the orientation of design content contained in the units. They can also admit diaper and step repeats as well as block repeat if additional conditions of unit boundaries are applied.

It is noted that the half-way translation in one direction of half-drop and brick repeats, the alternate arrangement of units and intervals of diaper repeat and the half-side sliding in every direction of step repeat may change symmetry operations or orientations between units from existing symmetry groups in block repeat. The resultant patterns may thus exhibit either the same or different symmetry classes as their generating symmetry groups depending upon individual symmetry features within and between units.

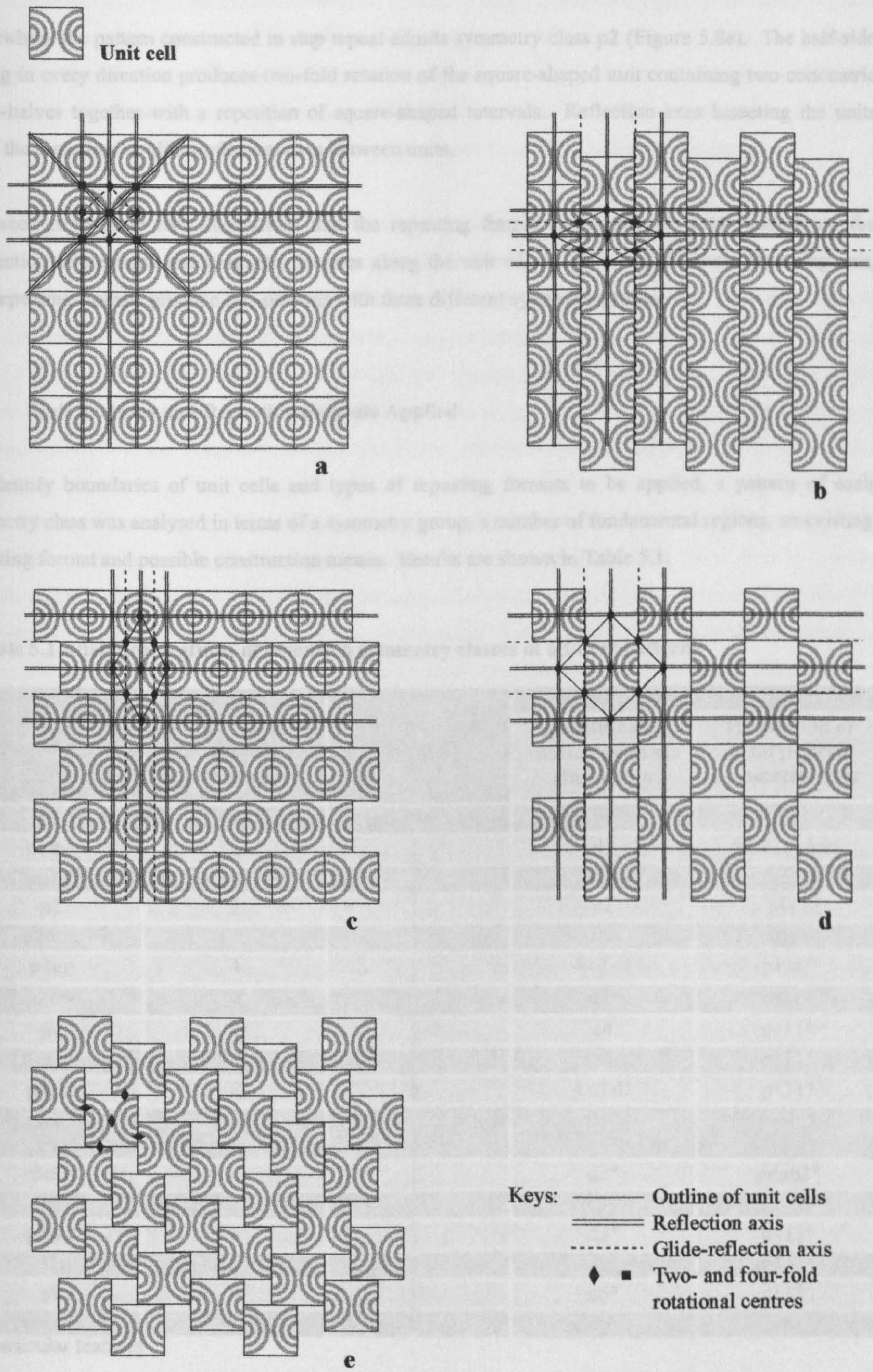
Five illustrations shown in Figure 5.8a-e exhibit five repeating formats generated from the square-shaped unit containing two concentric circle-halves or a finite design of class d2: a block repeat (Figure 5.8a), a half-drop repeat (Figure 5.8b), a brick repeat (Figure 5.8c), a diaper repeat (Figure 5.8d) and a step repeat (Figure 5.8e).

Different repeating formats produce different designs in terms of motif organisations and symmetry features. The pattern in block repeat (Figure 5.8a) presents a repetition of concentric circles symmetrical in vertical and horizontal directions as a result of the combination of perpendicular reflection axes within and between units. Since a block repeat provides a side-to-side connection along all four unit edges, the configuration of a circle at the vertical unit edges and bilateral intervals at the horizontal unit edges produces reflection axes along all sides of a unit. Two-fold rotational centres locate at the mid-sides of all four unit edges, while four-fold rotational centres locate at the unit centre and the unit corners. All of these symmetry properties identify symmetry group p4mm.

By using the same repeating unit as in a block repeat, the patterns constructed in three repeating formats, i.e., half-drop, brick and diaper repeats, generate three different designs of the same symmetry group c2mm. The half-drop repeat produces wavy series of circle-halve units in vertical direction (Figure 5.8b). The perpendicular reflections within units and the vertical half-way translation produce the intersection of two sets of glide-reflection axes, one set runs vertically along the unit edges and the other set runs horizontally and intersects the vertical set at ¼- and ¾- way on the vertical unit edges. Two-fold rotational centres locate at the mid-sides of the horizontal unit edges, ¼- and ¾-way on the vertical unit edges and at the unit centre.

The combination of perpendicular reflection axes within units and the horizontal half-way translation of brick repeat exhibits horizontal series of concentric circles, in which each row is arranged half-side sliding to the upper and the lower rows (Figure 5.8c). One set of glide-reflection axes running along horizontal unit edges intersects the other set of vertical glide-reflection axes at ¼- and ¾-way on the horizontal unit edges. Two-fold rotational centres locate at the mid-sides on the vertical unit edges and ¼- and ¾-way on the horizontal unit edges and the unit centre.

Figure 5.8a-e Five repeating formats constructed from identical square-shaped unit cells in a) block repeat, b) half-drop repeat, c) brick repeat, d) diaper repeat, and e) step repeat



The diaper repeat produces a checker-board pattern exhibiting an alternate repetition of square-shaped units of two concentric circle-halves and square-shaped intervals (Figure 5.8d). Glide-reflection axes run along all four sides of the unit. Two-fold rotational centres locate at the unit centre, the unit corners and the centres of intervals.

Meanwhile, the pattern constructed in step repeat admits symmetry class p2 (Figure 5.8e). The half-side sliding in every direction produces two-fold rotation of the square-shaped unit containing two concentric circle-halves together with a repetition of square-shaped intervals. Reflection axes bisecting the units share the same line as glide-reflection axes between units.

It is seen from these five illustrations that the repeating format is the critical factor underlying the connection of elements and symmetry features along the unit edges. By using the same repeating unit, five repeating formats produce five patterns with three different symmetry groups.

5.2.3 Unit Features and Repeating Formats Applied

To identify boundaries of unit cells and types of repeating formats to be applied, a pattern of each symmetry class was analysed in terms of a symmetry group, a number of fundamental regions, an existing repeating format and possible construction means. Results are shown in Table 5.1.

Table 5.1 Analytical features of seventeen symmetry classes of all-over patterns

Pattern symmetry class	Number of fundamental regions	Repeating format	Construction means	
			Repetition of finite design of class cn/dn	Translation of band pattern symmetry class
p1	1	Block repeat	c1	p111
p1m1	2		d1	pm11, p1m1
p1g1	2		2 of c1	p1a1
p2	2		c2	p112
p2mm	4		d2	pmm2
p2gg	4		4 of c1	p111*
p2mg	4		2 of d1	pma2
p4	4		c4	p111*
p4mm	8		d4	p111*
p4gm	8		4 of d1	p111*
c1m1	2	Half-drop or brick repeat	d1*	pm11*
c2mm	4		d2*	pmm2*
p3	3		c3*	p111*
p3m1	6		d3*	p111*
p31m	6		d3*	p111*
p6	6		c6*	p111*
p6mm	12		d6*	p111*

* particular features

In fact all-over patterns can be generated from either the repetition of unit cells or series of band patterns. It is found that patterns of all seventeen classes are possible to be constructed by the translation of their individual unit cells each of which may contain either an asymmetrical fundamental region or a group of fundamental regions and associated symmetry groups. However, some of them could not be constructed by one of seven band pattern classes because of limited features of symmetry groups obtained in linear repetition. If particular features are applied, for example, it is possible to produce all-over patterns classes p2gg, p4, p4mm or p4gm from series of band patterns containing linear repetition of a set of four fundamental regions admitting perpendicular glide-reflection, finite design of class c4, finite design of class d4 or four pairs of bilateral fundamental regions admitting glide-reflections respectively.

In two centre-celled cases, all-over pattern classes c1m1 and c2mm can also be constructed from the repetition of the centre-celled units containing finite designs of class d1 and d2 respectively, or from the translation of band pattern classes pm11 and p2mm containing linear repetition of centre-celled units of finite designs of classes d1 and d2 respectively. To generate hexagon-based patterns, i.e., all-over patterns classes p3, p31m, p3m1, p6 and p6mm, the construction of band patterns containing finite designs of classes c3, d3, c6 and d6 may need certain organisations which associate to their half-way translation formats of the hexagon-shaped units and their individual symmetry groups.

The five types of repeating formats widely employed by designers, as mentioned previously in chapter 4, are involved. Block repeat exhibits regular repetition of individual unit at the same time as regular construction of band patterns parallel in two directions. Half-drop and brick repeats apparently reflect the constructions of band patterns parallel in either horizontal or vertical direction rather than the repetitions of individual units. Whereas diaper and step repeats present the constructions of individual units only.

It is found that patterns from ten of seventeen symmetry classes, i.e., classes p1, p1m1, p1g1, p2, p2mm, p2gg, p2mg, p4, p4mm and p4gm, whose unit cells are bounded in either square, rectangle or parallelogram admit block repeats as their basic construction formats. Meanwhile, the other seven symmetry classes, i.e., two centre-celled all-over patterns classes c1m1and c2mm, and five hexagon-based all-over patterns classes p3, p3m1, p31m, p6 and p6mm, are constructed in half-drop and/or brick repeats. A centre-celled pattern exhibits both repeating formats in the same orientation due to the four-sided contact, while a hexagon-based pattern exhibits half-drop repeat in one direction and a brick repeat in a perpendicular direction due to the six-sided contact.

Seventeen symmetry classes provide seventeen individual unit cells according to the numbers of fundamental regions and the underlying symmetry groups. However, a pattern of one symmetry class may be constructed by different features of unit cells by successive translation. Symmetry groups p1 consists of only one fundamental region which undergoes repetition by translation, and as a result there is no variety of unit boundary. But, in cases where symmetry groups whose unit cells consist of more than one fundamental region, varieties of unit boundaries in terms of different organisations of fundamental regions or different orientations of symmetry group may produce patterns of different symmetry classes or different design outcomes of the same symmetry class within the same repeating format. Schematic illustrations in Figure 5.9 present varieties of unit boundaries derived from individual symmetry groups of seventeen symmetry classes, which will be developed on the construction of patterns in section 5.3-5.5.

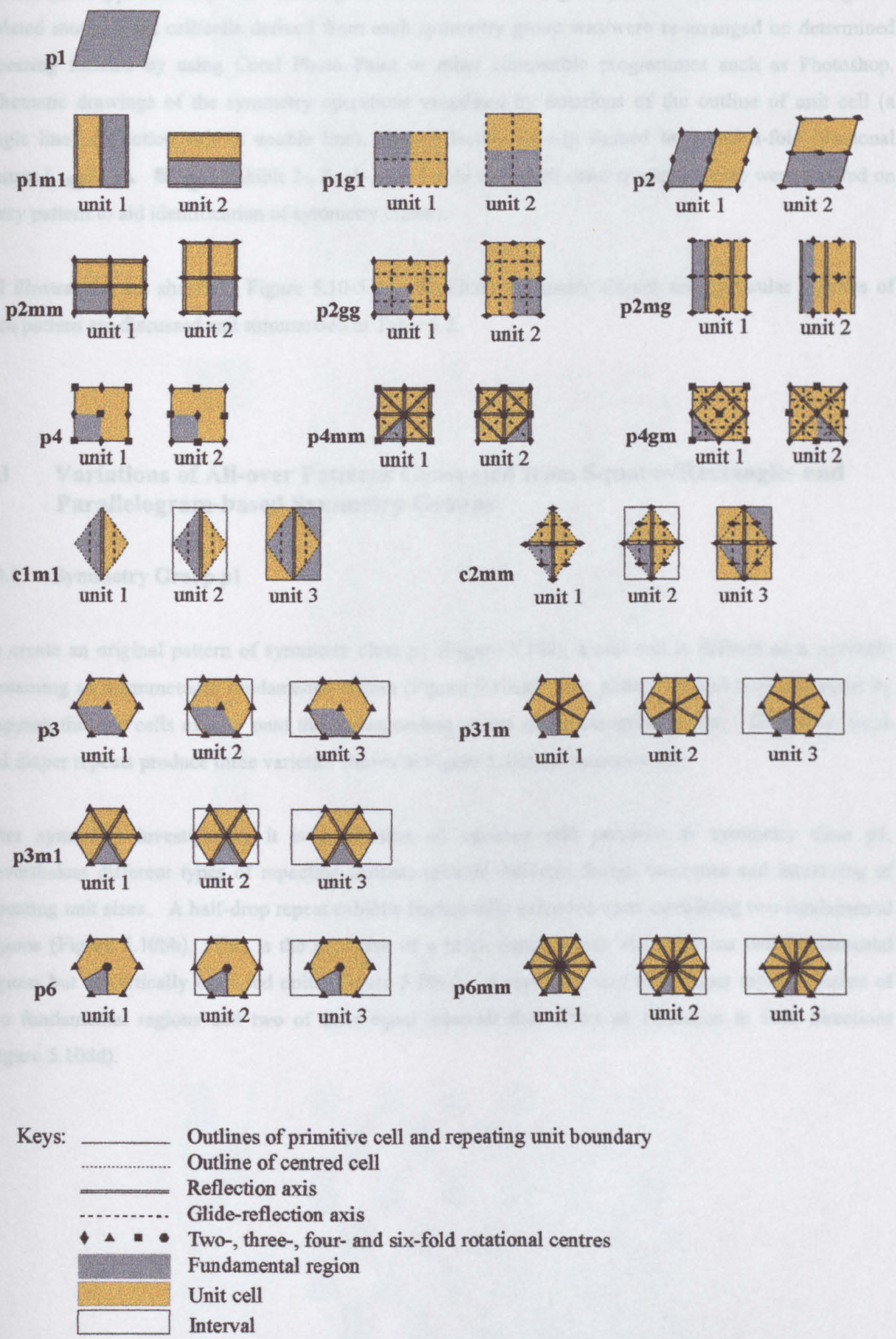
Varieties of ten square-/rectangle-/parallelogram-based symmetry groups whose original patterns admit block repeats are constructed in three repeating formats, i.e., half-drop, brick and diaper repeats. Symmetry group p1 produces only one set of varieties on these three repeating formats since the unit cell contains only one fundamental region. The other nine symmetry groups produce two sets of varieties generated using two alternative unit cells.

In cases of symmetry groups p1m1, p1g1 and p2, in which a unit cell contains two fundamental regions arranged along reflection axes, glide-reflection axes and about two-fold rotational centres respectively, varieties of unit boundaries can be created by vertical unit orientation: unit 1 or horizontal unit orientation: unit 2. In cases of symmetry groups p2mm and p2gg, the constructions of four fundamental regions bounded with perpendicular reflection and glide-reflection axes seem to be symmetrical in any direction. The 90° rotation together with horizontally half-side sliding of the unit boundary is applied to differentiate two alternative unit cells: unit 1 and unit 2. In cases of symmetry groups p2mg, p4, p4mm and p4gm, the horizontally half-side sliding of the unit boundary varies the orientation of symmetry operations of the symmetry group containing in unit 1 to unit 2.

In two centre-celled symmetry groups, three unit boundaries are defined, i.e., unit 1: an isolated centre-celled unit, unit 2: a square-/rectangle-shaped unit containing a centre-celled unit and its equal intervals, and unit 3: a square-/rectangle-shaped unit containing two centre-celled units. The first case provides an original pattern in half-drop and brick repeats and produces two varieties in a diaper repeat and a diagonal half-way translation. While the latter two produce two sets of varieties in half-drop and brick repeats.

For five hexagon-based symmetry groups, three features of unit cells are considered, i.e., unit 1: an isolated hexagon-shaped unit, unit 2: a rectangle-shaped unit containing a hexagon-shaped unit and intervals and unit 3: the special case of unit 2 where the intervals are equal to the hexagon-shaped unit. The first case generates an original pattern in half-drop or brick repeats when the pattern is turned 90° and a pattern in a half-side sliding format or step repeat. The second case produces varieties in block, half-drop, brick and diaper repeats. The third case produces a pattern in a diaper repeat.

Figure 5.9 Alternative features of unit cells of sixteen all-over symmetry groups (except class p1 which contains one fundamental region)



A total of 124 representative patterns: 76 patterns generated from ten square-/rectangle-/parallelogram-based symmetry groups, 13 patterns generated from two centre-celled symmetry groups and 35 patterns generated from five hexagon-based symmetry groups, were analysed and classified into their individual symmetry classes. The programme by the title of "Terrazzo", available in Corel Photo Paint version 7 onward [see Appendix A2], was used to generate all seventeen original patterns from scanned images of isolated motifs. Unit cell/cells derived from each symmetry group was/were re-arranged on determined repeating formats by using Corel Photo Paint or other compatible programmes such as Photoshop. Schematic drawings of the symmetry operations visualised by notations of the outline of unit cell (a single line), reflection axis (a double line), glide-reflection axis (a dashed line) and n-fold rotational centres (◆ ▲ ■ ⬢ exhibit 2-, 3-, 4-, and 6-fold rotational centres respectively) were applied on every pattern to aid identification of symmetry classes.

All illustrations are shown in Figure 5.10-5.26. Resultant symmetry classes and particular features of each pattern are discussed and summarised in Table 5.2.

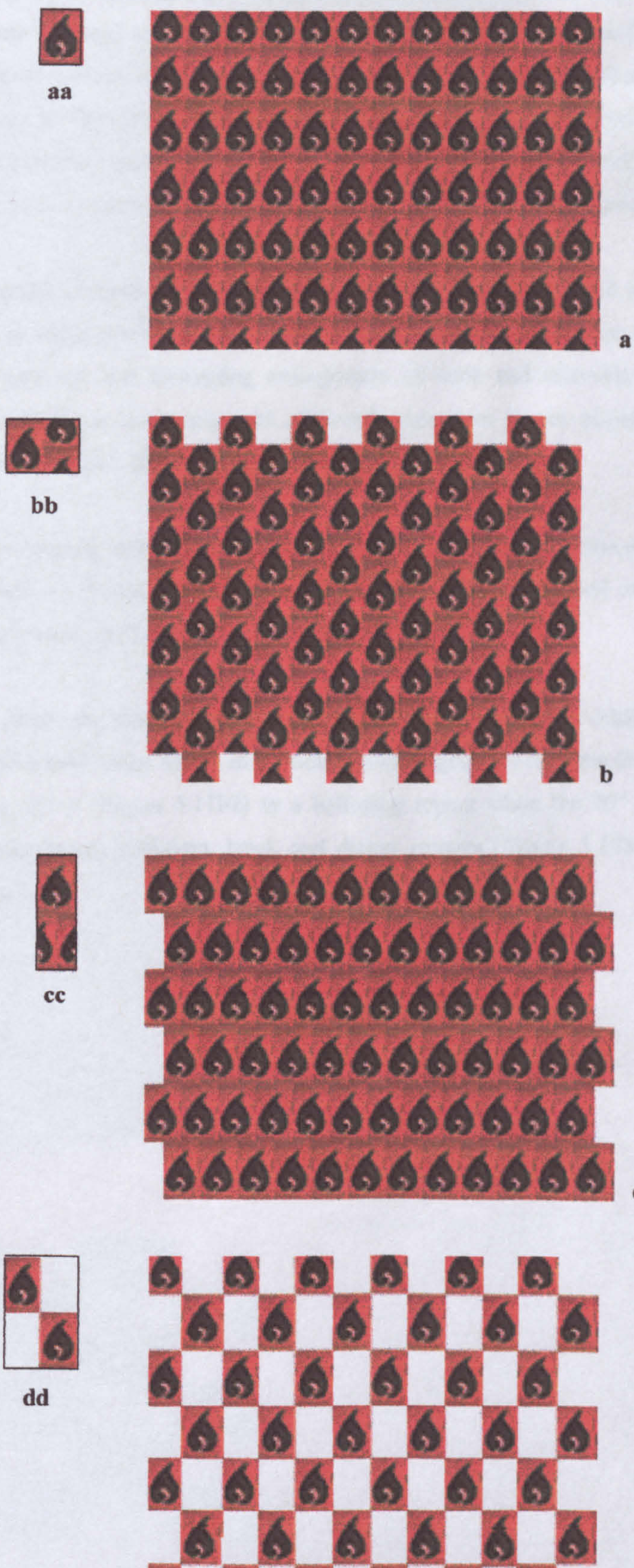
5.3 Variations of All-over Patterns Generated from Square-/Rectangle- and Parallelogram-based Symmetry Groups

5.3.1 Symmetry Group p1

To create an original pattern of symmetry class p1 (Figure 5.10a), a unit cell is defined as a rectangle containing an asymmetrical fundamental region (Figure 5.10aa). The pattern obtains a block repeat by mapping the unit cells exactly onto the corresponding points of a rectangular lattice. Half-drop, brick and diaper repeats produce three varieties shown in Figure 5.10b,c,d respectively.

After symmetry investigation, it is noted that all varieties still preserve as symmetry class p1. Nevertheless different types of repeating formats provide different design outcomes and increasing of repeating unit sizes. A half-drop repeat exhibits horizontally extended units containing two fundamental regions (Figure 5.10bb). This is the converse of a brick repeat which also contains two fundamental regions but in vertically extended units (Figure 5.10cc). A repeating unit of a diaper repeat consists of two fundamental regions and two of their equal intervals that affect an extension in both directions (Figure 5.10dd).

Figure 5.10 a-d Varieties generated from symmetry group p1 in a) block repeat, b) half-drop repeat, c) brick repeat and d) diaper repeat
aa Generating unit cell
bb-dd Resultant unit cells



5.3.2 Symmetry Group p1m1

Considered as a translation of a bilateral motif, all-over pattern class p1m1 contains two parallel reflection axes that run alternately in one direction. There is no rotation symmetry. A unit cell consisting of two fundamental regions is possible and this may have two features, i.e., unit 1: a unit admitting vertical reflection (Figure 5.11aa1) and unit 2: a unit admitting horizontal reflection (Figure 5.11aa2). In the left column, unit 1 produces an original pattern in a block repeat (Figure 5.11a1) and three varieties in a half-drop repeat (Figure 5.11b1), a brick repeat (Figure 5.11c1) and a diaper repeat (Figure 5.11d1). In the right column, unit 2 produces an original pattern in a block repeat (Figure 5.11aa2) and three varieties in a half-drop repeat (Figure 5.11b2), a brick repeat (Figure 5.11c2) and a diaper repeat (Figure 5.11d2).

It is found that all varieties generated using both unit cells in half-drop, brick and diaper repeats admit symmetry class c1m1 in stead of class p1m1 as the original patterns. Half-way translation in either vertical or horizontal direction and alternating arrangement of units and intervals produce symmetry groups where reflection axes run alternately with glide-reflection axes in one direction. A rectangular lattice is thus replaced by a centre-celled rectangular lattice.

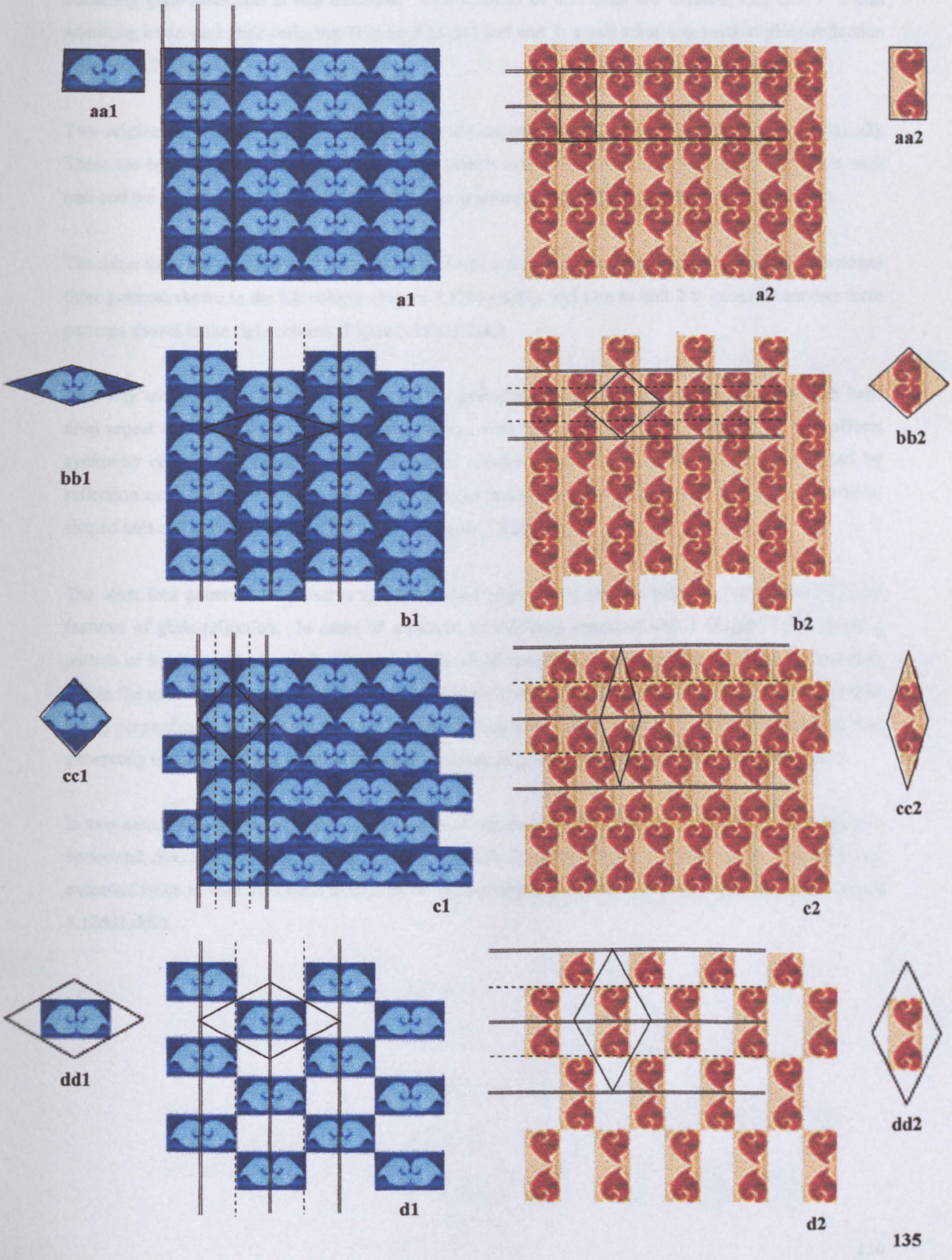
Each rhomboid-shaped repeating unit of half-drop and brick repeats contains a finite design of class d1 as its generating unit (Figure 5.11b1,b2,c1,c2), while each rhomboid-shaped unit cell of two diaper repeats encloses one generating unit at equal intervals (Figure 5.11dd1,dd2).

Although the patterns share the same symmetry class c2mm, they may also exhibit different design outcomes. A pattern generated using unit 1 in a brick repeat (Figure 5.11c1) has the same design as a pattern generated using unit 2 (Figure 5.11b2) in a half-drop repeat when the 90° rotation is applied. While the remaining patterns in half-drop, brick and diaper repeats (Figure 5.11b1,c2,d1,d2) present different design outcomes.

Figure 5.11 a-d Varieties generated using two unit cells of symmetry group p1m1: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats

aa Generating unit cells

bb-dd Resultant unit cells



5.3.3 Symmetry Group p1g1

All-over pattern class p1g1 admits glide-reflection in only one direction, where two parallel glide-reflection axes run alternately with each other. A unit cell is made up of two fundamental regions admitting glide-reflection in one direction. Two features of unit cells are defined, i.e., unit 1: a unit admitting horizontal glide-reflection (Figure 5.12aa1) and unit 2: a unit admitting vertical glide-reflection (Figure 5.12aa2).

Two original patterns generated from both unit cells are constructed in block repeats (Figure 5.12a1,a2). There are two sets of glide-reflections, one that bisects every unit produces glide-reflection within each unit and the other one that runs along the unit edge produces glide-reflection between adjacent units.

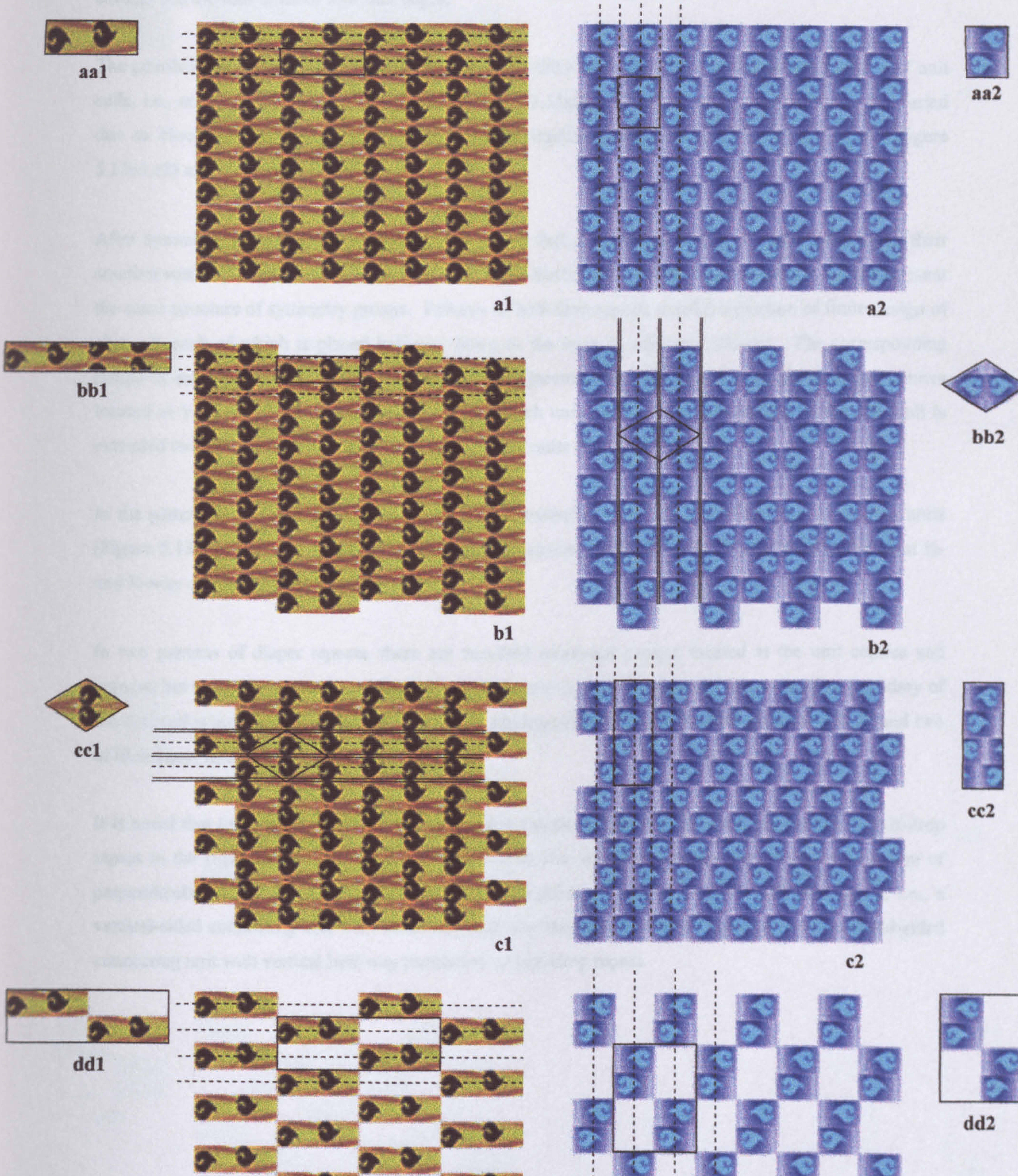
The other three repeating formats, i.e., half-drop, brick and diaper repeats are applied to unit 1 to generate three patterns shown in the left column (Figure 5.12b1,c1,d1), and also to unit 2 to generate another three patterns shown in the right column (Figure 5.12b2,c2,d2).

Half-way translation along the same direction as glide-reflection axes occurring in a pattern with half-drop repeat of unit 2 (Figure 5.12b2) and a pattern with brick repeat of unit 1 (Figure 5.12c1) affects symmetry class change from class p1g1 to c1m1. Glide-reflection axes between units are replaced by reflection axes. A rectangular lattice is thus changed to a centre-celled rectangular lattice. A rhomboid-shaped unit cell thus contains a bilateral motif (Figure 5.12bb2,cc1).

The other four patterns still preserve symmetry class p1g1 as the original patterns, but exhibit different features of glide-reflection. In cases of a pattern of half-drop repeat of unit 1 (Figure 5.12b1) and a pattern of brick repeat of unit 2 (Figure 5.12c2), all glide-reflection axes produce glide-reflection only within the unit. There are no glide-reflections between units due to the directions of half-way translations being perpendicular to the directions of glide-reflection axes within units. A unit cell containing two generating units is extended twice in the same direction as glide-reflection axes (Figure 5.12bb1,cc2).

In two cases of diaper repeats, glide-reflection axes run through units which alternate with intervals in a horizontal direction (Figure 5.12d1) and in a vertical direction (Figure 5.12d2). Both unit cells are extended twice in both directions to enclose two generating units and two of their equal intervals (Figure 5.12dd1,dd2).

Figure 5.12 a-d Varieties generated using two unit cells of symmetry group p1g1: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.4 Symmetry Group p2

A unit cell of all-over pattern class p2 consists of two fundamental regions obtaining two-fold rotation or a finite design of class c2. Two-fold rotational centres locate not only at the unit centre but also at four corners and the mid-sides of four unit edges.

The possibility to combine two fundamental regions with two-fold rotation provides two features of unit cells, i.e., unit 1 (Figure 5.13aa1) and unit 2 (Figure 5.13aa2). Four patterns in each column are varied due to block repeats (Figure 5.13a1,a2), half-drop repeats (Figure 5.13b1,b2), brick repeats (Figure 5.13c1,c2) and diaper repeats (Figure 5.13d1,d2).

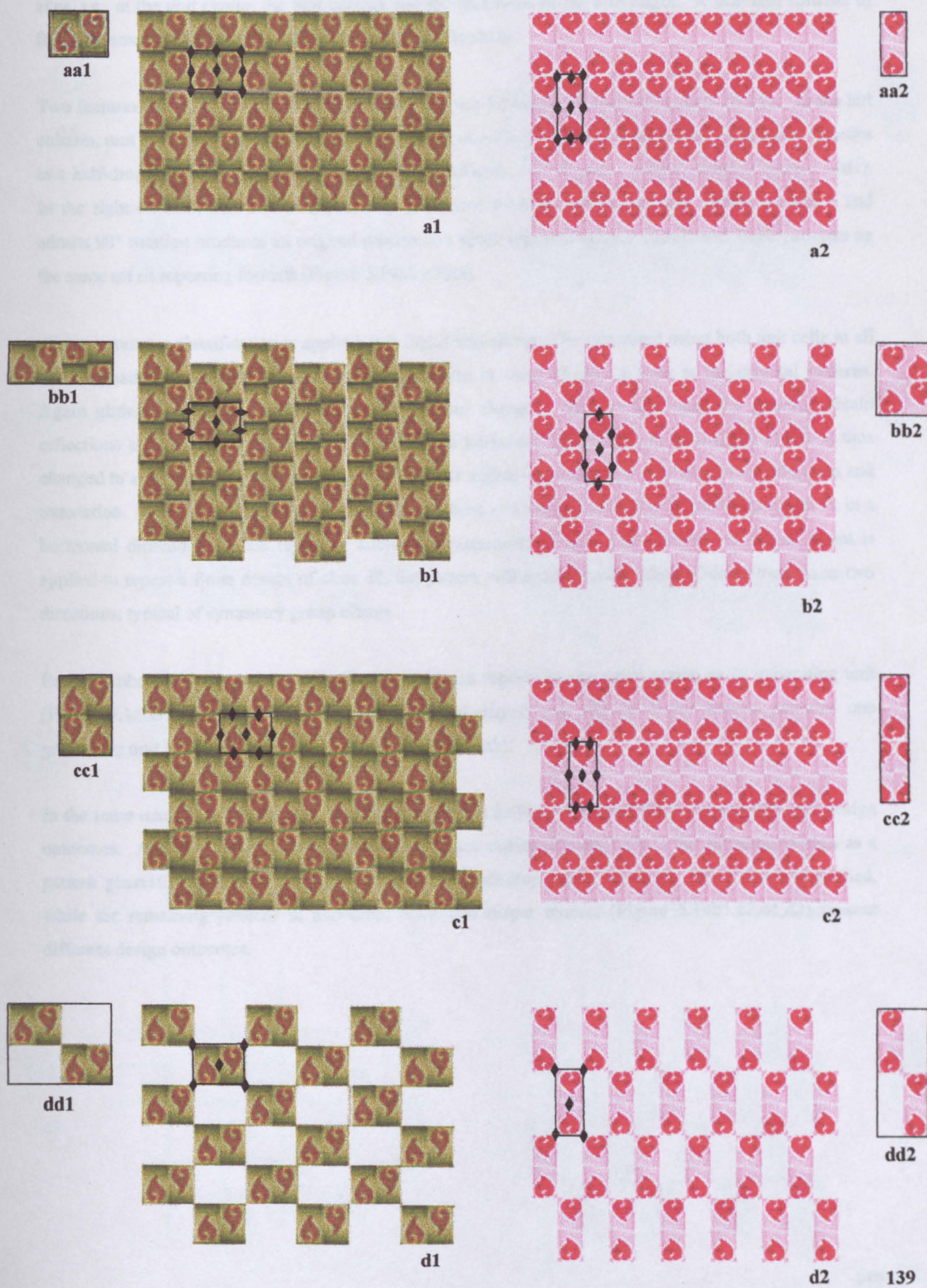
After symmetry classification is applied, it is found that all varieties obtain two-fold rotation as their smallest rotation, no reflections and glide-reflections. Both patterns of the same repeating formats present the same structure of symmetry groups. Patterns of half-drop repeats exhibit repetition of finite design of class c2, each of which is placed half-way down to the ones in adjacent columns. The corresponding points of each unit thus locate at the mid-side of adjacent units. This produces two rotational centres located at ¼- and ¾-way on the vertical sides of each unit. As a result a boundary of the unit cell is extended twice horizontally to enclose two generating units (Figure 5.13bb1,bb2).

In the patterns with brick repeats, the unit cells are extended vertically to enclose two generating units (Figure 5.13cc1,cc2), due to horizontal half-way translation. Two two-fold rotation centres locate at ¼- and ¾-way on the horizontal unit edges.

In two patterns of diaper repeats, there are two-fold rotational centres located at the unit centres and corners, but not at the mid-sides of the unit edges because of the emergence of intervals. The boundary of the unit cell is therefore extended both vertically and horizontally to enclose two generating units and two of their equal intervals (Figure 5.13dd1,dd2).

It is noted that the pattern of brick repeat in the left column (Figure 5.13c1) and the pattern of half-drop repeat in the right column (Figure 5.13b2) exhibit similar unit organisation due to the combination of perpendicular directions of the orientation of two fundamental regions and the unit translation, i.e., a vertical-sided connecting unit with horizontal half-way translation of brick repeat and a horizontal-sided connecting unit with vertical half-way translation of half-drop repeat.

Figure 5.13 a-d Varieties generated using two unit cells of symmetry group p2: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.5 Symmetry Group p2mm

Considered as a translation of a finite design of class d2, all-over pattern class p2mm admits reflection in two perpendicular directions. Two-fold rotation automatically occurs at every intersection of reflection axes, i.e., at the unit centre, the unit corners and the mid-sides of the unit edges. A unit cell consists of four fundamental regions obtaining perpendicular reflections.

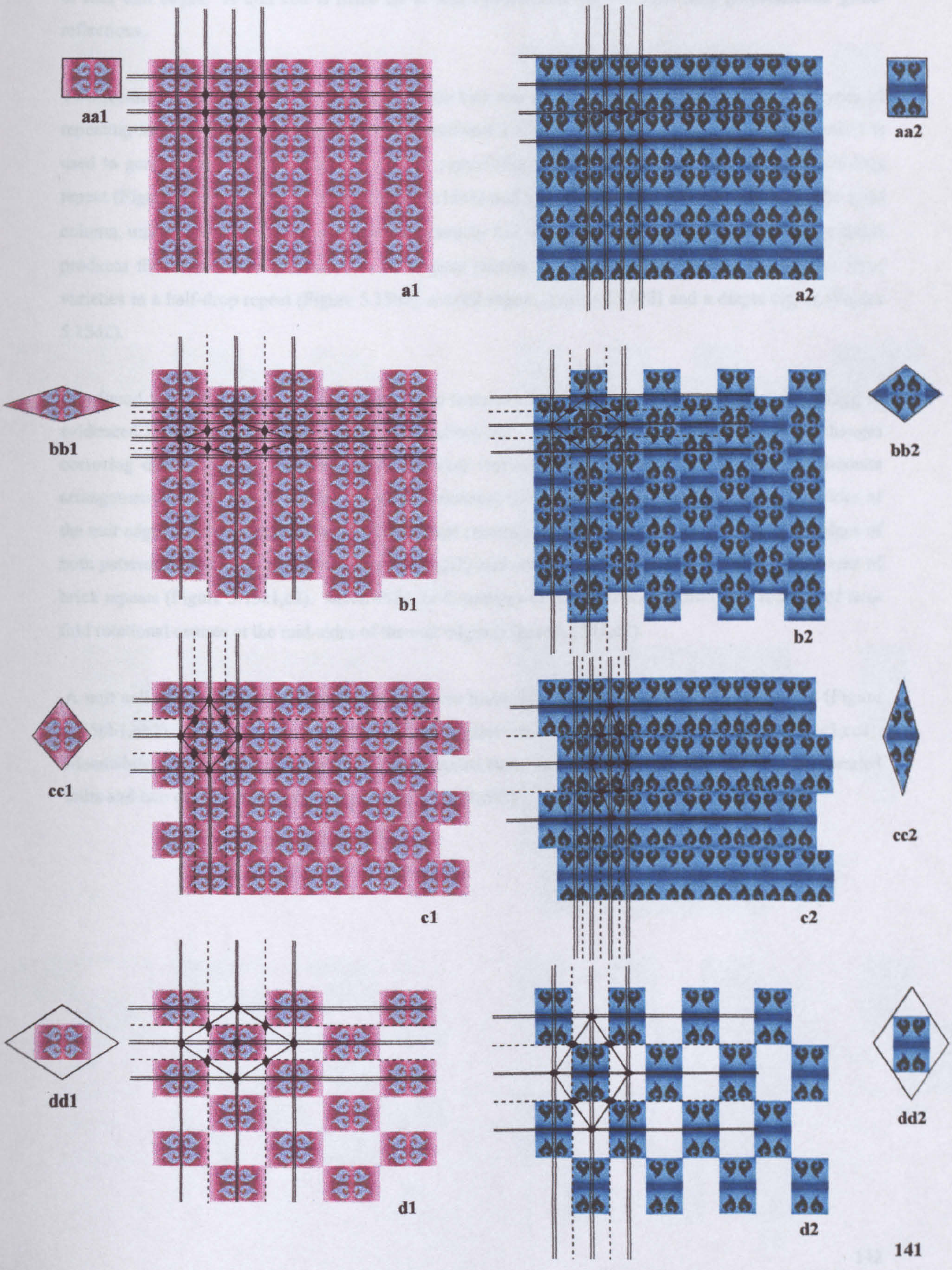
Two features of unit cells are defined, i.e., unit 1 (Figure 5.14aa1) and unit 2 (Figure 5.14aa2). In the left column, unit 1 is used to generate an original pattern in a block repeat (Figure 5.14a1) and three varieties in a half-drop repeat (Figure 5.14b1), a brick repeat (Figure 5.14c1) and a diaper repeat (Figure 5.14d1). In the right column, unit 2 whose boundary is defined by horizontally half-side sliding of unit 1 and admits 90° rotation produces an original pattern in a block repeat (Figure 5.14aa2) and three varieties on the same set of repeating formats (Figure 5.14b2,c2,d2).

Since symmetry classification is applied, it is found that all varieties generated using both unit cells in all three repeating formats obtain symmetry class c2mm in stead of class p2mm as the original patterns. Again glide-reflection is the key to symmetry class changes. But, in this case, the patterns obtain reflections and glide-reflections in both vertical and horizontal directions. A rectangular lattice is thus changed to a centre-celled rectangular lattice. In fact a glide-reflection is a combination of reflection and translation. Therefore, when either half-way translation in a vertical direction of half-drop repeat or in a horizontal direction of brick repeat or alternate arrangement of units and intervals of diaper repeat is applied to repeat a finite design of class d2, the pattern will automatically admit glide-reflections in two directions, typical of symmetry group c2mm.

Each rhomboid-shaped unit cell of half-drop and brick repeats has an equal region as its generating unit (Figure 5.14bb1,bb2,cc1,cc2), while each rhomboid-shaped unit cell of diaper repeats contains one generating unit and its equal intervals (Figure 5.14dd1,dd2).

In the same manner as pattern class p1m1, different unit cells provide both the same and different design outcomes. A pattern generated using unit 1 in a brick repeat (Figure 5.14c1) has the same design as a pattern generated using unit 2 (Figure 5.14b2) in a half-drop repeat when the 90° rotation is applied, while the remaining patterns in half-drop, brick and diaper repeats (Figure 5.14b1,c2,d1,d2) present different design outcomes.

Figure 5.14 a-d Varieties generated using two unit cells of symmetry group p2mm: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.6 Symmetry Group p2gg

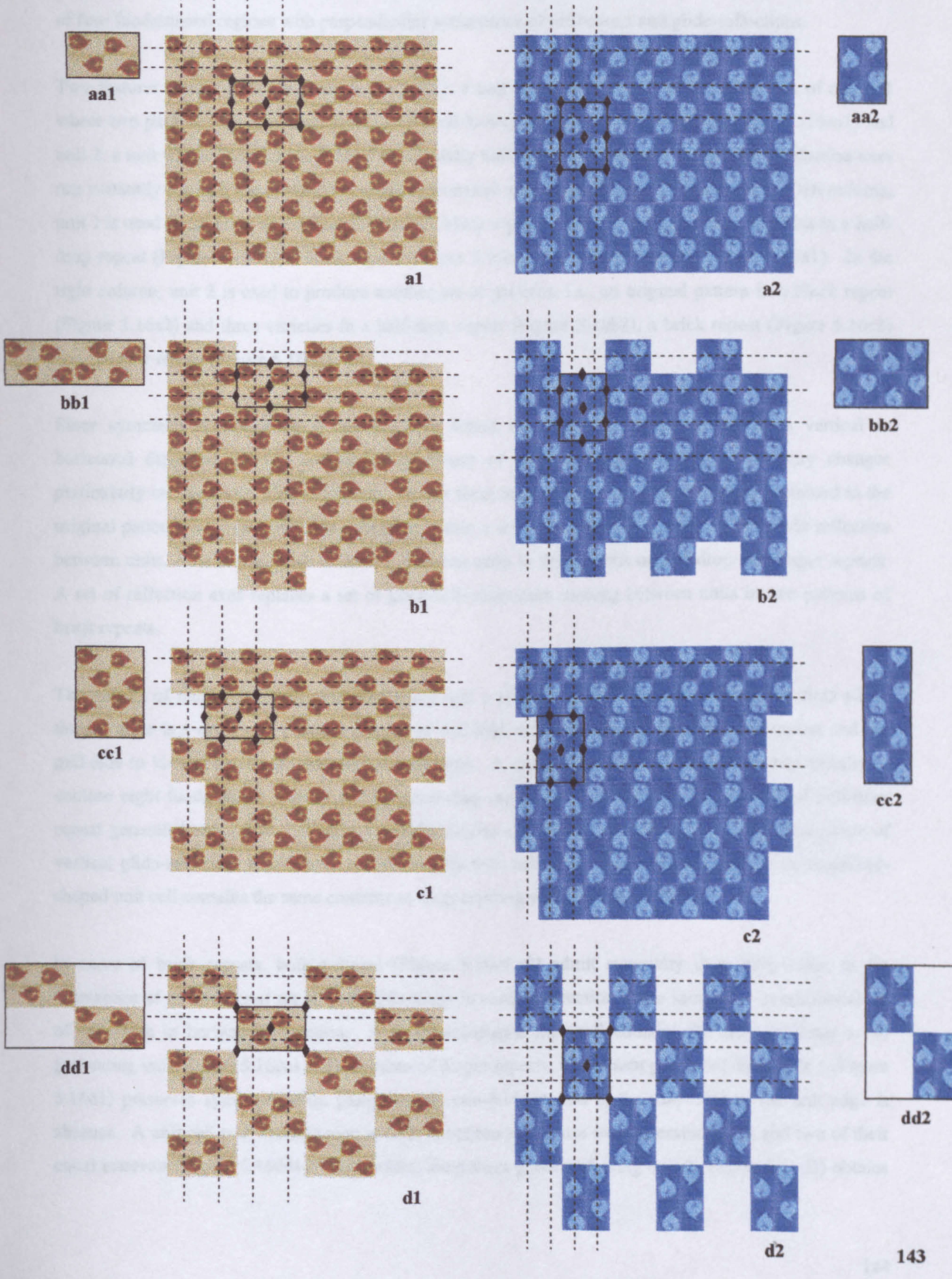
All-over pattern class p2gg admits two-directional glide-reflections where their axes intersect each other at right angles. Two-fold rotations simultaneously occur at the unit centre, four corners and the mid-sides of four unit edges. A unit cell is made up of four fundamental regions admitting perpendicular glide-reflections.

Two features of unit cells are defined to generate two sets of patterns in association with four types of repeating formats, i.e., unit 1 (Figure 5.15aa1) and unit 2 (Figure 5.15aa2). In the left column, unit 1 is used to generate an original pattern in a block repeat (Figure5.15a1) and three varieties in a half-drop repeat (Figure 5.15b1), a brick repeat (Figure 5.15c1) and a diaper repeat (Figure 5.15d1). In the right column, unit 2 whose boundary is defined horizontally half-side sliding of unit 1 and admits 90° rotation produces the other sets of patterns, i.e., an original pattern in a block repeat (Figure 5.15a2) and three varieties in a half-drop repeat (Figure 5.15b2), a brick repeat (Figure 5.15c2) and a diaper repeat (Figure 5.15d2).

It is found that all varieties generated from both features of unit cells preserve symmetry class p2gg as evidenced by two perpendicular sets of glide-reflections. There are some symmetry position changes occurring due to the half-way translation in either vertical and horizontal direction and the alternate arrangement of units and intervals. Two-fold rotational centres at the unit corners and the mid-sides of the unit edges are replaced with two-fold rotational centres at ¼- and ¾-way on the vertical unit edges of both patterns of half-drop repeats (Figure 5.15b1,b2) and on the horizontal unit edges of both patterns of brick repeats (Figure 5.15c1,c2). Meanwhile, both patterns of diaper repeats exhibit the absence of two-fold rotational centres at the mid-sides of the unit edges (Figure 5.15d1,d2).

A unit cell of a half-drop repeat is extended twice horizontally to enclose two generating units (Figure 5.15bb1,bb2). This is the converse of a brick repeat which extends vertically (Figure 5.15cc1,cc2). Meanwhile a unit cell of a diaper repeat is extended twice in both directions to enclose two generated units and two of their equal intervals (Figure 5.15dd1,dd2).

Figure 5.15 a-d Varieties generated using two unit cells of symmetry group p2gg: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.7 Symmetry Group p2mg

All-over pattern class p2mg exhibits reflection in a direction perpendicular to glide-reflection. Two parallel reflection axes intersect two parallel glide-reflection axes at right angles. Two-fold rotational centres locate at the unit centre, the unit corners and the mid-sides of four unit edges. A unit cell consists of four fundamental regions with perpendicular symmetries of reflections and glide-reflections.

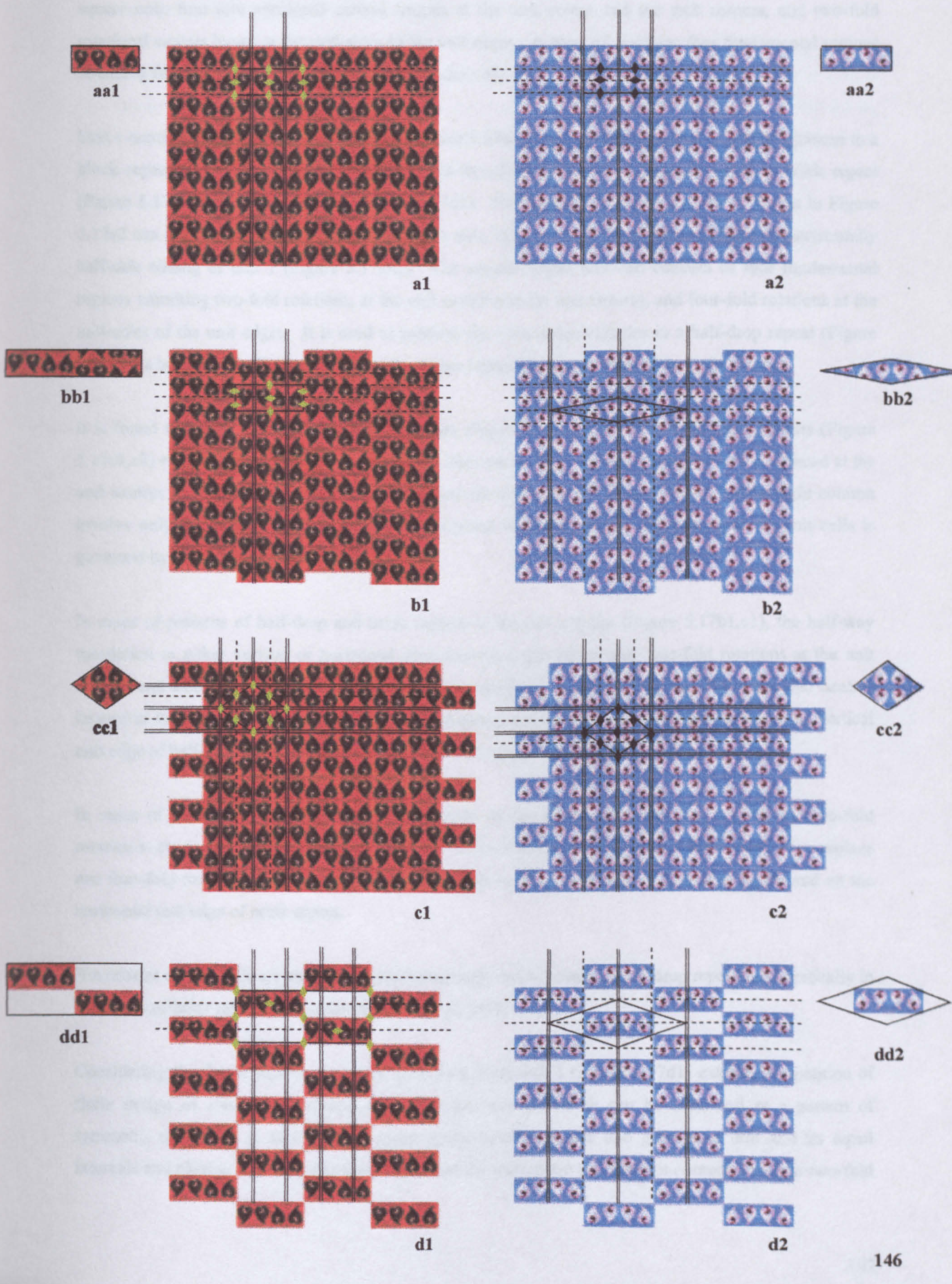
Two features of unit cells are defined, i.e., unit 1: a unit contains two pairs of finite designs of class d1 where two parallel reflection axes locate at ¼- and ¾-way on horizontal unit edges (Figure 5.16aa1) and unit 2: a unit whose boundary is defined horizontally half-side sliding of unit 1, therefore, reflection axes run vertically through the centres and along the vertical unit edges (Figure 5.16aa2). In the left column, unit 1 is used to generate an original pattern in a block repeat (Figure5.16a1) and three varieties in a half-drop repeat (Figure 5.16b1), a brick repeat (Figure 5.16c1) and a diaper repeat (Figure 5.16d1). In the right column, unit 2 is used to produce another set of patterns, i.e., an original pattern in a block repeat (Figure 5.16a2) and three varieties in a half-drop repeat (Figure 5.16b2), a brick repeat (Figure 5.16c2) and a diaper repeat (Figure 5.16d2).

Since symmetry classification is applied, it is found that half-way translation in either vertical or horizontal direction and the alternate arrangement of units and intervals cause symmetry changes particularly on horizontal glide-reflection. In fact there are two types of glide-reflections obtained in the original pattern, one produces glide-reflection within a unit while the other one involves glide-reflection between units. There is no glide-reflection between units in the patterns of half-drop and diaper repeats. A set of reflection axes replaces a set of glide-reflection axes running between units in two patterns of brick repeats.

The pattern of half-drop repeat generated using unit 1 (Figure 5.16b1) preserves symmetry class p2mg though there is a symmetry position change of two-fold rotational centres from the unit corner and the mid-side to ¼- and ¾-way on the vertical unit edge. A unit cell is thus extended twice horizontally to enclose eight fundamental regions of two generating units (Figure 5.16bb1). The pattern of half-drop repeat generated using unit 2 (Figure 5.16bb2) obtains symmetry class c1m1 due to the emergence of vertical glide-reflection whose axes run alternately with reflection axes (Figure 5.16b2). A rhomboid-shaped unit cell contains the same contents as its generating unit (Figure 5.16bb2).

In cases of brick repeats, both patterns (Figure 5.16c1,c2) admit symmetry class c2mm due to the emergence of an additional set of glide-reflections in vertical direction at the same time as additional set of reflection in horizontal direction. A rhomboid-shaped unit cell contains the same contents as its generating unit (Figure 5.16cc1,cc2). In cases of diaper repeats, the pattern generated using unit 1 (Figure 5.16d1) preserves symmetry class p2mg though two-fold rotation at the mid-sides of the unit edge is absence. A unit cell is extended twice in both directions to enclose two generating units and two of their equal intervals (Figure 5.16dd1). Meanwhile, the pattern generated using unit 2 (Figure 5.16d2) obtains symmetry class c1m1 as a result of an additional set of vertical glide-reflections whose axes run alternate with reflection axes. A rhomboid-shaped unit cell contains one generating unit and its equal intervals (Figure 5.16dd2).

Figure 5.16 a-d Varieties generated using two unit cells of symmetry group p2mg: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.8 Symmetry Group p4

All-over pattern class p4 admits four-fold rotation as the highest order of rotation symmetry. Based on a square unit, four-fold rotational centres locates at the unit centre and the unit corners, and two-fold rotational centres locate at the mid-sides of the unit edges. A unit cell contains four fundamental regions obtaining four-fold rotation or a finite design of class c4.

Unit 1 containing a finite design of class c4 (Figure 5.17aa1) is used to constructed an original pattern in a block repeat (Figure 5.17a1) and three varieties in a half-drop repeat (Figure 5.17b1), a brick repeat (Figure 5.17c1) and a diaper repeat (Figure 5.17d1). Nonetheless, the same pattern as shown in Figure 5.17a2 can also be generated from an alternative unit, unit 2, whose boundary is defined by horizontally half-side sliding of unit 1 (Figure 5.17aa2). The square-shaped unit cell consists of four fundamental regions admitting two-fold rotations, at the unit centre and the unit corners, and four-fold rotations at the mid-sides of the unit edges. It is used to produce the other three varieties in a half-drop repeat (Figure 5.17b2), a brick repeat (Figure 5.17c2) and a diaper repeat (Figure 5.17d2) in the right column.

It is found that both patterns of half-drop repeats (Figure 5.17b1,b2) and both of brick repeats (Figure 5.17c1,c2) exhibit symmetry class changes from class p4 to class p2. Four-fold rotations evidenced at the unit centres of two patterns in the left column and the mid-sides of the two patterns in the right column involve only the rotation of four fundamental regions within the units. Repetition of the unit cells is governed by two-fold rotation.

In cases of patterns of half-drop and brick repeats in the left column (Figure 5.17b1,c1), the half-way translation in either vertical or horizontal direction cause the absence of four-fold rotations at the unit corners and also the position changes of two-fold rotational centres. Two two-fold rotational centres located at ¼- and ¾-way replace one two-fold rotational centre located at every mid-side on the vertical unit edge of half-drop repeat and on the horizontal unit edge of brick repeat.

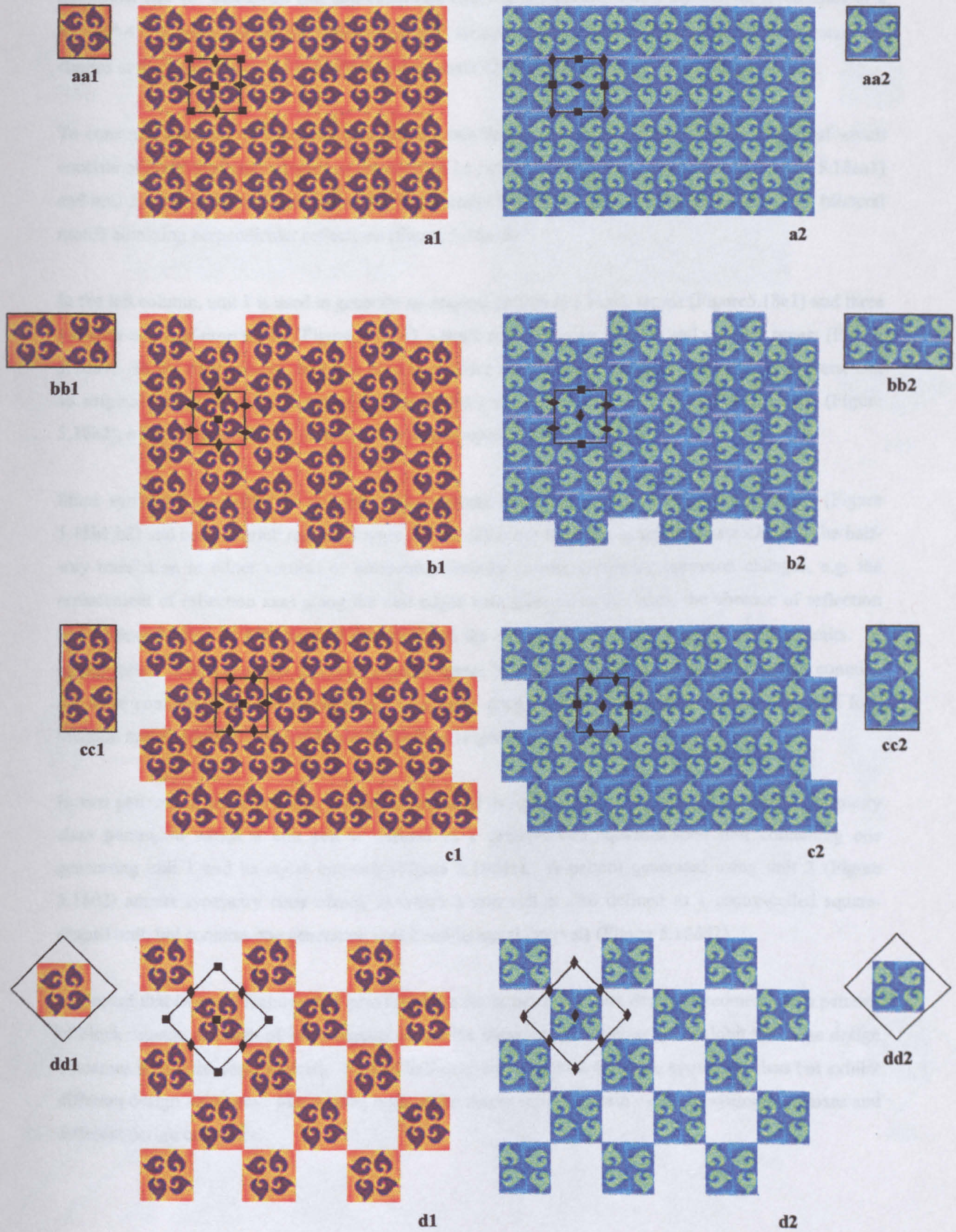
In cases of patterns of half-drop and brick repeats in the right column (Figure 5.17b2,c2), two-fold rotation at every unit corner is absence. Two two-fold rotational centres located at ¼- and ¾-way replace one four-fold rotational centre at every mid-side of the vertical unit edge of half-drop repeat and on the horizontal unit edge of brick repeat.

Boundaries of unit cells are extended twice horizontally in two cases of half-drop repeats and vertically in two cases of brick repeats to enclose two generating units.

Considering the diaper repeats, a pattern generated using unit 1 (Figure 5.17d1) exhibits a repetition of finite design of class c4 alternated with its equal interval, which can be identified as a pattern of symmetry class p4. A centre-celled square-shaped unit encloses one generating unit and its equal intervals and obtains four-fold rotational centres at the unit centre and the unit corners, and four two-fold rotational centres at the mid-sides of the unit edges (Figure 5.17dd1). Meanwhile, a pattern generated using unit 2 (Figure 5.17d2) admits symmetry class p2 as a repetition of a centre-celled square-shaped unit, which contains one generating unit of four fundamental regions admitting two-fold rotation and its equal intervals (Figure 5.17dd2).

It is noted that both patterns of brick repeats generated using whether a finite design of class c4 or a set of four fundamental regions admitting two-fold rotation have the same symmetry class and also exhibit the same design outcome. Both patterns of half-drop repeats preserve the same symmetry class p2 but produce different design outcomes. However, both patterns of diaper repeats exhibit different symmetry classes and design outcomes.

Figure 5.17 a-d Varieties generated using two unit cells of symmetry group p4: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.9 Symmetry Group p4mm

Considered as a translation of a finite design of class d4, all-over pattern class p4mm admits four-fold rotation as the highest order of rotation on a square lattice. Reflection axes run parallel in vertical, horizontal and 45° clockwise and anti-clockwise diagonal directions, which enclose all three sides of a 45°-90°-45° triangle-shaped fundamental region. Glide-reflection axes connect four two-fold rotational centres at the mid-sides of the unit edges and intersect 45° diagonal reflection axes at right angles.

To construct a pattern of symmetry class p4mm, two features of square-shaped unit cells each of which consists of eight fundamental regions are defined, i.e., unit 1: a finite design of class d4 (Figure 5.18aa1) and unit 2: a unit whose boundary locates horizontally half-side sliding of unit 1 contains four bilateral motifs admitting perpendicular reflections (Figure 5.18aa2).

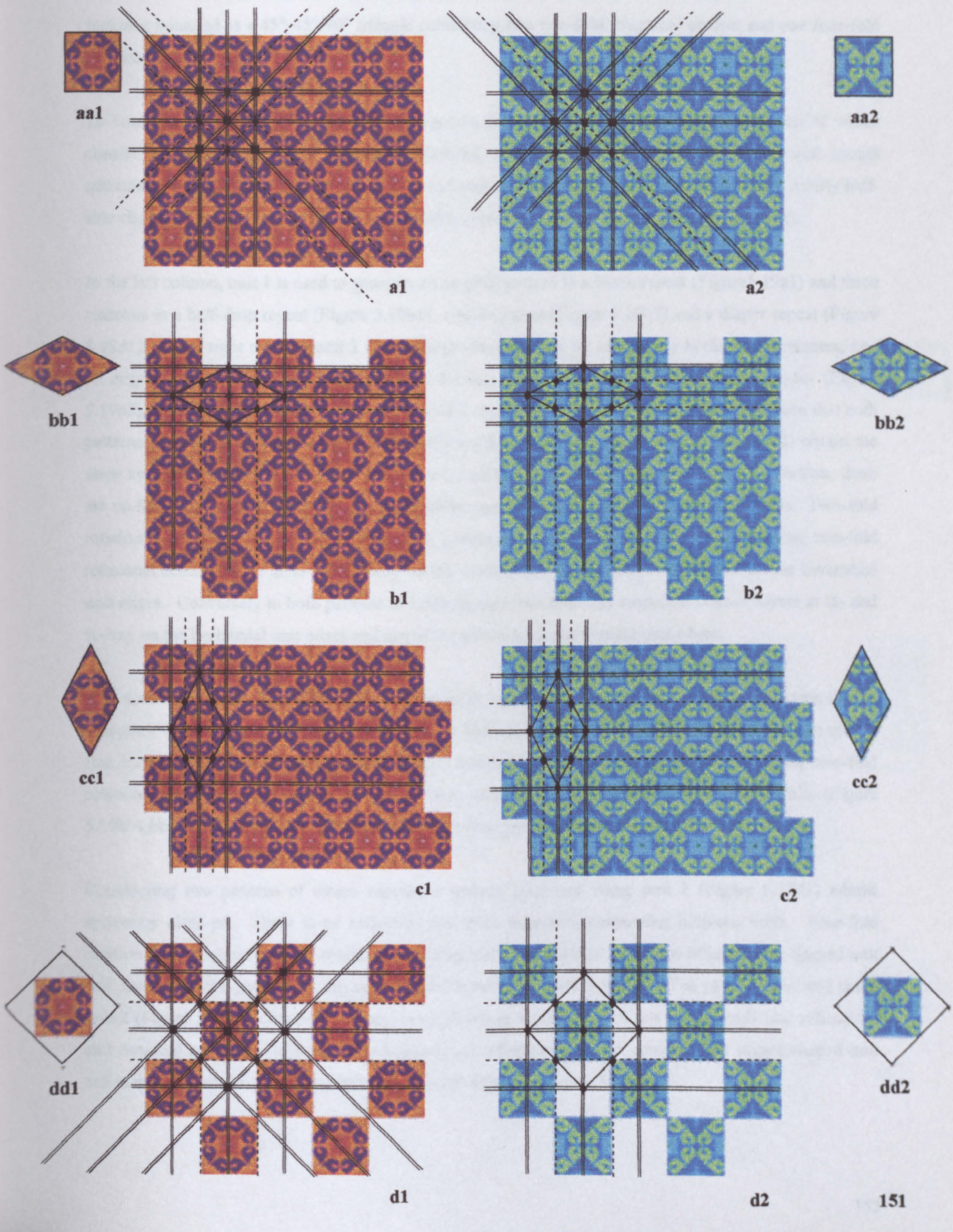
In the left column, unit 1 is used to generate an original pattern in a block repeat (Figure5.18a1) and three varieties in a half-drop repeat (Figure 5.18b1), a brick repeat (Figure 5.18c1) and a diaper repeat (Figure 5.18d1). In the right column, unit 2 is used to produce another set of patterns in the same manners, i.e., an original pattern in a block repeat (Figure 5.18a2) and three varieties in a half-drop repeat (Figure 5.18b2), a brick repeat (Figure 5.18c2) and a diaper repeat (Figure 5.18d2).

Since symmetry classification is applied, it is found that both patterns of half-drop repeats (Figure 5.18b1,b2) and both of brick repeats (Figure 5.18c1,c2) admit the same symmetry class c2mm. The half-way translation in either vertical or horizontal direction causes symmetry operation changes, e.g. the replacement of reflection axes along the unit edges with glide-reflection axes, the absence of reflection and glide-reflection in 45° diagonal direction and the absence of four-fold rotation between units. A square lattice is thus changed to a centre-celled square lattice. Each rhomboid-shaped unit cell contains the same contents as its generating unit, i.e., a finite design of class d4 (Figure 5.18bb1,cc1) and four bilateral motifs admitting perpendicular reflections (Figure 5.18bb2,cc2).

In two patterns of diaper repeats, a pattern generated using unit 1 (Figure 5.18d1) preserves symmetry class p4mm, in which a unit cell is defined as a centre-celled square-shaped unit containing one generating unit 1 and its equal intervals (Figure 5.18dd1). A pattern generated using unit 2 (Figure 5.18d2) admits symmetry class c2mm, in which a unit cell is also defined as a centre-celled square-shaped unit, but contains one generating unit 2 and its equal intervals (Figure 5.18dd2).

It is noted that two unit features may provide either the same or different design outcomes. Both patterns of block repeats and both of brick repeats obtain the same symmetry class and exhibit the same design outcomes within the same formats. Both of half-drop repeats admit the same symmetry class but exhibit different design outcomes. Meanwhile, both of the diaper repeats obtain different symmetry classes and different design outcomes.

Figure 5.18 a-d Varieties generated using two unit cells of symmetry group p4mm: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.3.10 Symmetry Group p4gm

Based on a square lattice, all-over pattern class p4gm exhibits the reflections of four-fold rotations. Reflection axes run through two-fold rotational centres at the mid-sides of the unit edges, therefore, four-fold rotational centres at the unit centres and unit corners are reflections of one another. A fundamental region is bounded as a 45°-45°-90° triangle connecting two two-fold rotational centres and one four-fold rotational centre together.

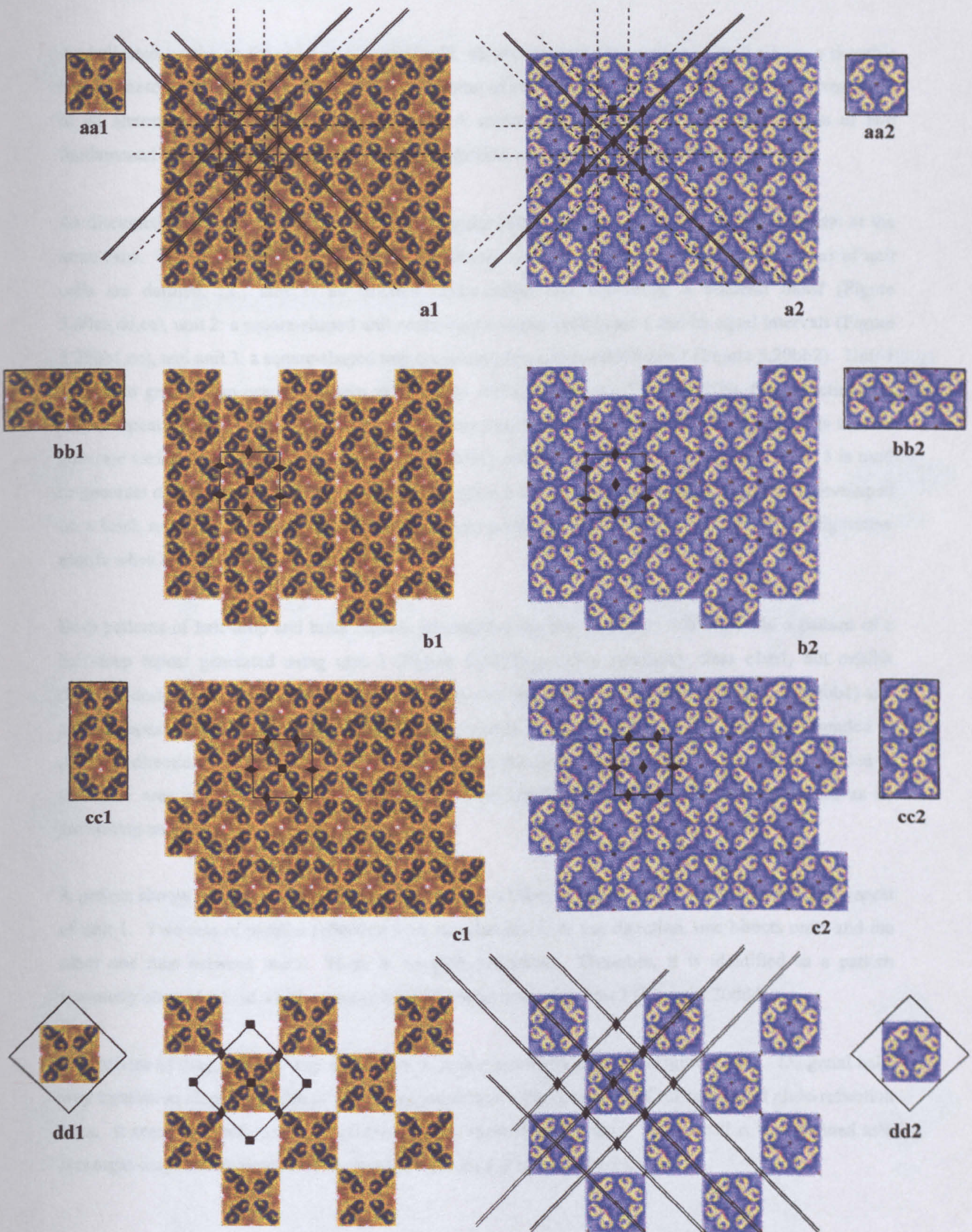
To construct pattern of symmetry class p4gm, two features of square-shaped unit cells each of which consists of eight fundamental regions are defined, i.e., unit 1: a unit contains four bilateral motifs admitting four-fold rotation (Figure 5.19aa1) and unit 2: a unit whose boundary locates horizontally half-side sliding of unit 1 contains four bilateral motifs admitting two-fold rotation (Figure 5.19aa2).

In the left column, unit 1 is used to generate an original pattern in a block repeat (Figure5.19a1) and three varieties in a half-drop repeat (Figure 5.19b1), a brick repeat (Figure 5.19c1) and a diaper repeat (Figure 5.19d1). In the right column, unit 2 is used to produce another set of patterns in the same manners, i.e., an original pattern in a block repeat (Figure 5.19a2) and three varieties in a half-drop repeat (Figure 5.19b2), a brick repeat (Figure 5.19c2) and a diaper repeat (Figure 5.19d2). It is apparently seen that both patterns of half-drop repeats (Figure 5.19b1,b2) and both of brick repeats (Figure 5.19c1,c2) obtain the same symmetry class p2. Due to the half-way translation in either vertical or horizontal direction, there are no four-fold rotation, reflection and glide-reflection that connect one unit to adjacent units. Two-fold rotation is merely symmetry underlying these patterns. In both patterns of half-drop repeats, two-fold rotational centres locate at ¼- and ¾-way on the vertical unit edges and the mid-sides on the horizontal unit edges. Conversely to both patterns of brick repeats, two two-fold rotational centres locate at ¼- and ¾-way on the horizontal unit edges and one at the mid-side on the vertical unit edges.

It is noted that these four varieties are of the same symmetry class and also exhibit the same design outcomes within the same formats. Therefore, all four unit cells contain the same content (i.e. one unit of four bilateral motifs admitting four-fold rotation and one unit of four bilateral motifs admitting two-fold rotation), but in different arrangements: both units of half-drop repeats are arranged horizontally (Figure 5.19bb1,bb2), while both units of brick repeats are arranged vertically (Figure 5.19cc1,cc2).

Considering two patterns of diaper repeats, a pattern generated using unit 1 (Figure 5.19d1) admits symmetry class p4. There is no reflection and glide-reflection connecting between units. Four-fold rotation is the highest order of rotation underlying unit organisation. A centre-celled square-shaped unit cell consists of one generating unit and its equal intervals (Figure 5.19dd1). The pattern generated using unit 2 (Figure 5.19d2) obtains symmetry class p2mm as a result of two sets of perpendicular reflections and two-fold rotation at every intersecting point of reflection axes. A centre-celled square-shaped unit cell is made up of one generating unit and its equal intervals (Figure 5.19dd2).

Figure 5.19 a-d Varieties generated using two unit cells of symmetry group p4gm: unit 1 in the left column and unit 2 in the right column, in a) block repeats, b) half-drop repeats, c) brick repeats and d) diaper repeats
aa Generating unit cells
bb-dd Resultant unit cells



5.4 Variations of All-over Patterns Generated from Centre-celled Symmetry Groups

5.4.1 Symmetry Group c1m1

As indicated by the preface "c" as a centred cell, all-over pattern class c1m1 is built up on a rhombic lattice where a bilateral motif is repeated at the centre of each rhomboid cell. Glide-reflection axes occur in an alternate manner with reflection axes. A centre-celled rectangular unit cell consists of two fundamental regions admitting reflection in one direction, or a finite design of class d1.

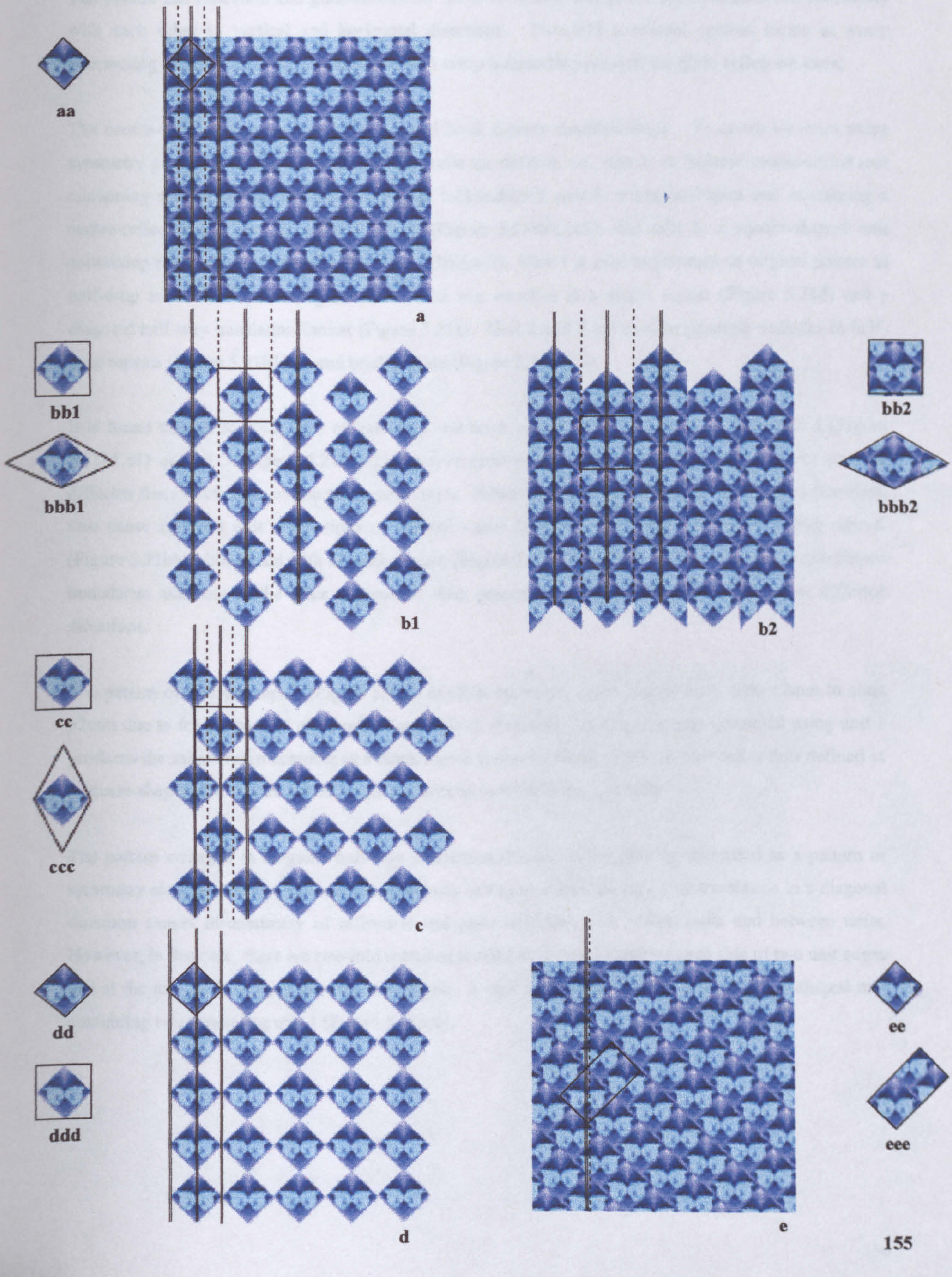
As discussed previously, the centre-celled rectangular lattice exhibits half-drop and brick repeats at the same time. To create varieties associated with different types of repeating formats, three features of unit cells are defined, i.e., unit 1: an isolated centre-celled unit containing a bilateral motif (Figure 5.20aa,dd,ee), unit 2: a square-shaped unit containing a centre-celled unit 1 and its equal intervals (Figure 5.20bb1,cc), and unit 3: a square-shaped unit containing two centre-celled unit 1 (Figure 5.20bb2). Unit 1 is used to generate an original pattern in half-drop and brick repeats (Figure 5.20a), two varieties in a diaper repeat (Figure 5.20d) and a diagonal half-way translation format (Figure 5.20e). Unit 2 is used to generate varieties in a half-drop repeat (Figure 5.20b1) and a brick repeat (Figure 5.20c). Unit 3 is used to generate only a pattern in a half-drop repeat (Figure 5.20b2). There is no variety of unit 3 developed on a brick repeat because of the horizontal half-way translation which causes incomplete configuration motifs when the units are connected.

Both patterns of half-drop and brick repeats generated using unit 2 (Figure 5.20b1,c) and a pattern of a half-drop repeat generated using unit 3 (Figure 5.20b2) preserve symmetry class c1m1, but exhibit different design outcomes. In the left column, both unit cells of a half-drop repeat (Figure 5.20bbb1) and a brick repeat (Figure 5.20ccc) contain the same contents as their generating unit 2, but are bounded in different directions. A unit cell of a half-drop repeat in the right column (Figure 5.20bbb2) is bounded in the same area and direction as the one of the same format but contains two bilateral motifs as its generating unit 3.

A pattern shown in Figure 5.20d may be considered as a block repeat of unit 2 rather than a diaper repeat of unit 1. Two sets of parallel reflection axes run alternately in one direction, one bisects units and the other one runs between units. There is no glide-reflection. Therefore, it is identified as a pattern symmetry class p1m1, in which a unit cell has the same content as unit 2 (Figure 5.20ddd).

The pattern of diagonal half-way translation indicates symmetry class p1 (Figure 5.20e). Diagonal half-way translation along two sides of the unit edges obstructs the continuity of reflection and glide-reflection axes. It seems that reflection and glide-reflection share the same axis. A unit cell is thus defined as a rectangle-shaped unit containing two generating units 1 (Figure 5.20eee).

Figure 5.20 a-e **Varieties generated using three unit cells of symmetry group c1m1 in a) centre-celled format, b1-b2) half-drop repeats, c) brick repeat, d) diaper repeat and e) diagonal half-way translation**
aa-ee **Generating unit cells**
bbb-eee **Resultant unit cells**



5.4.2 Symmetry Group c2mm

In the same manner as with symmetry class c1m1, all-over pattern class c2mm is also built up on the centre-celled rectangular lattice. However, in this case the unit cell contains a finite design of class d2. The pattern has reflection and glide-reflection. Both reflection and glide-reflection axes run alternately with each other in vertical and horizontal directions. Two-fold rotational centres locate at every intersecting point of the reflection axes and also every intersecting point of the glide-reflection axes.

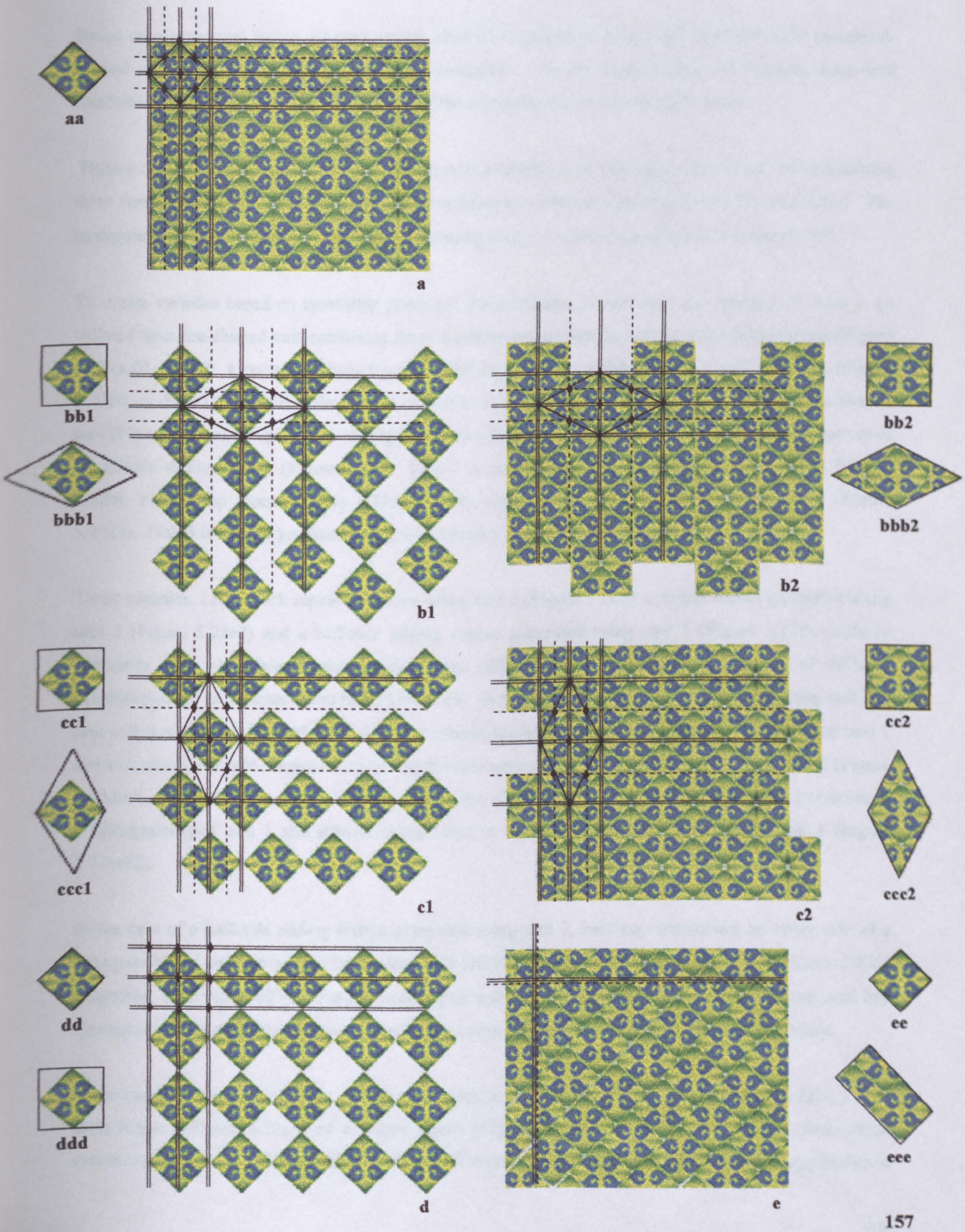
The centre-celled lattice exhibits half-drop and brick repeats simultaneously. To create varieties using symmetry group c2mm, three features of unit cells are defined, i.e., unit 1: an isolated centre-celled unit containing a finite design of class d2 (Figure 5.21aa,dd,ee), unit 2: a square-shaped unit containing a centre-celled unit 1 and its equal intervals (Figure 5.21bb1,cc1), and unit 3: a square-shaped unit containing two centre-celled unit 1 (Figure 5.21bb2,cc2). Unit 1 is used to generate an original pattern in half-drop and brick repeats (Figure 5.21a), and two varieties in a diaper repeat (Figure 5.21d) and a diagonal half-way translation format (Figure 5.21e). Unit 2 and 3 are used to generate varieties in half-drop repeats (Figure 5.21b1,b2) and brick repeats (Figure 5.21c1,c2).

It is found that all four varieties of half-drop and brick repeats generated from whether unit 2 (Figure 5.21b1,c1) or unit 3 (Figure 5.21b2,c2) preserve symmetry class c2mm. Two unit features provide different design outcomes within the same formats. However, half-way translations in different directions also cause different unit orientations of the same unit features. Both unit cells of half-drop repeats (Figure 5.21bbb1,bbb2) and both of brick repeats (Figure 5.21ccc1,ccc2) have the same rhomboid-shaped boundaries and contain the same contents as their generating units, but they are extended in different directions.

The pattern of a diaper repeat (Figure 5.21d) exhibits symmetry class change from class c2mm to class p2mm due to the absence of glide-reflections in both directions. A diaper repeat generated using unit 1 produces the same design outcome as a block repeat generated using unit 2. A unit cell is thus defined as a square-shaped unit containing the same contents as unit 2 (Figure 5.21ddd).

The pattern arranged in diagonal half-way translation (Figure 5.21e) may be identified as a pattern of symmetry class p2. Again, as occurred previously in Figure 5.20e, the half-way translation in a diagonal direction causes discontinuity of reflection and glide-reflection axes within units and between units. However, in this case, there are two-fold rotations located at ¼- and ¾-way on each side of two unit edges and at the mid-sides of the other two unit edges. A unit cell is thus defined as a rectangle-shaped unit containing two generating unit 1 (Figure 5.21eee).

Figure 5.21 a-e Varieties generated using three unit cells of symmetry group c2mm in a) centre-celled format, b1-b2) half-drop repeats, c1-c2) brick repeats, d) diaper repeat and e) diagonal half-way translation
aa-ee Generating unit cells
bbb-eee Resultant unit cells



5.5 Variations of All-over Patterns Generated from Hexagon-based Symmetry Groups

5.5.1 Symmetry Group p3

Based on a hexagonal lattice, all-over pattern class p3 is generated from a 60°-120°-60°-120° rhomboid-shaped unit, which contains two equilateral triangles. As the highest order of rotation, three-fold rotational centres are located at every centre of the triangular cell and every unit corner.

Pattern of symmetry class p3 (Figure 5.22a) reveals a repetition of a hexagon-shaped unit cell containing three fundamental regions admitting three-fold rotation or a finite design of class c3 (Figure 5.22aa). The hexagonal lattice exhibits a half-drop repeat simultaneously as a brick repeat when it is turned 90°.

To create varieties based on symmetry group p3, three features of unit cells are defined, i.e., unit 1: an isolated hexagon-shaped unit containing three fundamental regions admitting three-fold rotation (Figure 5.22aa,ff), unit 2: a rectangle-shaped unit containing a hexagon-shaped unit 1 and intervals (Figure 5.22bb, cc,dd,ee1), and unit 3: the special case of unit 2 in which intervals are equal to a hexagon-shaped unit (Figure 5.22ee2). Apart from an original pattern (Figure 5.22a), unit 1 is used to generate a pattern in a half-side sliding format (Figure 5.22f). Unit 2 is used to produce varieties in a block repeat (Figure 5.22b), a half-drop repeat (Figure 5.22c), a brick repeat (Figure 5.22d) and a diaper repeat (Figure 5.22e1). Unit 3 is used to generate a pattern in a diaper repeat (Figure 5.22e2).

Three varieties, i.e., a brick repeat generated using unit 2 (Figure 5.22d), a diaper repeat generated using unit 3 (Figure 5.22e2) and a half-side sliding format generated using unit 1 (Figure 5.22f), preserve symmetry class p3. Three of them exhibit three different design outcomes as the result of different combinations of unit contents and related intervals. In the case of a brick repeat generated using unit 2, a unit cell is identified as a 60°-120°-60°-120° rhomboid-shaped unit containing a hexagon-shaped unit 1 and two related triangle-shaped intervals whose areas are equal to ⅓ of the hexagon-shaped unit 1 (Figure 5.22ddd). Meanwhile the unit cell of the same shape of a diaper repeat generated using unit 3 consists of a hexagon-shaped unit 1 and interval whose area is equal to a three hexagon-shaped unit 1 (Figure 5.22eee2).

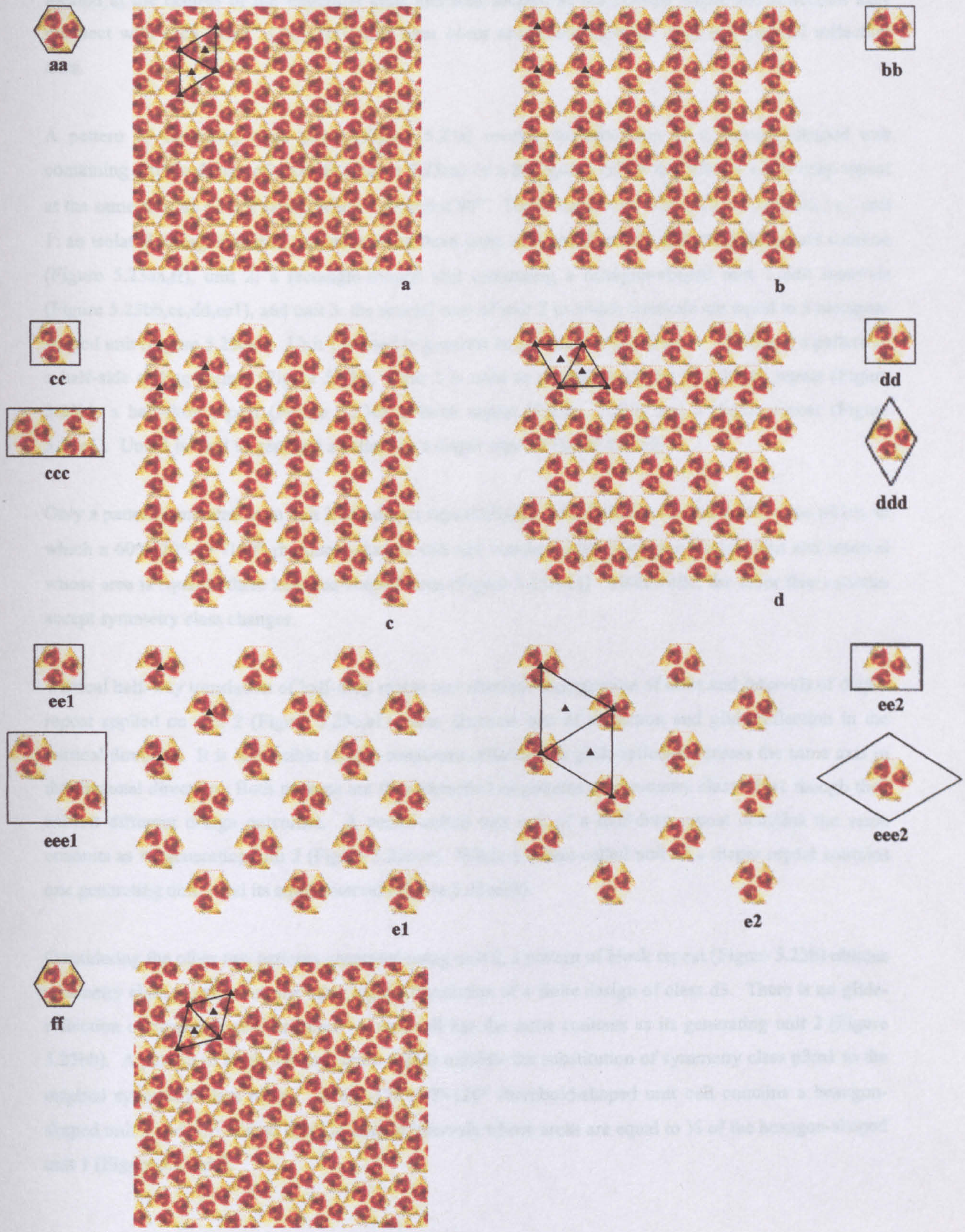
In the case of a half-side sliding format generated using unit 1, half-way translation on every side of a hexagon-shaped unit produces a large-sized unit cell (larger than entire pattern shown in Figure 5.22f). Therefore, a 60°-120°-60°-120° rhomboid-shaped unit containing three configuration regions and two tiny equilateral triangle-shaped intervals may be used as a unit cell arranged on a hexagonal lattice.

In the case of the remaining three varieties generated using unit 2 in a block repeat (Figure 5.22b), a half-drop repeat (Figure 5.22c) and a diaper repeat (Figure 5.22e1), it is found that all of them admit symmetry class p1. Although there is three-fold rotation occurring within a unit, it is inapplicable to achieve a 60°-120°-60°-120° rhomboid-shaped unit cell by connecting four three-fold rotational centres. A block repeat generated using unit 2 is possibly considered as a translation of a finite design of class c3, in which a unit cell consists of the same contents as its generating unit 2 (Figure 5.22bb). While a half-drop repeat and a diaper repeat generated using unit 2 may be considered as a translation of two finite designs of class c3 since the unit cell of a half-drop repeat contains two generating units 2 (Figure 5.22ccc) and the unit cell of a diaper repeat contains two generating units 2 and two of their equal intervals (Figure 5.22eee1).

Figure 5.22 a-f **Varieties generated using three unit cells of symmetry group p3 in a) hexagonal format, b) block repeat, c) half-drop repeat, d) brick repeat, e1-e2) diaper repeats and f) half-side sliding format**

aa-ff **Generating unit cells**

ccc, ddd, eee **Resultant unit cells**



5.5.2 Symmetry Group p31m

Constructed on a hexagonal lattice, all-over pattern class p31m exhibits reflections of three-fold rotation, the highest order of rotation. A 60°-120°-60°-120° rhomboid-shaped unit cell containing two equilateral triangles obtains reflection axes along all sides of the triangular cell. Two three-fold rotational centres located at the centres of the triangular cells and four located at the corners where the reflection axes intersect with each other. Glide-reflection axes occur alternately between each two parallel reflection axes.

A pattern of symmetry class p31m (Figure 5.23a) reveals the repetition of a hexagon-shaped unit containing a finite design of class d3 (Figure 5.23aa) on a hexagonal lattice that admits a half-drop repeat at the same time as a brick repeat when it is turned 90°. Three features of unit cells are defined, i.e., unit 1: an isolated hexagon-shaped unit containing three pairs of bilateral motifs admitting three-fold rotation (Figure 5.23aa,ff), unit 2: a rectangle-shaped unit containing a hexagon-shaped unit 1 and intervals (Figure 5.23bb,cc,dd,ee1), and unit 3: the special case of unit 2 in which intervals are equal to a hexagon-shaped unit (Figure 5.23ee2). Unit 1 is used to generate an original pattern (Figure 5.23a) and a pattern in a half-side sliding format (Figure 5.23f). Unit 2 is used to produce varieties in a block repeat (Figure 5.23b), a half-drop repeat (Figure 5.23c), a brick repeat (Figure 5.23d) and a diaper repeat (Figure 5.23e1). Unit 3 is used to generate a pattern in a diaper repeat (Figure 5.23e2).

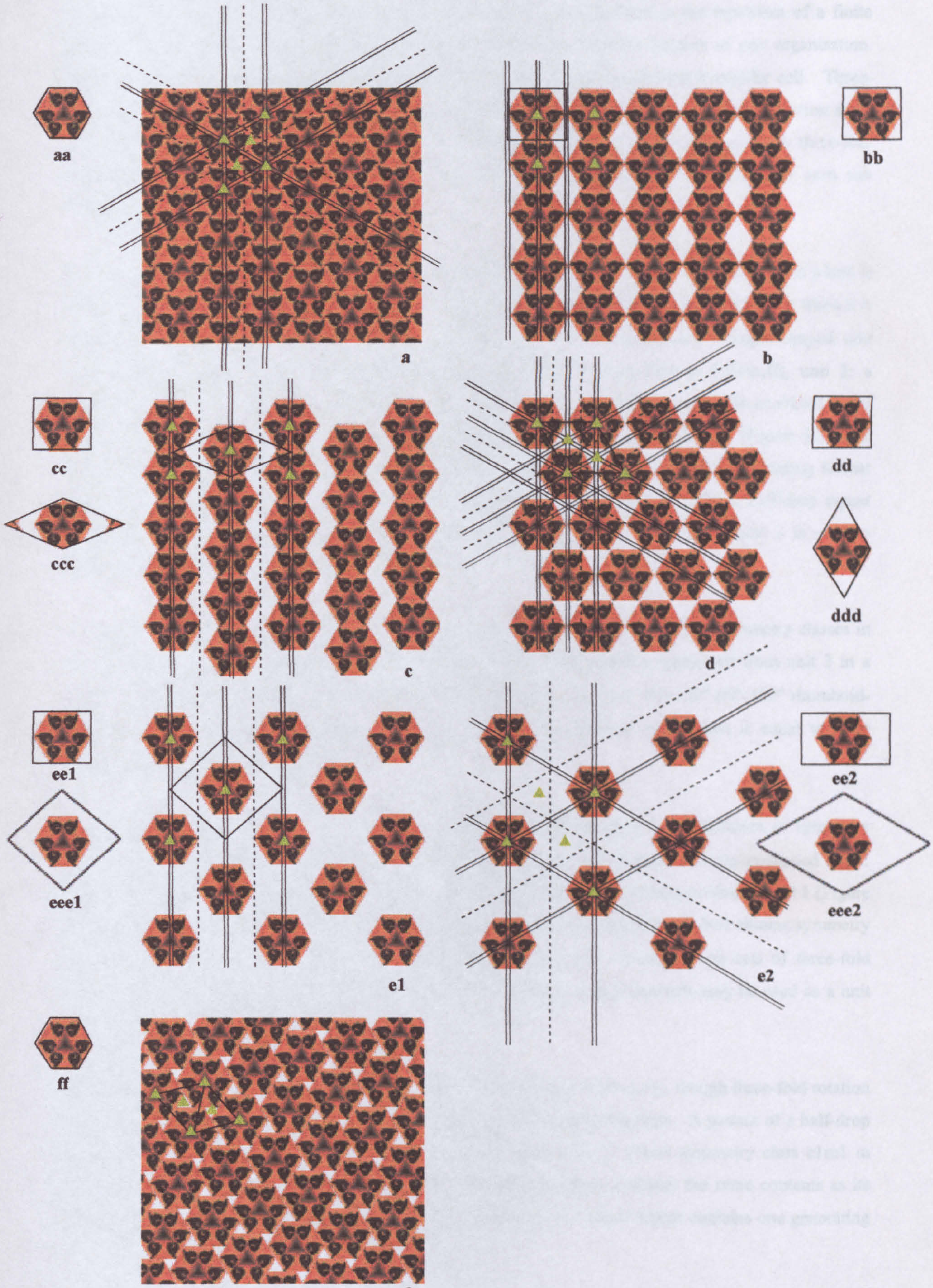
Only a pattern generated from unit 3 in a diaper repeat (Figure 5.23e2) preserves symmetry class p31m, in which a 60°-120°-60°-120° rhomboid-shaped unit cell consists of one hexagon-shaped unit and interval whose area is equal to three hexagon-shaped units (Figure 5.23eee2). Meanwhile, the other five varieties accept symmetry class changes.

Vertical half-way translation of half-drop repeat and alternate arrangement of units and intervals of diaper repeat applied on unit 2 (Figure 5.23c,e1) cause alternate sets of reflection and glide-reflection in the vertical direction. It is impossible to gain consistent reflection or glide-reflection across the same axis in the diagonal direction. Both patterns are thus identified as patterns of symmetry class c1m1 though they exhibit different design outcomes. A centre-celled unit cell of a half-drop repeat contains the same contents as its generating unit 2 (Figure 5.23ccc). While a centre-celled unit of a diaper repeat contains one generating unit 2 and its equal interval (Figure 5.23eee1).

Considering the other two patterns generated using unit 2, a pattern of block repeat (Figure 5.23b) obtains symmetry class p1m1, or on the other hand a translation of a finite design of class d3. There is no glide-reflection occurring in any direction. A unit cell has the same contents as its generating unit 2 (Figure 5.23bb). A pattern of brick repeat (Figure 5.23d) exhibits the substitution of symmetry class p3m1 to the original symmetry class p31m. A 60°-120°-60°-120° rhomboid-shaped unit cell contains a hexagon-shaped unit 1 and two related triangle-shaped intervals whose areas are equal to ⅓ of the hexagon-shaped unit 1 (Figure 5.23ddd).

A pattern generated using unit 1 in a half-side sliding format (Figure 5.23f) admits symmetry class p3. Half-way translation on all sides of a hexagon-shaped unit cell causes discontinuity of reflection and glide-reflection axes in any direction and also produces a large-sized unit cell. In this case, a 60°-120°-60°-120° rhomboid-shaped unit containing two sets of three-fold rotational configuration regions and two tiny equilateral triangle-shaped intervals may be used as a unit cell arranged on a hexagonal lattice.

Figure 5.23 a-f Varieties generated using three unit cells of symmetry group p31m in a) hexagonal format, b) block repeat, c) half-drop repeat, d) brick repeat, e1-e2) diaper repeats and f) half-side sliding format
aa-ff Generating unit cells
ccc, ddd, eee Resultant unit cells



5.5.3 Symmetry Group p3ml

Despite both patterns of symmetry classes p31m and p3ml are described as the repetition of a finite design of class d3 on a hexagonal lattice, however, they obtain different features of unit organisation. The pattern of symmetry class p3ml exhibits reflections within each equilateral-triangular cell. Three-fold rotation centres, the highest order of rotation, locate on every intersecting point of reflection axes, i.e., at the centre of the triangular cell and the unit corner. Reflection axes run through every three-fold rotational centre and are perpendicular with every triangular unit edge. Glide-reflection axes run alternately between two parallel reflection axes.

In the same manner as patterns of symmetry classes p3 and p31m, a pattern of symmetry class p3ml is constructed on a hexagonal lattice that obtains half-drop repeat at the same time as brick repeat when it is turned 90°. Three features of unit cells are defined, i.e., unit 1: an isolated hexagon-shaped unit containing three pairs of bilateral motifs admitting three-fold rotation (Figure 5.24aa,ff), unit 2: a rectangle-shaped unit containing a hexagon-shaped unit 1 and intervals (Figure 5.24bb,cc,dd,ee1), and unit 3: a special case of unit 2 in which intervals are equal to a hexagon-shaped unit (Figure 5.24ee2). Unit 1 is used to generate an original pattern (Figure 5.24a) and a pattern in a half-side sliding format (Figure 5.24f). Unit 2 is used to produce varieties in a block repeat (Figure 5.24b), a half-drop repeat (Figure 5.24c), a brick repeat (Figure 5.24d) and a diaper repeat (Figure 5.24e1). Unit 3 is used to generate a pattern in a diaper repeat (Figure 5.24e2).

It is amazingly found that all varieties of symmetry class p3ml produce a variety of symmetry classes in the same manner as all varieties of symmetry class p31m. Only a pattern generated from unit 3 in a diaper repeat (Figure 5.24e2) preserves symmetry class p3ml, in which a 60°-120°-60°-120° rhomboid-shaped unit cell consists of one hexagon-shaped unit 1 and an interval whose area is equal to three hexagon-shaped units (Figure 5.24eee2).

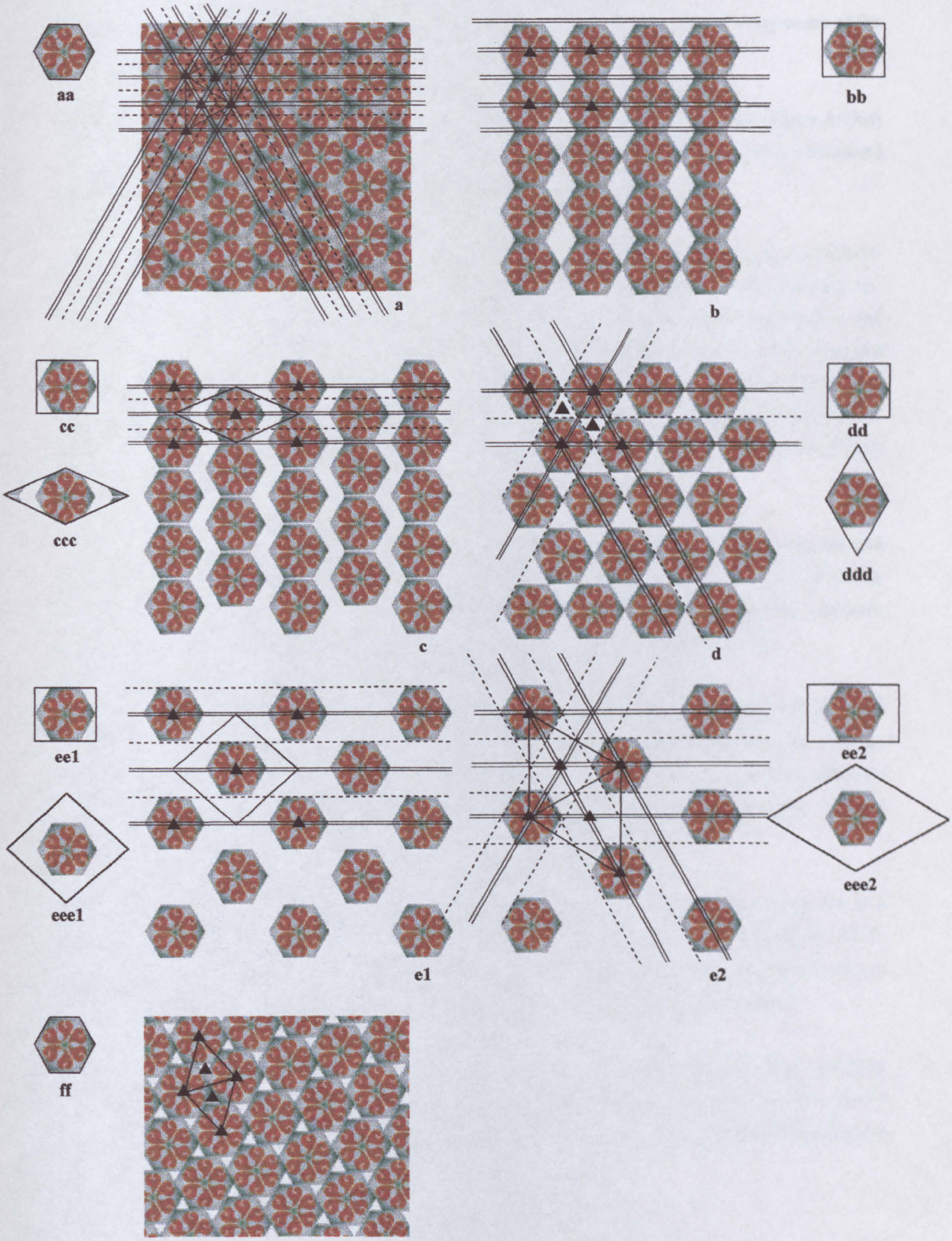
A pattern generated using unit 2 in a brick repeat (Figure 5.24d) accepts the substitution of symmetry class p31m, in which a 60°-120°-60°-120° rhomboid-shaped unit cell contains a hexagon-shaped unit 1 and two related triangle-shaped intervals whose areas are equal to ⅓ of the hexagon-shaped unit 1 (Figure 5.24ddd). A pattern generated using unit 1 in a half-side sliding format (Figure 5.24f) obtains symmetry class p3, in which a 60°-120°-60°-120° rhomboid-shaped unit cell containing two sets of three-fold rotational configuration regions and two tiny equilateral triangle-shaped intervals may be used as a unit cell arranged on a hexagonal lattice.

In the other three remaining patterns generated using unit 2 (Figure 5.24b,c,e1), though three-fold rotation is found within units, the patterns are not constructed on the hexagonal lattices. A pattern of a half-drop repeat (Figure 5.24c) and a pattern of a diaper repeat (Figure 5.24e1) obtain symmetry class c1m1 in different design outcomes. A centre-celled unit of a half-drop repeat contains the same contents as its generating unit 2 (Figure 5.24ccc). While a centre-celled unit of a diaper repeat contains one generating unit 2 and its equal interval (Figure 5.24eee1). A pattern of a block repeat (Figure 5.24b) admits symmetry class p1m1 due to two sets of reflection occurring parallel on one direction, or in the other hand it may be considered as a translation of a finite design of class d3. A unit cell thus contains the same contents as its generating unit 2 (Figure 5.24bb).

Figure 5.24 a-f Varieties generated using three unit cells of symmetry group p3m1 in a) hexagonal format, b) block repeat, c) half-drop repeat, d) brick repeat, e1-e2) diaper repeats and f) half-side sliding format

aa-ff Generating unit cells

ccc, ddd, eee Resultant unit cells



5.5.4 Symmetry Group p6

All-over pattern class p6 is also built up on a hexagonal lattice, however, it accepts the highest order of rotation as a six-fold rotation. A 60°-120°-60°-120°rhomboid-shaped unit cell whose area is equal to two equilateral triangles admits six-fold rotation at every unit corner, three-fold rotation at every centre of the triangular cell, and two-fold rotation at the mid-side of every triangular cell-edge.

Considering a pattern of symmetry class p6 (Figure 5.25a), a hexagon-shaped unit cell (Figure 5.25aa) containing six fundamental regions admitting six-fold rotation or a finite design of class c6 is constructed on a half-drop repeat simultaneously on a brick repeat when it is turned 90°.

To generate varieties associated with different types of repeating formats, three features of unit cells are defined, i.e., unit 1: an isolated hexagon-shaped unit containing six fundamental regions admitting six-fold rotation (Figure 5.25aa,ff), unit 2: a rectangle-shaped unit containing a hexagon-shaped unit 1 and intervals (Figure 5.25bb,cc,dd,ee1), and unit 3: the special case of unit 2 in which intervals are equal to a hexagon-shaped unit (Figure 5.25ee2). Unit 1 is used to generate an original pattern (Figure 5.25a) and a pattern in a half-side sliding format (Figure 5.25f). Unit 2 is used to produce varieties in a block repeat (Figure 5.25b), a half-drop repeat (Figure 5.25c), a brick repeat (Figure 5.25d) and a diaper repeat (Figure 5.25e1). Unit 3 is used to generate a pattern in a diaper repeat (Figure 5.25e2).

In the same manner as varieties of all-over pattern class p3, three patterns of a brick repeat generated using unit 2 (Figure 5.25d), a diaper repeat generated using unit 3 (Figure 5.25e2) and a half-side sliding format generated using unit 1 (Figure 5.25f) preserve symmetry class p6. However, three different combinations of unit contents and related intervals produce three different design outcomes.

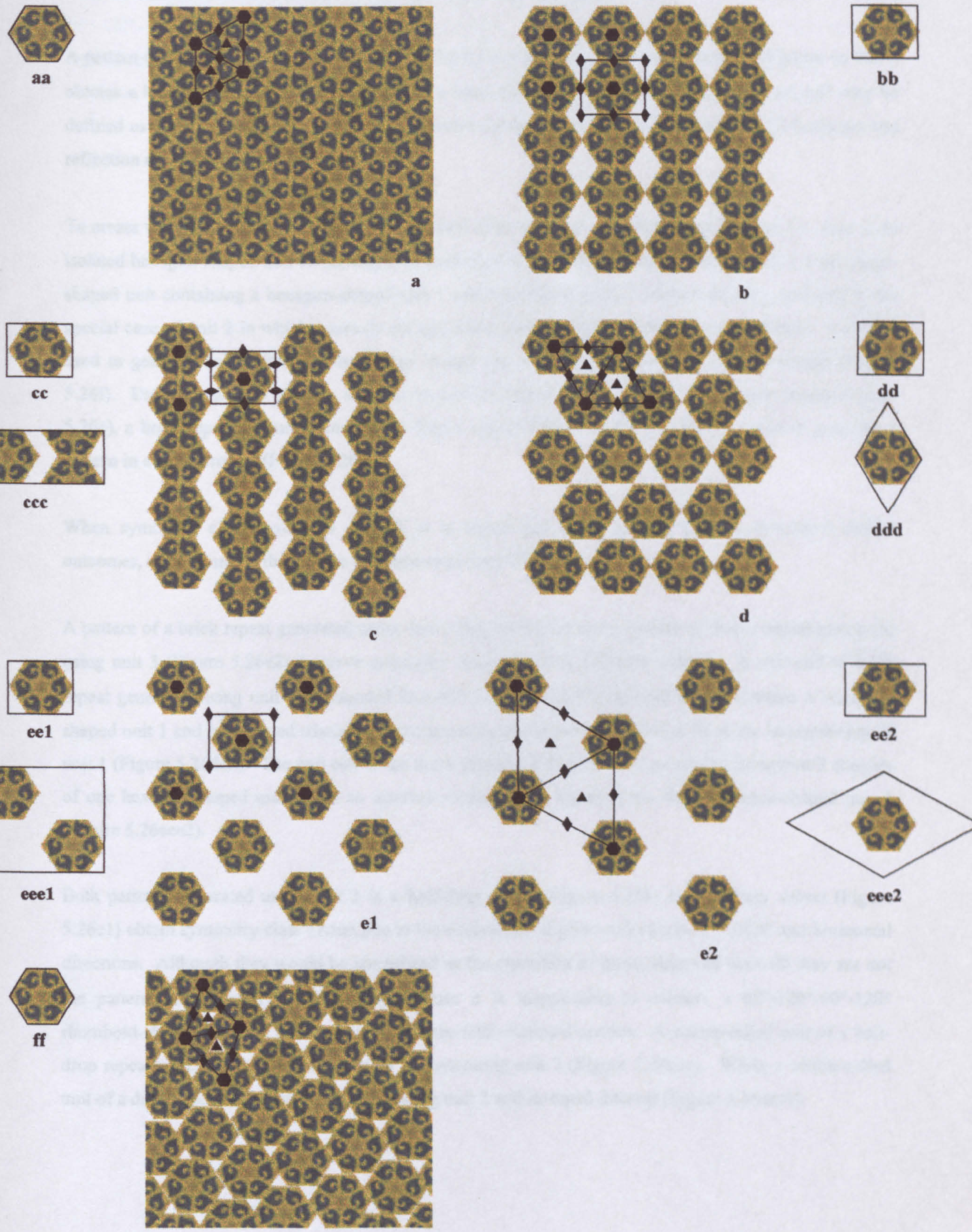
In the case of a brick repeat generated using unit 2, a unit cell is identified as a 60°-120°-60°-120° rhomboid-shaped unit containing a hexagon-shaped unit 1 and two related triangle-shaped intervals whose areas are equal to ⅓ of the hexagon-shaped unit 1 (Figure 5.25ddd). While the unit cell of the same shape of a diaper repeat generated using unit 3 consists of one hexagon-shaped unit 1 and an interval whose area is equal to three hexagon-shaped unit 1 (Figure 5.25eee2).

In the case of half-side sliding format generated using unit 1, half-way translation on every side of a hexagon-shaped unit produces a large-sized unit cell (larger than entire pattern shown in Figure 5.25f). Therefore, a 60°-120°-60°-120° rhomboid-shaped unit containing six configuration regions and two tiny equilateral triangle-shaped intervals may be used as a unit cell arranged on a hexagonal lattice.

The other three remaining patterns generated using unit 2 in a block repeat (Figure 5.25b), a half-drop repeat (Figure 5.25c) and a diaper repeat (Figure 5.25e1) exhibit symmetry class p2. Despite six-fold rotation occurs at every unit centre it is inapplicable to gain a 60°-120°-60°-120° rhomboid-shaped unit by connecting four six-fold rotational centres. They may be considered as a translation of finite design of class c6, in which two-fold rotation is the symmetry that underlies the unit organisation.

A unit cell of a block repeat generated using unit 2 contains the same contents as its generating unit (Figure 5.25bb) and obtains two-fold rotations at the unit corners and the mid-sides of the unit edges. The vertical half-way translation of a half-drop repeat generated using unit 2 produces two-fold rotations at ¼- and ¾-way on the vertical unit edges and the mid-sides of the horizontal unit edges, therefore, a unit cell is extended twice horizontally to enclose two generating units 2 (Figure 5.25ccc). The alternate arrangement of units and intervals of a diaper repeat generated using unit 2 exhibits two-fold rotation only at the unit corner, therefore, a unit cell is extended twice in both directions to enclose two generating units 2 and two of their equal intervals (Figure 5.25eee1).

Figure 5.25 a-f **Varieties generated using three unit cells of symmetry group p6 in a) hexagonal format, b) block repeat, c) half-drop repeat, d) brick repeat, e1-e2) diaper repeats and f) half-side sliding format**
aa-ff **Generating unit cells**
ccc, ddd, eee **Resultant unit cells**



5.5.5 Symmetry Group p6mm

All-over pattern class p6mm obtains six-fold rotation as the highest order of rotation together with reflection. Reflection axes enclose all three sides of every fundamental region by connecting three different n-fold rotational centres, i.e., two-fold, three-fold and six-fold.

A pattern of symmetry class p6mm (Figure 5.26a) is basically constructed on a hexagonal lattice so that it obtains a half-drop repeat at the same time as a brick repeat when it is turned 90°. A unit cell may be defined as a hexagon-shaped unit containing twelve fundamental regions admitting six-fold rotation and reflection or a finite design of class d6.

To create varieties based on symmetry class p6mm, three features of unit cells are defined, i.e., unit 1: an isolated hexagon-shaped unit containing a finite design of class d6 (Figure 5.26aa,ff), unit 2: a rectangle-shaped unit containing a hexagon-shaped unit 1 and intervals (Figure 5.26bb,cc,dd,ee1), and unit 3: the special case of unit 2 in which intervals are equal to a hexagon-shaped unit (Figure 5.26ee2). Unit 1 is used to generate an original pattern (Figure 5.26a) and a pattern in a half-side sliding format (Figure 5.26f). Unit 2 is used to produce varieties in a block repeat (Figure 5.26b), a half-drop repeat (Figure 5.26c), a brick repeat (Figure 5.26d) and a diaper repeat (Figure 5.26e1). Unit 3 is used to generate a pattern in a diaper repeat (Figure 5.26e2).

When symmetry classification is applied, it is found that every pattern exhibits individual design outcomes, while some of them share the same symmetry classes.

A pattern of a brick repeat generated using unit 2 (Figure 5.26d) and a pattern of diaper repeat generated using unit 3 (Figure 5.26e2) preserve symmetry class p6mm in different features. A unit cell of brick repeat generated using unit 2 is bounded in a 60°-120°-60°-120° rhomboid which contains a hexagon-shaped unit 1 and two related triangle-shaped intervals whose area are equal to ⅓ of the hexagon-shaped unit 1 (Figure 5.26ddd). The unit cell of the same shape of a diaper repeat generated using unit 3 consists of one hexagon-shaped unit 1 and an interval whose area is equal to the three hexagon-shaped unit 1 (Figure 5.26eee2).

Both patterns generated using unit 2 in a half-drop repeat (Figure 5.26c) and a diaper repeat (Figure 5.26e1) obtain symmetry class c2mm due to the occurrence of glide-reflections in vertical and horizontal directions. Although they would be considered as the repetition of finite design of class d6 they are not the patterns of symmetry class p6mm because it is inapplicable to achieve a 60°-120°-60°-120° rhomboid-shaped unit cell by connecting four six-fold rotational centres. A centre-celled unit of a half-drop repeat contains the same contents as its generating unit 2 (Figure 5.26ccc). While a centre-celled unit of a diaper repeat consists of one generating unit 2 and its equal interval (Figure 5.26eee1).

A pattern generated using unit 2 in a block repeat (Figure 5.26b) obtains symmetry class p2mm, or is considered as a translation of finite design of class d6 having two-fold rotation between units. A unit cell contains the same contents as its generating unit 2 (Figure 5.26bb). Meanwhile, a pattern generated using unit 1 in a half-side sliding format (Figure 5.26f) exhibits symmetry class p6. It is inapplicable to gain reflection across the same axis due to the half-way translation on every side of a hexagon-shaped unit. A 60°-120°-60°-120° rhomboid-shaped unit containing twelve configuration regions and two tiny equilateral triangle-shaped intervals may thus be used as a unit cell arranged on a hexagonal lattice.

Figure 5.26 a-f **Varieties generated using three unit cells of symmetry group p6mm in a) hexagonal format, b) block repeat, c) half-drop repeat, d) brick repeat, e1-e2) diaper repeats and f) half-side sliding format**
aa-ff **Generating unit cells**
ccc, ddd, eee **Resultant unit cells**

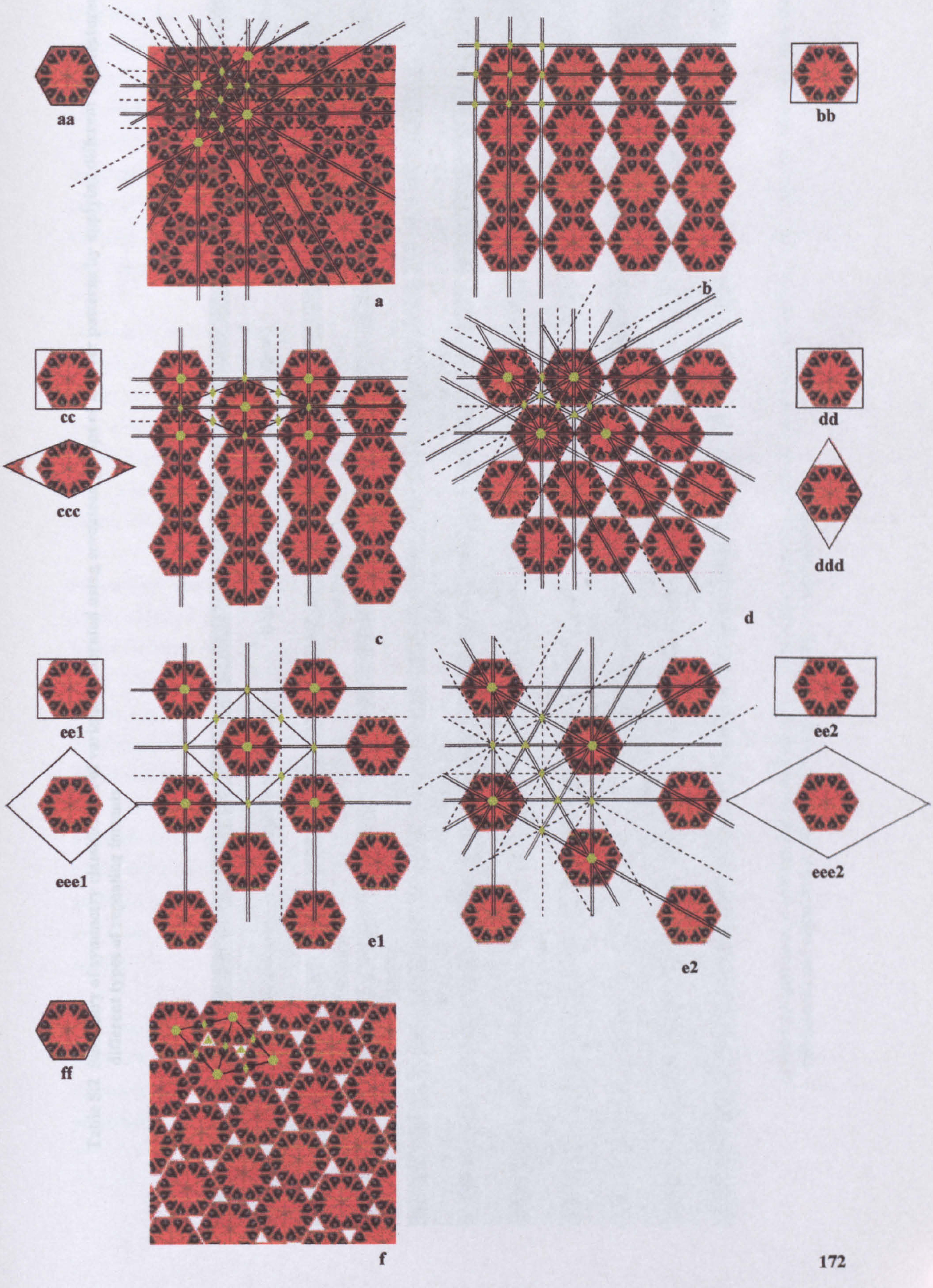


Table 5.2 Summary of symmetry classification of varieties generated using seventeen groups of all-over patterns by applying different unit features and different types of repeating formats

Pattern symmetry class	Repeating format					
	Block repeat	Half-drop repeat	Brick repeat	Diaper repeat	Diagonal half-way translation	Half-side sliding format
p1	**p1**	p1	p1	p1	-	-
p1m1	**p1m1/*p1m1***	c1m1/*c1m1*	c1m1/*c1m1*	c1m1/*c1m1*	-	-
p1g1	**p1g1/*p1g1***	p1g1/*c1m1*	c1m1/*p1g1*	p1g1/*p1g1*	-	-
p2	**p2/*p2***	p2/*p2*	p2/*p2*	p2/*p2*	-	-
p2mm	**p2mm/*p2mm***	c2mm/*c2mm*	c2mm/*c2mm*	c2mm/*c2mm*	-	-
p2gg	**p2gg/*p2gg***	p2gg/*p2gg*	p2gg/*p2gg*	p2gg/*p2gg*	-	-
p2mg	**p2mg/*p2mg***	p2mg/*c1m1*	c2mm/*c2mm*	p2mg/*c1m1*	-	-
p4	**p4/*p4***	p2/*p2*	p2/*p2*	p4/*p2*	-	-
p4mm	**p4mm/*p4mm***	c2mm/*c2mm*	c2mm/*c2mm*	p4mm/*c2mm*	-	-
p4gm	**p4gm/*p4gm***	p2/*p2*	p2/*p2*	p4/*p2mm*	-	-
c1m1	-	**c1m1**/*c1m1*/c1m1	**c1m1**/*c1m1*	p1m1	p1	-
c2mm	-	**c2mm**/*c2mm*/c2mm	***c2mm***/*c2mm*/c2mm	p2mm	p2	-
p3	*p1*	**p3**/*p1*	**p3**/*p3*	*p1*/p3	-	p3
p31m	*p1m1*	**p31m**/*c1m1*	**p31m**/*p3m1*	*c1m1*/p31m	-	p3
p3m1	*p1m1*	**p3m1**/*c1m1*	**p3m1**/*p31m*	*c1m1* / p3m1	-	p3
p6	*p2*	**p6**/*p2*	**p6**/*p6*	*p2*/p6	-	p6
p6mm	*p2mm*	**p6mm**/*c2mm*	**p6mm**/*p6mm*	*c2mm*/p6mm	-	p6

(Noted: normal cases = patterns generated from unit 1, italic cases = patterns generated from unit 2, underlying cases = patterns generated from unit 3, emboldened and emboldened italic cases = original patterns, - = no experiment)

5.6 Discussion of Results

Data from Table 5.2 and all illustrations of 124 representative patterns provided a further understanding of the nature of symmetry in pattern. All varieties which were generated from sixteen symmetry groups (except symmetry group p1) indicated that patterns constructed on the same repeating format but generated using different features of unit cells may preserve the same symmetry class as their generating symmetry group but probably in different design outcomes, or exhibit different symmetry classes from the generating symmetry group. In cases of symmetry groups p2 and p2gg, two features of unit cells produced two varieties of patterns in every repeating format, all of which preserved symmetry classes p2 and p2gg as their generating symmetry groups. This means that no symmetry class change has occurred in all repeating formats of these two symmetry groups.

Meanwhile, two/three features derived from symmetry groups p1m1, p2mm, c1m1 and c2mm produced patterns of the same symmetry classes in some repeating formats. In cases of symmetry groups p1m1 and p2mm, all varieties in three formats, i.e., half-drop, brick and diaper repeats generated using both unit cells exhibited symmetry classes c1m1 and c2mm respectively. In cases of symmetry groups c1m1 and c2mm, patterns generated using three unit cells in half-drop and brick repeats preserved symmetry classes c1m1 and c2mm as their generating symmetry groups, while patterns generated using unit 1 in diaper repeats and diagonal half-way translation formats exhibited different symmetry classes.

Nonetheless, varieties of the ten remaining symmetry groups, i.e., p1g1, p2mg, p4, p4mm, p4gm, p3, p31m, p3m1, p6 and p6mm, generated using two/three unit cells, admitted different symmetry classes within some repeating formats.

In the context of design outcomes, both original patterns in block repeats of square-/rectangle-/ parallelogram-based symmetry groups generated using two features of unit cells exhibited identical design outcomes. Due to the combinations of symmetries in parallel directions (horizontal half-side sliding of unit boundaries and horizontal half-way translation), both patterns of symmetry groups p2mg, p4 p4mm and p4gm in brick repeats presented identical design outcomes of pattern classes c2mm, p2, c2mm and p2 respectively. However, in cases of symmetry groups p1m1 and p2mm, the patterns generated using unit 2 in half-drop repeats exhibited identical design outcomes of patterns classes c1m1 and c2mm respectively as the patterns generated using unit 1 in brick repeats when 90° rotation was applied.

Focusing attention on symmetry groups without reflection and glide-reflection (i.e. groups p1, p2, p3, p4 and p6) it was found that these were applicable to any determined repeating formats without producing symmetry class changes. In addition, not every pattern generated from one of these symmetry groups preserved the same symmetry class in every format.

Varieties in all repeating formats of symmetry groups p1 and p2 preserved their own symmetry classes. In cases of symmetry group p4, only a pattern generated using unit 1 in a diaper repeat preserved symmetry class p4 while the others obtained symmetry class p2 as the two-fold rotation of a finite design of class c4 or a set of four motifs admitting two-fold rotation. Meanwhile, the hexagon-based varieties of symmetry groups p3 and p6 obtained symmetry class changes in the same manner. Patterns of brick repeats generated using unit 2, diaper repeats generated using unit 3 and half-side sliding formats generated using unit 1, preserved their own symmetry classes p3 and p6 respectively. However, varieties that were not constructed on the hexagonal lattices, i.e., patterns generated using unit 2 in block repeats, half-drop repeats and diaper repeats, exhibited symmetry class changes from class p3 to class p1 and class p6 to class p2. It was found that certain combinations of unit features and repeating formats exhibited symmetry operation changes of n-fold rotations underlying the unit organisation.

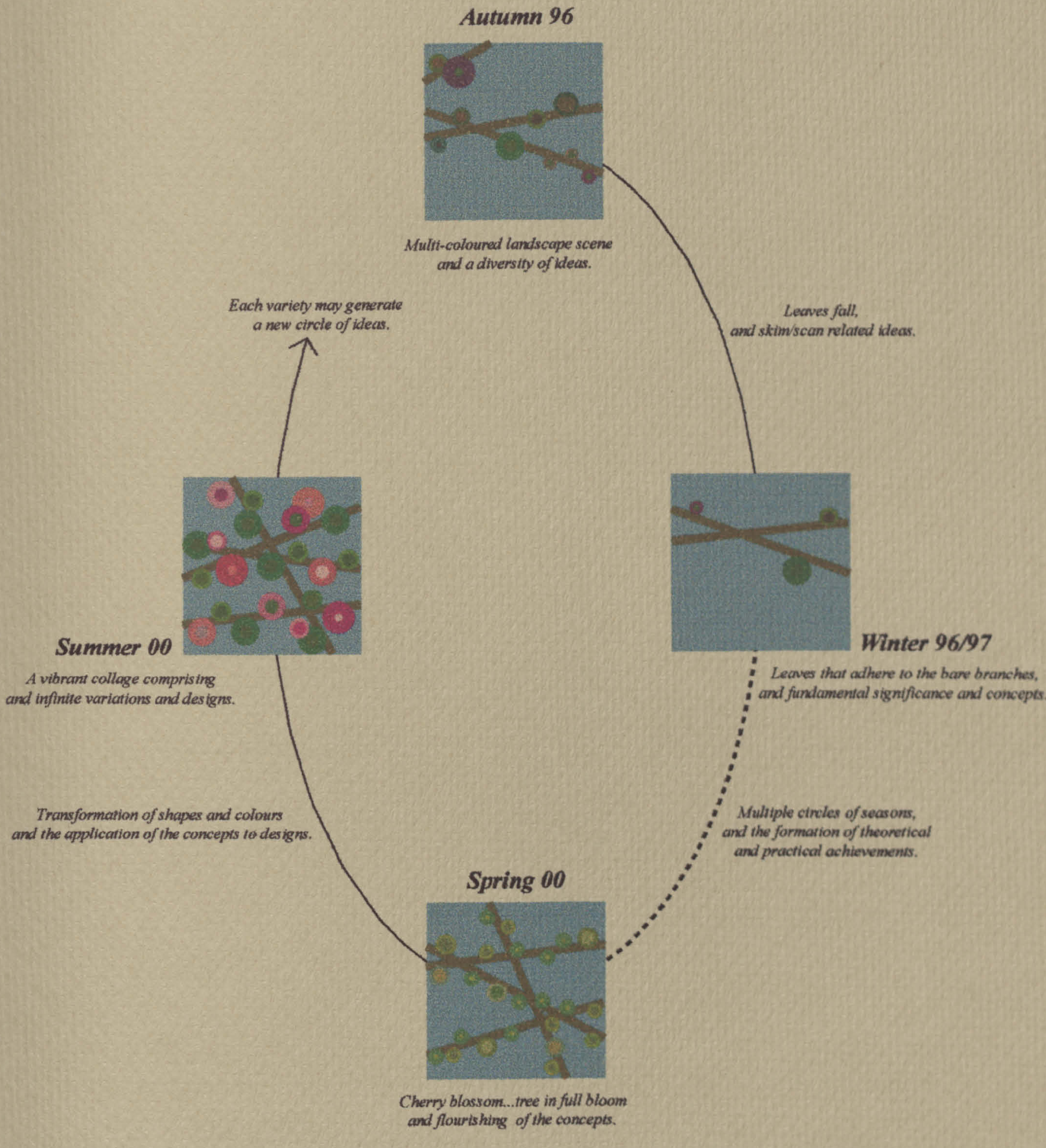
There were some interchanges of symmetry groups occurring between patterns from two symmetry classes when certain combinations of unit features and repeating formats were applied. The first case was found in brick repeats generated using unit 2 of symmetry groups p31m and p3m1. A pattern of symmetry group p31m in a brick repeat obtained symmetry class p3m1 at the same time as a pattern of symmetry group p3m1 in a brick repeat admitted symmetry class p31m.

The second case was found in diaper repeats of symmetry groups p1m1 and c1m1. The alternate arrangement of bilateral motifs (finite designs of class d1) and intervals produced glide-reflection axes along the unit edges instead of reflection axes, by which affected symmetry class change from class p1m1 to class c1m1. While the alternate arrangement of centre-celled bilateral motifs (finite designs of class d1) and intervals replaced glide-reflection axes between units with reflection axes that affected symmetry class change from class c1m1 to class p1m1.

There was an interchange between symmetry groups p2mm and c2mm in diaper repeats where the alternate arrangement of finite designs of class d2 and intervals and centre-celled finite designs of class d2 and intervals caused the replacement between reflection axes and glide-reflection axes in two directions. Both patterns of symmetry group p2mm generated using both features of repeating units in diaper repeats obtained symmetry class c2mm, while a pattern of symmetry group c2mm in a diaper repeat admitted symmetry class p2mm. The fourth case was found within the same class of symmetry class p1g1. The interchange between symmetry groups p1g1 and c1m1 occurred in two pairs of patterns of half-drop and brick repeats. In the half-drop repeats, a pattern generated using unit 1 preserved symmetry group p1g1 and a pattern generated using unit 2 admitted symmetry group c1m1, while, in the brick repeats, a pattern generated using unit 1 admitted symmetry group c1m1 and a pattern generated using unit 2 preserved symmetry group p1g1.

Varieties of unit translation (e.g. half-way translation in a vertical direction of a half-drop repeat and in horizontal direction of a brick repeat or alternate arrangement of units and intervals of a diaper repeat) may result in the absence or the interchange of symmetry operations between units not only reflection and glide-reflection but also n-fold rotations. Certain relationships between n-fold rotations were found when certain combinations of unit features and repeating formats were applied. Patterns admitting three-fold rotation with or without reflection and glide-reflection (i.e. classes p3, p3ml and p31m) exhibited symmetry operation changes to patterns without rotation (i.e. classes pl, plml and clml). Patterns admitting four- and six-fold rotations with or without reflection and glide-reflection (i.e. classes p4, p4mm, p4gm, p6 and p6mm) exhibited symmetry operation changes to patterns admitting two-fold rotations (i.e. classes p2, p2mm and c2mm).

It should be noted that various combinations of symmetry groups contained in the unit cells and symmetry operations of unit organisation governed by individual repeating formats may not only determine different symmetry classes or different design outcomes of resultant patterns, but may also produce configuration designs which affect the extension of repeating unit boundaries.



The development of each stage of the research synchronised to four orderly seasons

Chapter 6 Summary and Conclusion

Whilst undertaking this research programme, it has been necessary for the author to go through a series of methodological and empirical cycles and iterations. At the beginning, the view was unordered and unclear and as various ideas and avenues were examined some fell aside like the leaves in autumn. However, some buds adhered, survived the austere winter period and begun to blossom in spring and summer. This process was repeated over the years and finally the healthiest buds were identified, nurtured and the investigation bore fruit. The author has attempted to visualise this important process in a manner which will be meaningful within the culture of the design community (see figure on previous page).

Symmetry is a natural and inevitable property inherent in every repeating pattern. It not only represents the visual effects of harmony, balance and order, but also exhibits the geometrical rules of space. Since pattern is generated from the unity of mathematical principles and the aesthetic input of its creator, the question that of whether the beauty of a pattern is due to the craft of the designer or intrinsic to its geometry arises. Kappraff addressed the issue as follows:

> *"... beautiful designs must both exhibit a free flow of creative energy from the designer to the work and obey the invisible hand restraining design due to the geometric constraints of space. More often than not, the designer is not conscious of these constraints; however, the success of a design depends to a large degree on how well the artist is attuned to the problems and possibilities presented by these constraints.... Symmetry is a concept that has inspired the creative works of artists and scientists; it is the common root of artistic and scientific endeavour. To an artist or architect symmetry conjures up feelings of order, balance, harmony, and an organic relation between the whole and its parts. On the other hand, making these notions useful to a mathematician or scientist requires a precise definition. Although such a definition may make the idea of symmetry seem less flexible than the artist's intuitive feeling of it, that precision can actually help designers unravel the complexities of a design and see greater possibilities for symmetry in their own work. It can also lead to practical techniques for generating patterns."*
>
> [Kappraff, 1991, p.405]

An understanding of geometric symmetry would appear to offer the designer a useful potential tool enhancing the systematic understanding of regular repeating patterns and at the same time offering infinite possibilities for pattern creation. Abas and Salman [1995, pp.33-34] recognised the importance of symmetry in the context of human perception. They maintained that symmetry not only offers efficiency in grasping, storing and recalling information but also gives the brain the power to predict and this in turn enhances sense of confidence and security. Moreover, it implies economy in manufacture by which the uniform components are used repeatedly to create a variety of combinations, as is the case with a modular construction system.

An important aspect of this thesis is to explore the potential of geometric symmetry in the context of pattern analysis and pattern synthesis. Chapter 2 has unlocked the theoretical principles of geometric symmetry and the classification system. One design may be distinguished from others by its inherent symmetry characteristics. Four types of transformations, i.e., translation, rotation, reflection and glide-reflection, known also as symmetry operations, are associated with regular repeating designs. A motif undergoes repetition with respect to a collection of one or more of these four symmetry operations, the combination of which will determine its symmetry group or class.

The classification of designs with reference to their symmetry characteristics is dependent upon dividing designs into one of three categories, i.e., finite designs, band patterns or all-over patterns. A finite design is further classified into two categories, i.e., a finite design of class cn or dn, where n is an integer. Meanwhile, a band pattern and an all-over pattern are sub-divided into seven and seventeen classes respectively. Each class is denoted by a widely accepted notation which has its origin in the discipline of crystallography.

Chapter 3 verified that symmetry classification offers potential as an analytical tool for pattern analysis in the context of traditional Thai textile patterns and in the further understanding of Thai cultural significance in these patterns. Hann observed that precise classificatory tools, such as symmetry group analysis, aid hypothesis formation and systematic data testing [Hann, 1993, p.47].

Data tested from six categories of Thai textile patterns revealed that patterns from different categories share particular symmetry preferences. The non-random distribution of symmetry classes is of fundamental significance for it reflects that symmetry classification is in some way culturally sensitive. Certain symmetry characteristics exhibited may be associated with certain patterning techniques and aspects of Thai culture. It is therefore reasonable to accept the two hypotheses which are firstly, that while varieties of Thai textiles may fulfil different functions and will be produced using different patterning techniques, symmetry characteristics are nonetheless broadly shared and secondly, the symmetry characteristics exhibited may be closely associated with and explained in the context of Thai culture and may in some way be manifestations of traditional Thai beliefs and Buddhist philosophy. Certain similarities with Indian or other Southeast Asian textiles seem apparent although this avenue has not been explored at this stage and offers an opportunity for further research on cross-cultural comparison among Southeast Asian countries and the primarily influential Indian culture.

In the context of Thai contemporary pattern design, traditional patterns have been transformed to new designs probably in different arrangements. Certain motifs may require particular structures to preserve Thai characteristic. For example, if the divinity figures are not constructed either in a stepped square shape of symmetry class p4mm or p2mm, or in a lotus-bud or rhombic shape of symmetry class c1m1 or p1m1, will the pattern be identified as a Thai pattern? This may lead to a subjective area involving visual identity which may underpin the creation of new designs in which designers develop their own creativity onto traditional resources. Examples are given in Figure 6.1-6.2.

Figure 6.1a-g Six patterns (b-g) generated using *prajumyam* motifs from the decorative ceiling of Wat Phrasri-rattana-saddharam, Bangkok (a).

Figure 6.2a-g Six patterns generated using divinity motifs from *pha lai-yang* pattern (a)
Source: (a) was reproduced from Sachio, 1993

a b c d e f g

Chapters 4 and 5 have explored how an extensive variety of designs can be developed based on the unvarying principles of geometric symmetry. Chapter 4 offered a discussion of geometric symmetry in the context of pattern construction by designers. It has been emphasised that designers must take account of the rules of geometric symmetry and associated techniques in order to effectively create a new design. The process of pattern construction may be initiated either i) from the space sub-dividing approach by which a pattern structure is built up by applying the point connection technique on one of the two fundamental lattices (i.e. a square lattice or an isometric lattice), or ii) from the space filling approach by which an existing structure is modified and transformed by filling design contents into each repeating unit.

It should be noted that not all of the construction techniques presented in this chapter can be applied directly to every design as some of them require more than just the direct application of existing techniques. A method that is practical for one pattern may be inapplicable for another pattern. Meanwhile the combination of more than one method can thus be developed as a special case.

A rough design layout allows designers to see coverage relationship of all parts of elements connected from one repeat to succeeding repeats in order to avoid gaps, optical stripes or other visual defects which would occur in the final designs. There is a range of repeating formats which determine the construction of square-/rectangle-shaped units. Varieties of design contents can then be regularly developed on every unit boundary.

The distribution of motifs within a unit could be governed by one or a collection of the four symmetry operations. It is noted that different designs may fit different symmetry characteristics. For example, the figurative interlocking patterns whose units are not bounded by straight lines at all sides may be applicable to translation, rotation and glide-reflection rather than reflection, as evidenced from the absence of patterns of classes p1m1, p4mm and p6mm among 144 of Escher's designs [Schattschneider, 1986, pp.94-95]. However, the greater degree of rotation together with reflection (e.g. symmetry classes p4mm and p6mm) will produce the more dynamic kaleidoscopic-type designs. A combination of upright motifs with reflection or glide-reflection may work well on the continuous designs where two or more widths are linked and hung as a curtain or stretched as a wall-covering. Meanwhile, in the case of carpet tiles, reflection along four sides of a square unit produces the configuration designs which may benefit the extension of repeats in all directions.

Designers may also employ a regular geometric structure (e.g. tiling patterns or Islamic polygonal networks) as a guideline aiding motif distribution. Not every uniform unit is necessarily filled by the same contents. One structure may thus be able to be transformed to a wide range of patterns (e.g. interlocking, connected or isolated patterns) dependent upon the relationship between motifs and background applied.

A range of weave structure formats may exhibit irregular arrangement of marked points within the repeating units but produce regular repetition in the all-over patterns. Each square marked point which is symmetric in all directions may be substituted by any kind of motifs in any alignments. A set of marked points is moreover not necessarily represented by the motifs with the same shape, size or orientation. Additional details can subsequently be filled in the intervals between the motifs to create a connected design. .

Chapter 5 has investigated variation of designs possibly generated from each of seventeen symmetry groups of all-over patterns by applying alternative features of unit cells and unit translations. Combination of symmetry group within a repeating unit and symmetry operation between units underpinned by the repeating format identified the symmetry class of a resultant pattern. Some patterns preserve the same symmetry classes as their generating symmetry groups, but in different design outcomes. While some are totally different in terms of symmetry groups and design outcomes.

Symmetry group p1 produced three varieties of symmetry class p1 in three repeating formats (i.e. half-drop repeat, brick repeat and diaper repeat). Combinations of two features of unit cells derived from each of nine square-/rectangle- and parallelogram-based symmetry groups (i.e. p1m1, p1g1, p2, p2mm, p2gg, p2mg, p4, p4mm and p4gm) and three repeating formats (i.e. half-drop repeat, brick repeat and diaper repeat) generated fifty-four design varieties of nine symmetry classes (i.e. p1g1, p2, p2mm, p2gg, p2mg, p4, p4mm, c1m1 and c2mm). Combinations of three features of unit cells derived from each of two centre-celled symmetry groups (i.e. c1m1 and c2mm) and four repeating formats (i.e. half-drop repeat, brick repeat, diaper repeat and diagonal half-way translation) produced eleven design varieties of six symmetry classes (i.e. p1, p1m1, p2, p2mm, c1m1 and c2mm). Combinations of three features of unit cells derived from each of five hexagon-based symmetry groups (i.e. p3, p3m1, p31m, p6 and p6mm) and five repeating formats (i.e. block repeat, half-drop repeat, brick repeat, diaper repeat and half-side sliding format) produced thirty design varieties of eleven symmetry classes (i.e. p1, p1m1, p2, p2mm, c1m1, c2mm, p3, p3m1, p31m, p6 and p6mm).

In some cases, the half-way translation in either the vertical or the horizontal direction and the alternate arrangement of units and intervals caused changes in symmetry operations between units (e.g. from reflection to glide-reflection and vice versa, three-fold to one-fold rotation and six- or four-fold to two-fold rotation), and also in symmetry orientations, that resulted either in symmetry class changes (e.g. from class p1m1 to c1m1 and vice versa, p2mm to c2mm and vice versa, p3 to p1, p4 and p6 to p2), or in particular features of the same symmetry classes. Changes of symmetry orientation sometimes caused an ambiguity in symmetry classification.

It was found that different features of unit cells produced varieties either of the same symmetry class as their generating symmetry group or in different symmetry classes within the same repeating format, and also that there were some interchanges of symmetry groups occurring between patterns from two symmetry classes, which were evidenced in four cases, i.e., between symmetry classes p31m and p3m1 in brick repeats, p1m1 and c1m1 in diaper repeats, p2mm and c2mm in diaper repeats, and p1g1 and c1m1 in half-drop and brick repeats.

It was not always the case that symmetry groups without reflection and glide-reflection could admit any of the repeating formats without producing symmetry class changes. Only varieties generated from symmetry groups p1 and p2 preserved their own symmetry classes in all repeating formats. Whereas varieties generated from symmetry group p3 exhibited symmetry class changes to class p1, and so do varieties generated from symmetry groups p4 and p6 to class p2, since certain repeating formats are applied.

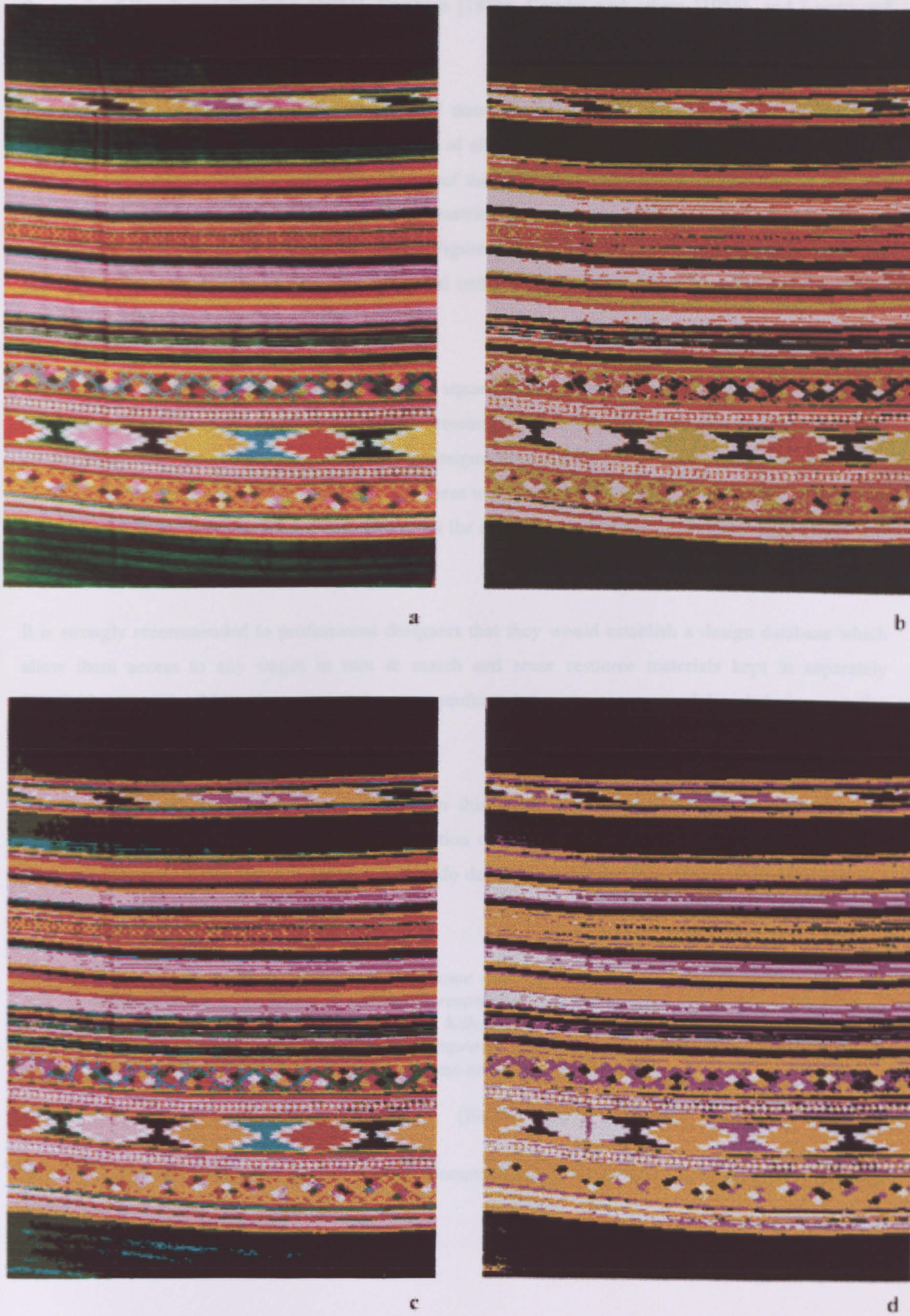
Varieties of unit contents and unit translations discussed in chapter 5 represent only a small amount of all possible features that may create design varieties of each symmetry group. There are also other features (e.g. combinations of multiple symmetry groups contained within a unit cell and other fractional sliding of unit translation in either vertical, horizontal or diagonal direction, e.g., ¼-, ¾-way drop or brick formats), that may require further investigation.

Developments in computer technology has tremendously impacted on the contemporary design sphere. Metropolis Design Group predicted the major change in the 21st century that people tend to work from home, where they can connect to others around the world by the global networks [Metropolis, 2000]. Designers may then communicate with their clients and manufacturers through digital data or images on screens instead of hard prints or tactile models.

Computers have become standard pieces of equipment aiding the design process and allowing development from concept to manufacture. Laborious time-consuming tasks during the process of repeating pattern design has also been executed by the use of CAD, which is available from general computer graphics in multi-disciplinary design agencies to particular manufacturing programmes in high-technology factories. The more sophisticated that the programme is, the greater ranges of facilities are installed for specific functions. As seen from CAD for various textile techniques (e.g. printing, knitting and weaving), there is a range of automatic functions relating particularly to repeat arrangement and colour modification contained. The exploitation of CAD has a potential not only to enhance creativity in a technical viewpoint but also to attune design perspective to production possibility. Diverse designs may occur by applying some modification functions. Figure 6.3, for example, exhibits three design possibilities when automatic four- and eight-colour reductions are applied.

Although CAD provides design a feasibility to be produced practically and economically, but at the same time it may impose some design constraints. As John Maeda, the director of the MIT Media Lab's Aesthetics and Computation Group, commented, if designers are only the computer users, their designs may be influenced by the computer programmers' imagination. Therefore, a new hybrid generation of designers could be computer programmers whose *goals are to make art* [Jacobs, 1998, p.62]. This may be evidenced from a number of innovative designs developed from the geometric principles of repeating patterns mentioned in chapter 4 (e.g. fractal designs, Stereograms, Photomosaics and Maeda's computer-code graphics), all of which have been produced by particular software programmes.

Figure 6.3a-d Illustrations showing three design varieties generated from an original pattern (a): (b) four-colour reduction by Sirix, (c) and (d) eight- and four-colour reduction by Tex-Design.



The potential future of CAD development in the context of algorithm aiding regular repeating pattern construction and graphic manipulation would appear to be the significant issue. Related research includes the work of Kosek and Koskova [1984], Dunham [1986], Cervini and others [1986], and Loreto and others [1993].

Terrazzo (available in Corel PhotoPaint version 7 onward), for example, is a programme enabling the instant production of each of the seventeen groups of all-over patterns from a variety of graphic sources. However, the constraint imposed by given shapes of the polygonal units may limit variation of resultant patterns and simultaneously produce strong geometric characteristics. This could be applicable for isolated and patchwork-liked patterns as seen in Figure 5.3-5.5, but not for interlocking patterns whose unit-edges do not fit straight edges of the polygonal units. Further discussion related to CAD for pattern design is presented in Appendix A1-A2.

The principles of dotted configuration shared by a square lattice, a computer pixel grid and a diagram of stitch/raised yarn identify avenues for further research in the aspect of design/textile technology. Designers need to be aware of a limitation of designs imposed by production techniques and also a potential of technology employed in order to use them to advantage. For example, geometric shapes (e.g. triangle, square and hexagonal) may not appear on the actual knitted fabric as straight-boundary designs as a result of tensions between stitches.

It is strongly recommended to professional designers that they would establish a design database which allow them access to any stages to mix & match and reuse resource materials kept in separately files/folders, and is able to be connected to a manufacturing system, as a model and design samples suggested in Appendix A3.

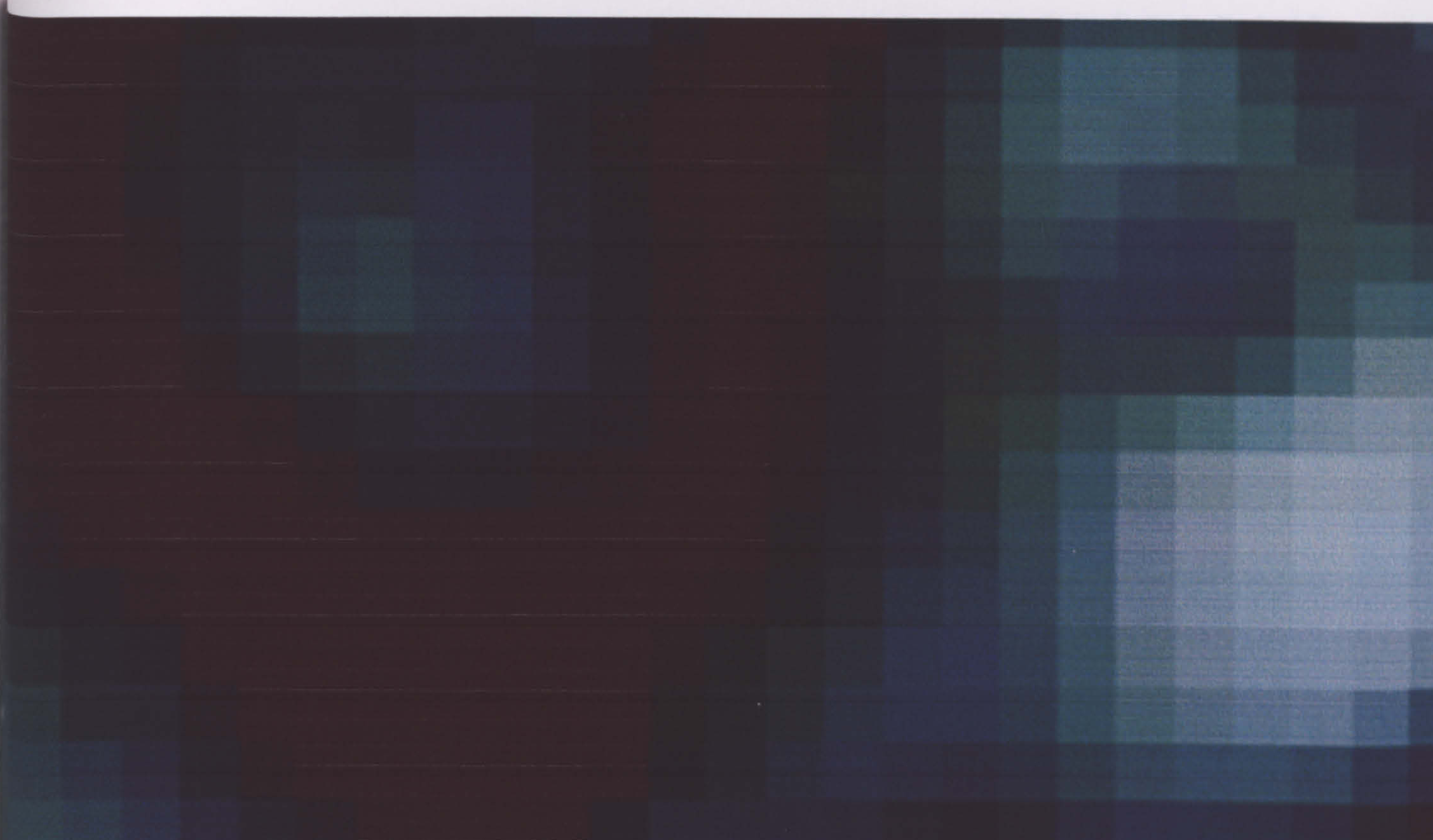
The integration of all considerations presented in this thesis and the intuitive artistic inclination of designer lead to an avenue which allows the creation of new designs and the exploration of unlimited design variations. In the 15th-16th century, Leonardo da Vinci gave advice to stimulate imagination and fantasy, as Escher translated below:

> *" If you have to depict a scene, look at some walls daubed with marks or built from stones of different kinds. In them you will see a resemblance to diversity of mountainous landscapes, rivers, rocks, trees, sweeping plains and hills. You can also see battles and human figures, whose shapes you could straighten and improve. These crumbling walls are like the peals of church bells in which you can hear any name or word you choose."*
>
> [Escher, cited by Bool, 1986, pp. 20-21]

In the 21st century, technological innovation in computer-aided design will revolutionise the working methods as well as the vision of designers.

In the 15^{th}-16^{th} century what did artists see on the stone wall?

In the 21st century what do designers perceive on the computer screen?



References

Abas, S.J. and Salman, A.S. (1995) *Symmetries of Islamic Geometrical Patterns*, London: World Scientific Publishing.

Albarn, K. and Others (1974) *The Language of Pattern*, London: Thames and Hudson.

Archambault, M. (1988) *Traditional Patterns of Textiles from Thailand*, Tissus Royaux, Tissus Villageois De Thailande, France: Musee De L'impression Sur Etoffes.

Bool, F. (ed.) (1986) *M.C. Escher (1898-1972): Regular Divisions of the Plane at the Haag Gemeentemuseum*, Translated by K. Ruth, Gent: Snoeck-Ducaju & Zoon.

Bourgoin, J. (1973) *Arabic Geometrical Pattern & Design*, New York: Dover Publications.

Briggs, J. (1992) *Fractals the Patterns of Chaos*, London: Thames and Hudson.

Buhler, A. and Fischer, E. (1979) *The Patola of Gujarat: Double Ikat in India (Vol.1-2)*, Basle: Krebs AG.

Cervini, L., Farinato, R. and Loreto, L. (1986) The Interactive Computer Graphic (ICG) Production of the 17 2D Crystallographic Groups and Other Related Topics, In: Coxeter, H.S.M. (ed. al.), *M.C. Escher: Art and Science*, Proceedings of the International Congress on M.C. Escher, Rome, Italy, 26-28 March, 1985, 3rd Printing (1988), Amsterdam: Elsevier Science Publishers B.V., pp.269-284.

Cheeseman, P. (1988) *Lao Textiles: Ancient Symbols - Living Art*, Bangkok: White Lotus.

Christie, A.H. (1910) *Traditional Methods of Pattern Designing*, Oxford: Clarendon Press, Reprinted as *Pattern Design: An Introduction to the Study of Formal Ornament*, 2nd ed. (1969), London: Dover Publication.

Chuintaranond, S. (1990) Mandala: Segmentary State and Politics of Centralization in Medieval Ayudhaya, *the Journal of the Siam Society*, Vol. 78, Part 1, pp.89-99.

Conway, S. (1992) *Thai Textiles*, London: British Museum Press.

Coxeter, H.S.M. (ed. al.) (1986) *M.C. Escher: Art and Science*, Proceedings of the International Congress on M.C. Escher, Rome, Italy, 26-28 March, 1985, 3rd Printing (1988), Amsterdam: Elsevier Science Publishers B.V.

Critchlow, K. (1969) *Order in Space*, London: Thames and Hudson.

Critchlow, K. (1976) *Islamic Patterns: An Analytical and Cosmological Approach*, London: Thames and Hudson.

Dachatiwong Na Ayuthaya, K., *Thai textiles* (in Thai: *Sing Tor Thai*), Bangkok: Magic Line.

Day, L.F. (1903) *Pattern Design*, Reprinted (1979), London: B.T. Batsford.

Department of Industrial Promotion, *Ikat Textiles* (in Thai: *Pha Mat-mi*), Bangkok.

Department of Industrial Promotion, *Khit Textiles* (in Thai: *Pha Thor Lai Khit*), Bangkok.

Dunham, D.J. (1986) Creating Hyperbolic Escher Patterns, In: Coxeter, H.S.M. (ed. al.), *M.C. Escher: Art and Science*, Proceedings of the International Congress on M.C. Escher, Rome, Italy, 26-28 March, 1985, 3rd Printing (1988), Amsterdam: Elsevier Science Publishers B.V., pp.241-248.

El-Said, I. and Parman, A. (1976) *Geometric Concepts in Islamic Art*, London: World of Islam Festival Publishing Company.

Emmer, M. (ed.) (1993) *The Visual Mind: Art and Mathematics*, London: the MIT Press.

Ernst, B. (1994) *The Magic Mirror of M.C. Escher*, Taschen America L.L.C.

Fraser-Lu, S. (1988) *Hand Woven Textiles of South-East Asia*, Oxford/New York: Oxford University Press.

Ginsburg, H. (1989) *Thai Manuscript Painting*, London: British Library.

Gittinger, M. (1989) A Reassesment of the Tampan of South Sumatra, In: *To Speak with Cloth: Studies in Indonesian Textiles*, Museum of Cultural History, University of California, pp.225-239.

Gittinger, M. and Lefferts, H.L., Jr. (1992) *Textiles and the Tai Experience in Southeast Asia*, Washington D.C.: the Textile Museum.

Gombrich, E.H. (1979) *The Sense of Order. A Study in the Psychology of Decorative Art*, Reprinted (1998), London: Phaidon Press.

Grünbaum, B. and Shephard, G.C. (1987) *Tilings and Patterns*, New York: W.H. Freeman and Company.

Grünbaum, B. and Shephard, G.C. (1993) Interactive Patterns in Islamic and Moorish Art, In: Emmer, M. (ed.), *The Visual Mind: Art and Mathematics*, London: the MIT Press, pp.147-155.

Guy, J. (1989) Sarasa and Patola: Indian Textiles in Indonesia, *Orientations*, Vol. 20, Part 1, pp.48-60.

Guy, J. (1992) Indian Textiles for the Thai Market - A Royal Prerogative, *The Textile Museum Journal*, Washington D.C., Vol.31, pp.82-96.

Guy, J. (1998) *Woven Cargoes: Indian Textiles in the East*, London: Thames and Hudson.

Haake, A. (1996) *Symmetry of Javanese Batik Patterns*, Paper Presented at the International Symposium on Indonesian Textiles, Jambi, Indonesia, Nov. 6-9.

Hall, P. (1999) Repeat After Me, *I.D. Magazine*, September/October, 1999, p.27.

Hammond, C. (1997) *The Basics of Crystallography and Diffraction*, International Union of Crystallography, New York: Oxford University Press.

Hann, M.A. (1991) *Classification and Analysis of Regular Geometric Patterns with Particular Reference to Textile*, PhD Thesis, School of Textile Industries, The University of Leeds.

Hann, M.A. (1992) Symmetry in Regular Repeating Pattern: Case Studies from Various Cultural Settings, *Journal of Textile Institute*, Vol. 83, No. 4, pp.579-590.

Hann, M.A. and Thomson, G.M. (1992) The Geometry of Regular Repeating Pattern, *Textile Progress*, Vol.22, No.1.

Hann, M.A. (1993) The Symmetry Preferences Exhibited by Japanese Textile Patterns Produced During the Edo Period (1604-1867*), ARS Textrina*, Vol. 19, pp.37-59.

Hargittai, I. (ed.) (1986) Symmetry: Unifying Human Understanding (Part 1), *An International Journal of Computers & Mathematics with Applications*, Part B, Vol.12B, Oxford: Pergamon Press.

Hargittai, I. (ed.) (1989) *Symmetry 2: Unifying Human Understanding*, Oxford: Pergamon Press.

Hawley, M. (ed.) (1997) *Robert Silvers: Photomosaics*, New York: Henry Holt and Company.

Hoggar, S.G. (1992) *Mathematics for Computer Graphics*, Cambridge University Press.

Horne, C.E. (1997) *The Application of the Principles of Geometric Symmetry to the Construction of Regularly Repeating Patterns and Tilings,* PhD Thesis, School of Textile Industries, The University of Leeds.

Jacobs, K. (1998) The Maeda Touch, *I.D. Magazine*, November, 1998, pp.58-65.

Jones, O. (1856) *The Grammar of Ornament*, Reprinted (1987/1988), London: Parkgate Books.

Joray, M. (1965) *Plastic Arts of the 20th Century: Vasarely*, Editions Du Griffon Neuchalel, Switzerland.

Jumsai, S. (1982) Water (Part 3), In: *Thai Characteristics: Vol.1 Background* (in Thai: Nam (Part 3), In: *Luk-sa-na Thai: Vol.1 Phum-lang*), Bangkok: Thai Wattana-panich Press.

Jumsai, S. (1997) *Naga: Cultural Origins in Siam and the West Pacific*, Bangkok: Chalermnit Press and DD Books.

Kalyanamitra, C. (1982) Traditional Thai Architectures (Part 4), In: *Thai Characteristics: Vol.1 Background* (in Thai: Satapattayakhum Bab Thai-Duem (Part 4), In: *Luk-sa-na Thai: Vol.1 Phum-lang*), Bangkok: Thai Wattana-panich Press.

Kappraff, J. (1991) *Connections: The Geometric Bridge between Art and Science*, New York: McGraw-Hill.

Kosek, M. and Koskova, B. (1984) Computer-aided Patterning of Surface Design, In: *Computer in the World of Textiles,* Papers Presented at the Annual World Conference, September, 26-29, Hong Kong, The Textile Institute.

Larsen, J.L. (1976) *The Dyer's Art, Ikat, Plangi*, London: Van Nostrand Reinhold Company.

Leesuwan, V. (ed.) (1987) *Thai Textiles* (in Thai: *Pha Thai*), Bangkok: IFCT.

Lin, X. (1995) *The Application of the Principles of Symmetry to the Synthesis of Multi-coloured Counterchange Patterns*, PhD Thesis, School of Textile Industries, The University of Leeds.

Locher, J.L. (ed.) (1982) *M.C. Escher: His Life and Complete Graphic Work*, New York: Harry N. Abrams.

Loeb, A.L. (1971) *Colour and Symmetry*, USA: John Wiley & Sons.

Loreto, L., Farinato, R. and Tonetti, M. (1993) Crystallography and Plane Ornaments: Interactive Multi-Window Computer Graphics, In: Emmer, M. (ed.), *The Visual Mind: Art and Mathematics*, London: the MIT Press, pp.187-191.

MacGillavry, C.H. (1976) *Fantasy and Symmetry: The Periodic Drawings of M.C. Escher*, New York: Harry N. Abrams.

Mandelbrot, B.B. (1993) Fractals and an Art for the Sake of Science, In: Emmer, M. (ed.), *The Visual Mind: Art and Mathematics*, London: the MIT Press, pp.11-14.

McNamara, A. and Snelling, P. (1995) *Design and Practice for Printed Textiles*, Melbourne: Oxford University Press.

Metropolis (2000) Retail 2010, *FW*, Spring/Summer 2000.

Nakayama, S. (1994) Poly 2000, In: *Stereogram*, London: Boxtree, p.16.

Padwick, R. and Walker, T. (1977) *Pattern: Its Structure and Geometry*, Sunderland Arts Centre: Ceolfrith Press.

Phillips, P. and Bunce, G.E. (1993) *Repeat Pattern: A Manual for Designers, Artists and Architects*, London: Thames and Hudson.

Phra Devabhinimmit (1994) *Thai Pattern Books* (in Thai: *Samud Tham-ra Lai Thai*), Bangkok: Karu-sapha.

Pramoj, K. (1982) Thai Society (Part 1), In: *Thai Characteristics: Vol.1 Background* (in Thai: Sung-khom Thai (Part 1), In: *Luk-sa-na Thai: Vol.1 Phum-lang*), Bangkok: Thai Wattana-panich Press.

Prangwatthanakun, S. and Naenna, P. (1990) *Lan-na Textiles: Yuan, Lue, Lao*, Chiang Mai: The Centre for the Promotion of Arts and Culture, Chiang Mai University.

Prangwatthanakun, S. and Naenna, P. (1994) *Thai Textiles: Thread of a Cultural Heritage*, Bangkok: National Identity Board.

Rheingold, H. (1994) Foreword, In: *Stereogram*, London: Boxtree, pp.6-9.

Rosen, J. (1975) *Symmetry Discovered: Concepts and Application in Nature and Science*, Cambridge University Press.

Sachio, Y. (1993) *Sarasa, Printed and Painted Textiles*, Japan: Kyoto Shi: Kyoto Shoin

Schattschneider, D. (1978) The Plane Symmetry Groups: Their Recognition and Notation, *American Mathematical Monthly*, Vol.85, pp.439-450.

Schattschneider, D. (1986) M.C. Escher's Classification System for His Coloured Periodic Drawings, In: Coxeter, H.S.M. (ed. al.), *M.C. Escher: Art and Science*, Proceedings of the International Congress on M.C. Escher, Rome, Italy, 26-28 March, 1985, 3rd Printing (1988), Amsterdam: Elsevier Science Publishers B.V., pp.82-96.

Schattschneider, D. (1986) In Black and White: How to Create Perfectly Coloured Symmetric Patterns, In: Hargittai, I. (ed.), Symmetry: Unifying Human Understanding (Part 2), *An International Journal of Computers & Mathematics with Applications*, Part B, Vol.12B, Oxford: Pergamon Press, pp.673-695.

Schattschneider, D. (1990) *Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M.C. Escher*, 5th Printing (1998), New York: W.H. Freeman.

Schattschneider, D. (1993) The Fascination of Tiling, In: Emmer, M. (ed.), *The Visual Mind: Art and Mathematics*, London: the MIT Press, Chapter 23, pp.157-164.

Shubnikov, A.V. and Belov, N.V. (1964) *Coloured Symmetry*, Oxford/London: Pergamon Press.

Spies, W. (1971) *Victor Vasarely*, London: Thames and Hudson.

Stevens, P.S. (1981) *Handbook of Regular Patterns. An Introduction to Symmetry in Two Dimensions*, 5th Printing (1996), Cambridge: the MIT Press.

Summerfield, A. (1996) *Symbolic Meanings in Minangkabau Ceremonial Cloths*, Paper Presented at the International Symposium on Indonesian Textiles, Jambi, Indonesia, Nov. 6-9.

Taylor, P.Y. (1994) *Breasts, Birds, and Blossoms in Thai Art*, Oxford University Press.

Tyler, C.W. (1994) The Birth of Computer Stereograms for Unaided Stereovision, In: *Stereogram*, London: Boxtree, pp.83-89.

Van Esterik, P. (1979) Symmetry and Symbolism in Ban Chiang Painted Pottery, *Journal of Anthropological Research*, Vol. 35, No. 4, pp.495-508.

Wade, D. (1976) *Pattern in Islamic Art*, London: StudioVista.

Wade, D. (1982) *Geometric Patterns & Borders*, London: Wildwood House.

Wanamas, S. (1991) *Northeastern Thai Textiles* (in Thai: *Pha Thai Lai Esarn)*, Bangkok: Odean Store.

Walter, C. (1992) The Impact of Computer Graphics on Clothing Design, In: Aldrich, W. (ed.) *CAD in Clothing and Textiles*, 2nd ed., London: Blackwell Science, pp. 82-89.

Warren, W. and Tettoni, L.I. (1994) *Arts and Crafts of Thailand*, London: Thames and Hudson.

Washburn, D.K. and Crowe, D.W. (1988) *Symmetries of Culture. Theory and Practice of Plane Pattern Analysis*, Seattle/London: University of Washington Press.

Watson, W. (1954) *Textile Design and Colour: Elementary Weaves and Figured Fabrics*, 6th ed. (1996), UK: Ariel Books.

Wiltshire, A. (1989) *Symmetry Patterns: The Art of Making Beautiful Patterns from Special Lattices*, Diss Norfolk: Tarquin Publications.

Woods, H.J. (1935) The Geometrical Basis of Pattern Design, Part I: Point and Line Symmetry in Simple Figures and Borders, *Journal of the Textile Institute*, Transaction 26, T.197-210.

Woods, H.J. (1935) The Geometrical Basis of Pattern Design, Part II: Nets and Sateens, *Journal of the Textile Institute*, Transaction 26, T.293-308.

Woods, H.J. (1935) The Geometrical Basis of Pattern Design, Part III: Geometrical Symmetry in Plane Patterns, *Journal of the Textile Institute*, Transaction 26, T.341-357.

Woods, H.J. (1936) The Geometrical Basis of Pattern Design, Part IV: Counterchange Symmetry in Plane Patterns, *Journal of the Textile Institute*, Transaction 27, T.305-320.

Yu, L.X. (1989) Symmetry in Chinese Arts and Crafts, *Journal of Computers & Mathematics with Applications*, Vol. 17, No. 4-6, pp.1009-1026.

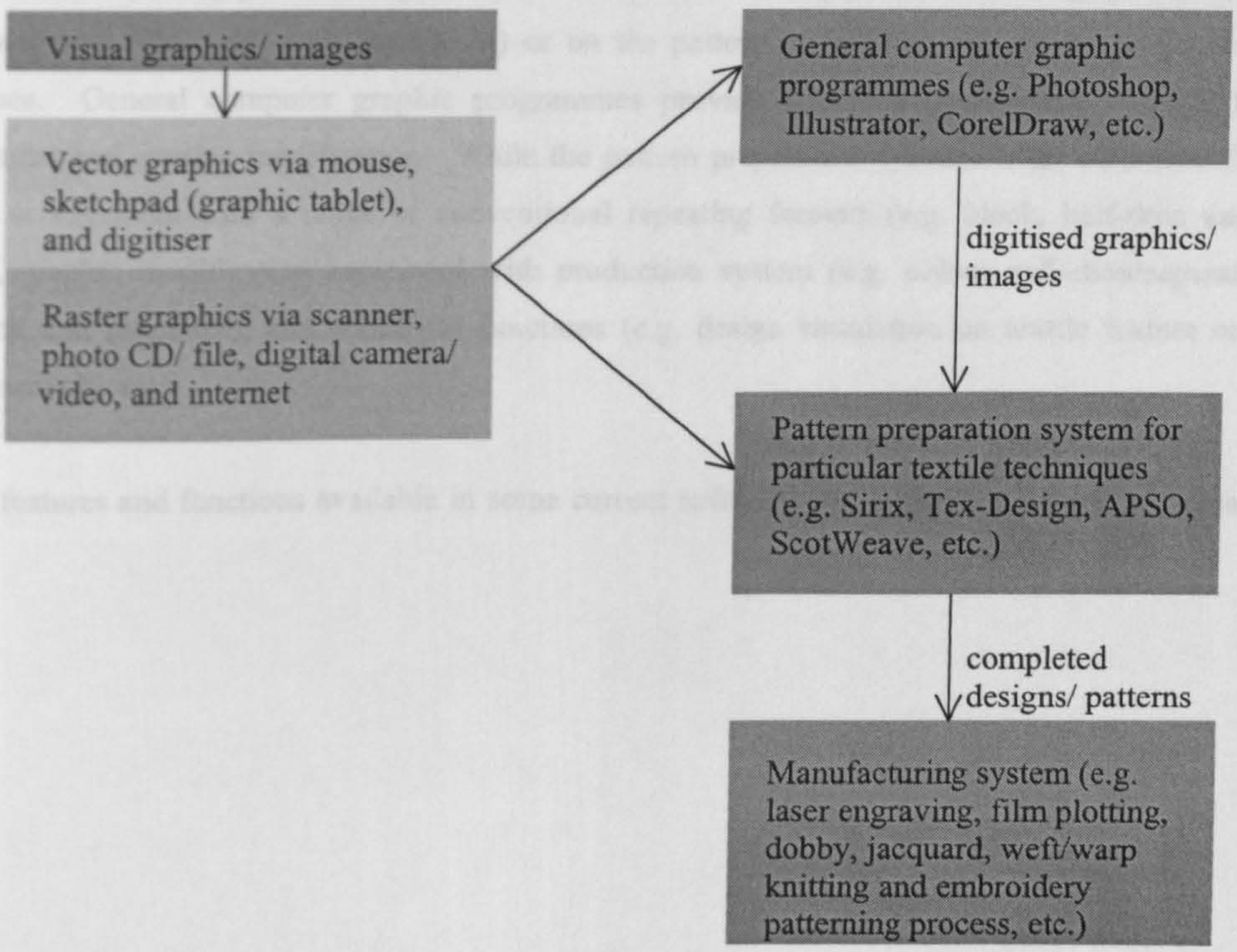
Electronic references

http://www.apso.com (09/03/99)
http://www.colorado-int.com (09/03/99)
http://www.crinson.demon.ac.uk (10/11/99)
http://www.digitile.ac.uk (10/11/99)
http://www.maedastudio.com (18/11/99)
http://www.photomosaic.com (18/11/99)
http://www.prima.com.hk (08/03/99)
http://www.scotweave.com (09/03/99)
http://www.tex-data.com (14/06/00)

Appendix A1 CAD for repeating pattern design

The integration of CAD into the design process aids designers to investigate a number of design options before manufacturing takes place. Time-consuming jobs, e.g., re-drawing, re-arranging and re-colouring have been executed rapidly and accurately. As a user interface tool, CAD provides advantage features for repeating pattern design as summarised in Figure A1.1 and discussed below.

Figure A1.1 Flow-chart showing a process of repeating pattern design by employing computer



All software facilities and design data are displayed in a window system on a monitor. A user can select a function to perform a particular operation through a series of pull-down menus, dialog boxes, command icons and keyboard. Visual graphic is generated not only by applying drawing and other manipulating functions available, but also from the combination of graphics from various sources, e.g., photographic images and artworks produced by conventional methods through scanner, digitiser or digital camera/video. These graphics are automatically transformed into digitised data. A resultant design is also visualised on the monitor in form of a surface image, simulated fabric or a three-dimensional design mapping on object in various scales and colour ways, and is able to be transferred to manufacturing system.

In fact a pattern generated either on a computer screen or on printed/knitted/woven textiles exhibit the basis of dotted configuration or pixelated graphic corresponding to x and y axes of a square lattice. As Walter [1992, p.83] pointed out, the principle of patterning knitted textiles was determined by a selection of needles, by which the sequence of selection was programmed and simulated on a square lattice corresponding to courses and wales. This can also be applied to the selection of raised yarn in the case of woven textiles, in which warp is represented by a pixel column and weft is represented by a pixel row. Theoretically, each pixel is individual which can be selected and manipulated separately. However, in some cases of woven and knitted textiles (e.g. yarn-dyed jacquard or intrasia), manufacturing system may underpin the manipulation of pixels.

The process of repeating pattern design can be developed either on general computer graphic programmes (e.g. Photoshop, Illustrator and CorelDraw) or on the pattern preparation system for particular textile techniques. General computer graphic programmes provide manually open-ended solution for unit arrangement and graphic modification. While the pattern preparation systems offer automatic facilities for unit arrangement with a range of conventional repeating formats (e.g. block, half-drop and brick repeats), graphic modification associated with production system (e.g. colour reduction/separation/co-ordination and reference), and additional functions (e.g. design simulation on textile texture or three-dimensional object).

Related features and functions available in some current software programmes are summarised in Table A1.1.

Table A1.1 Features and functions available in some current software programmes

Software	Platform		General functions			Particular functions					
	PC	MacIntosh	Drawing	Transformation	Graphic modification	Repeat formation	Colour separation	Colour reduction	Colour co-ordination & reference	Stitch/texture simulation	Manufacturing connection
General computer graphic programmes - ***Aldus Freehand*** (version 5.5)	✓	✓	✓	✓	✓						
- ***Adobe Illustrator*** (version 5.5)	✓	✓	✓	✓	✓						
- ***Adobe Photoshop*** (version 4)	✓	✓	✓	✓	✓						
- ***Corel Draw & Photo Paint*** (version 7)	✓		✓	✓	✓						
Pattern preparation systems for particular textile techniques - ***Sirix/Stoll VDU Selectanit Pattern Unit***: for weft-knitted textiles	✓		✓	✓	✓	✓	✓	✓		✓	✓
- ***ScotWeave Professional Suite***: *ScotColour* (for accurate colour matching), *ScotPaint* (for image manipulation), *Yarn Design, Dobby Design, Jacquard Design*, and separated programme for texture mapping (*ScotDrape*)	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
- ***APSO***: *Dobby/Jacquard/Carpet/Lance/Tufted Designer, Textile-DTP, Digital Assistant, 3Di, Archive,* and *Image Explorer*	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
- ***Colorado Textile Software***: *Multicolor* (for printing and colouring), *ColorKnit, ColorJacq, ColorWeave/PCweave* (for dobby fabrics), *ColorMode* (for mapping 3-D), *ColorTex* (for texture simulation) and *Prism* (for colour reference)	✓	✓ Color Weave	✓	✓	✓	✓	✓	✓	✓	✓	✓
- ***Tex-Design***: for printed textiles and fashion, and additional plug-in programmes, e.g., *Tex-Screen, Tex-Print, Tex-Check, Tex-Knit, Tex-Dress* (for 3-D mapping), *Tex-Define, Tex-Line*, and *Tex-Store*	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓
- ***PrimaVision***: for printed textiles and fashion, and additional plug-in programmes, e.g., *Maskfilm* (for printing preparation and colour separation), *Colour Knit, Weave Expert, Textile Prints*, and *PrimaVision-TCX* (for pre-sale catalogues)	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓

Sources: http://www.apso.com, http://www.colorado-int.com, http://www.prima.com.hk. http://www.scotweave.com and http://www.tex-data.com.

Appendix A2 **Terrazzo**

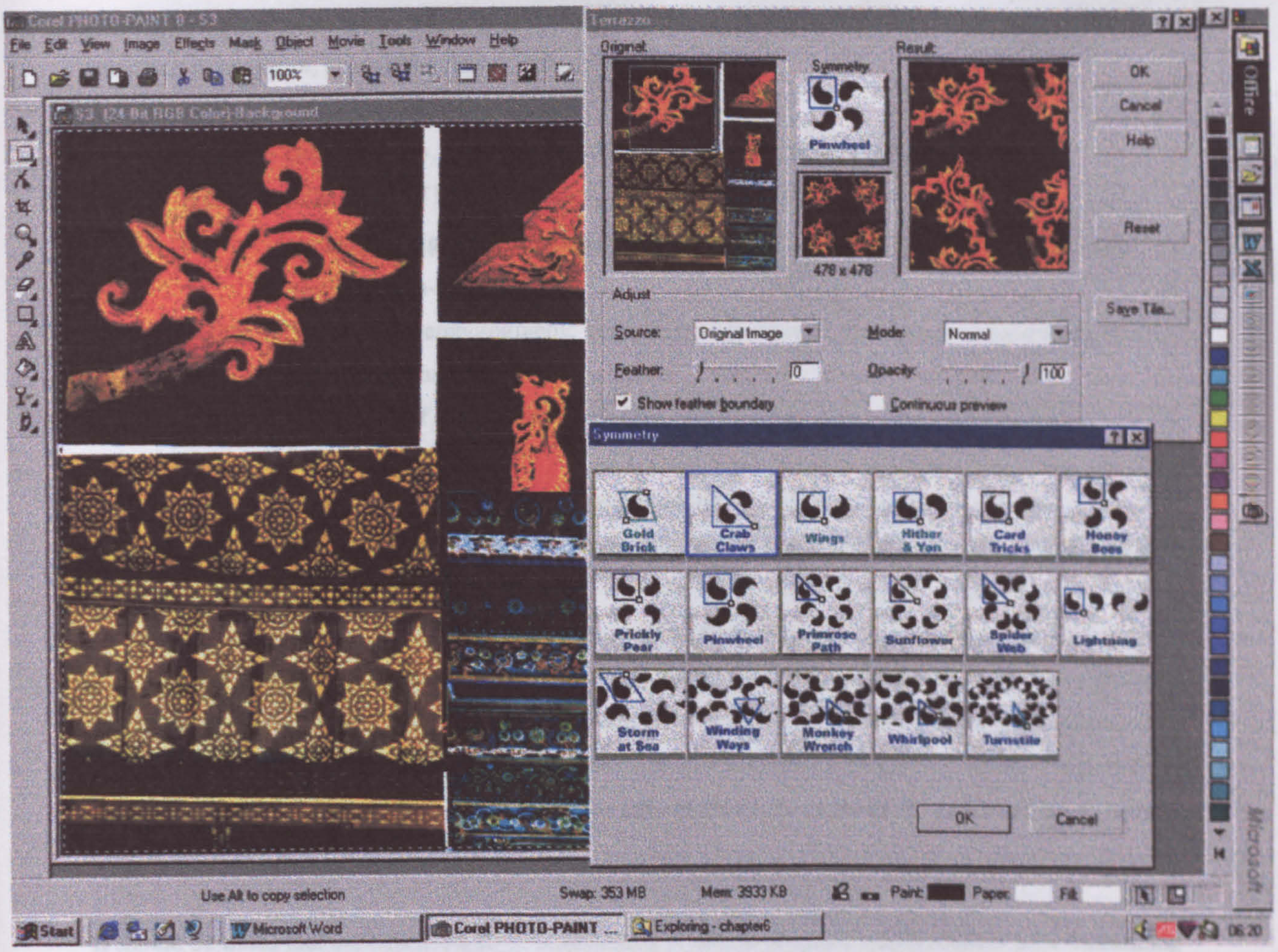
Corel Photo-Paint (version 7 onward) contains the programme called "Terrazzo" which is aimed particularly for repeating pattern permutation based on a choice of seventeen all-over symmetry groups. Relevant information including an original graphic or a generating motif, a symmetry group, a repeating-unit boundary and a resultant pattern is displayed in a dialog box as shown in Figure A2.1.

A series of a menu selection, Effects→Fancy→Terrazzo, enables a user to produce a wide range of patterns associated with seventeen symmetry classes, i.e., "Gold Brick" as class p1, "Crab Claws" as class c1m1, "Wings" as class pm11, "Hither&Yon" as class p2, "Card Tricks" as class p1g1, "Honey Bees" as class p2gg, "Prickly Pear" as class p2mm, "Pinwheel" as class p4, "Primrose Path" as class p4gm, "Sunflower" as class p4mm, "Spider Web" as class c2mm, "Lightning" as class p2mg, "Storm at Sea" as class p3, "Winding Ways" as class p3m1, "Monkey Wrench" as class p31m, "Whirlpool" as class p6 and "Turnstile" as class p6mm.

After a boundary of a fundamental region and a symmetry group are identified the programme automatically arranges an all-over pattern, which is then displayed as a resultant pattern. Variation can be achieved by re-locating a fundamental-region boundary and also applying some options of graphic modification, i.e., opacity, graphic mode and feature value.

Due to individual symmetry groups obtain particular polygonal shapes of the fundamental regions, that fit precisely on their underlying structures. Users can manipulate only sizes of the fixed shapes, i.e., a square shape of symmetry class p4, 45°-90°-45° triangular shapes of symmetry classes c1m1, c2mm, p4mm and p4gm, a 60°-120°-60°-120° rhombic shape of symmetry class p3, an equilateral triangular shape of symmetry class p3m1, a 30°-90°-60° triangular shape of symmetry class p6mm and a particular polygonal shape of symmetry class p31m. There is an alternative between a square and a rectangular shape for symmetry classes p2, pm11, p1g1, p2mm, p2gg and p2mg. However, in a case of "Gold Brick" (symmetry class p1), a parallelogram-shaped unit can obtain any angle and size.

Figure A2.1 Terrazzo screen



Appendix A3 Interactive Repeating Pattern Database

The establishment of the interactive repeating pattern database may aid designers to explore un-limited varieties of pattern permutation and simultaneously reduce operating time during creating and developing new designs. By working on a window system and multiple software programmes, designers have alternatives to innovate, modify or mix and match any resource kept in separated filed or folders. In fact they can build up their own database to fit individual design features generated by various techniques (e.g. printing, warp/weft knitting and jacquard/dobby weaving). Based on the concept of pattern construction discussed in chapter 4, four folders may be required:

Folder 1: Dotted lattices, e.g., a square, an isometric and other parallelogram lattices

Folder 2: Collections of linear structures, e.g., square-based structures (e.g. curvilinear, half-circled, 30° zigzag, etc.), isometric-based structures, rhombic-based structures and parallelogram-based structures.

Folder 3: Graphic/image/photographic collections, e.g., natural, ethnical, abstract and traditional motifs/patterns.

Folder 4: Collections of design outcomes.

Designers can access any stage of pattern permutation by employing these resources. A dotted lattice in folder 1 offers not only a freedom to innovate new structures through the point connection means, but also being as a series of corresponding points aiding unit placement. A regular linear structures in folder 2 provides a wide variety of design transformation ranging from interlocking patterns to isolated patterns with uniform distribution. Each repeating unit can be modified into any shape with respect to symmetry group underlying unit construction. Graphic materials from folder 3 can be matched and cropped in a determined repeating unit, which is then repeated on the underlying structure. Some designs from previous collections/seasons kept in folder 4 may have potential to be adapted, combined or re-used as graphic materials to generate new designs.

It should be noted that each material in these folders is re-usable resource which reduces time-consuming task and at the same time may enhance designers' creativity. Examples of design variations are shown in Figure A3.1-A3.7.

Figure A3.1 Four square-based varieties of scale and ogee patterns generated from identical motifs in different arrangement

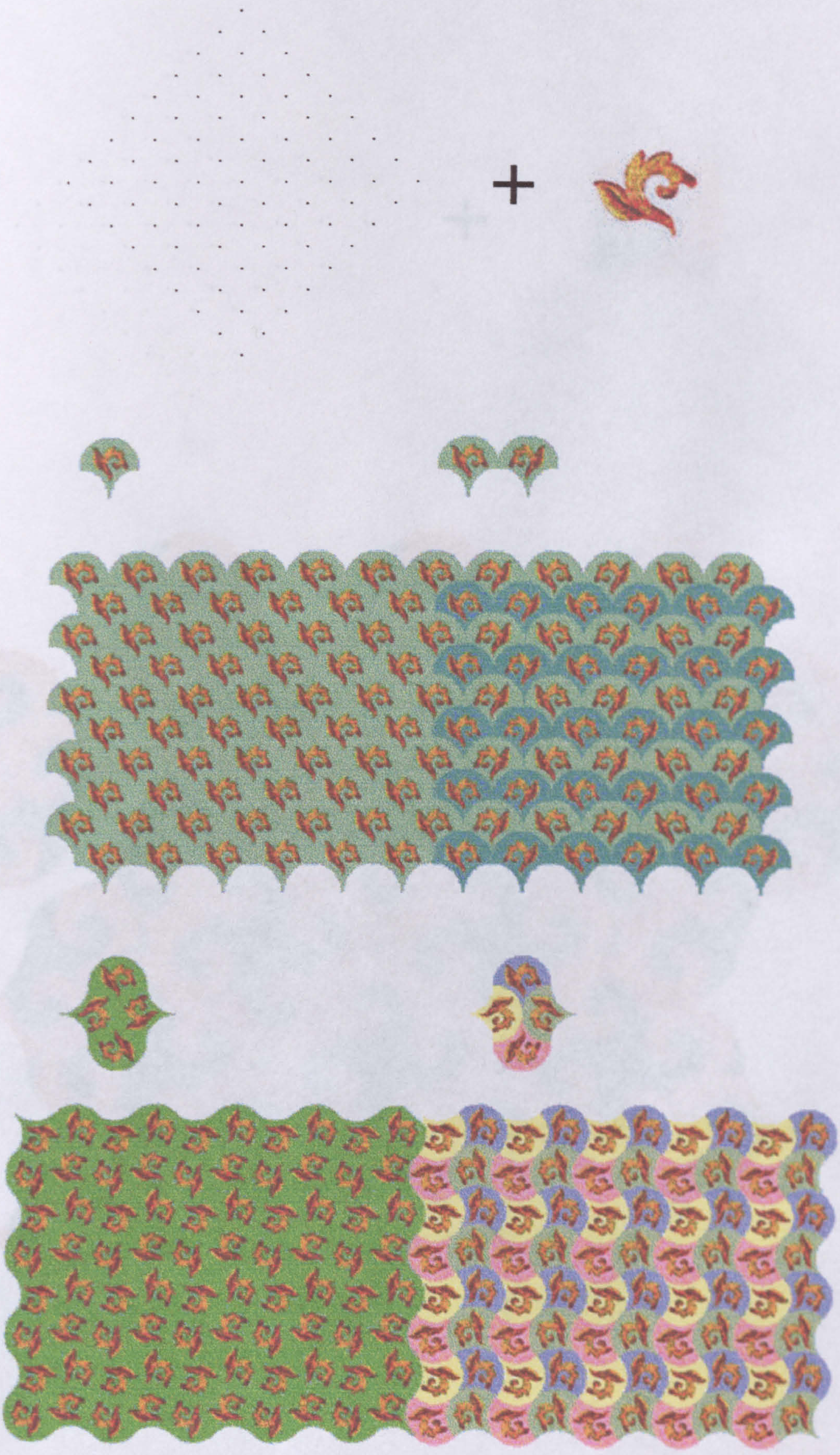


Figure A3.2 A square-based pattern generated from a motif which is cropped in a scale shape and arranged on an ogee structure

Figure A3.3 Two square-based varieties generated from fish motifs arranged on scale structures

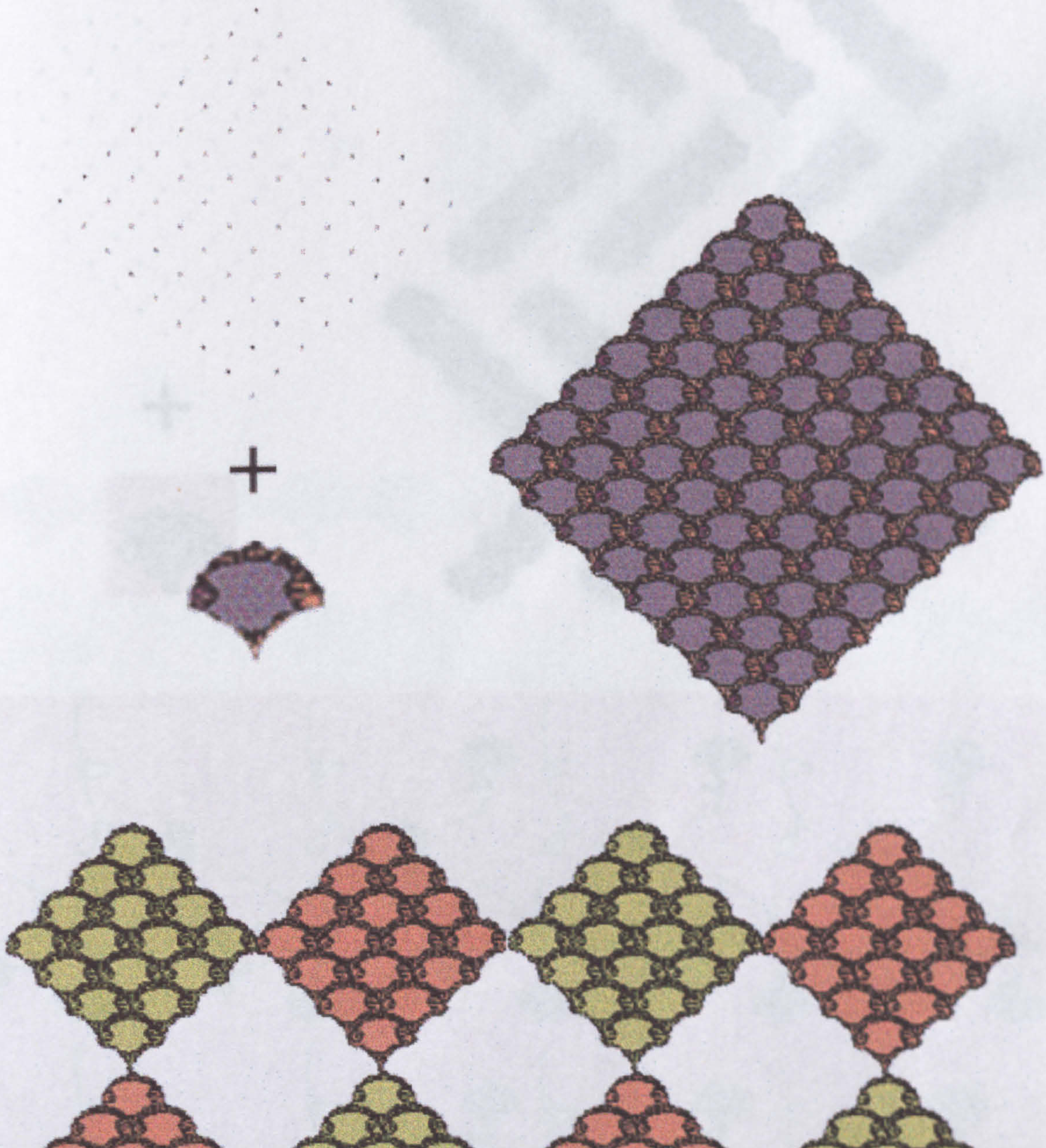


Figure A3.4 **Two isolated patterns generated from identical scale-shaped motifs: the upper one is developed on a scale structure, while the lower one is arranged on an ogee structure with additional elements**

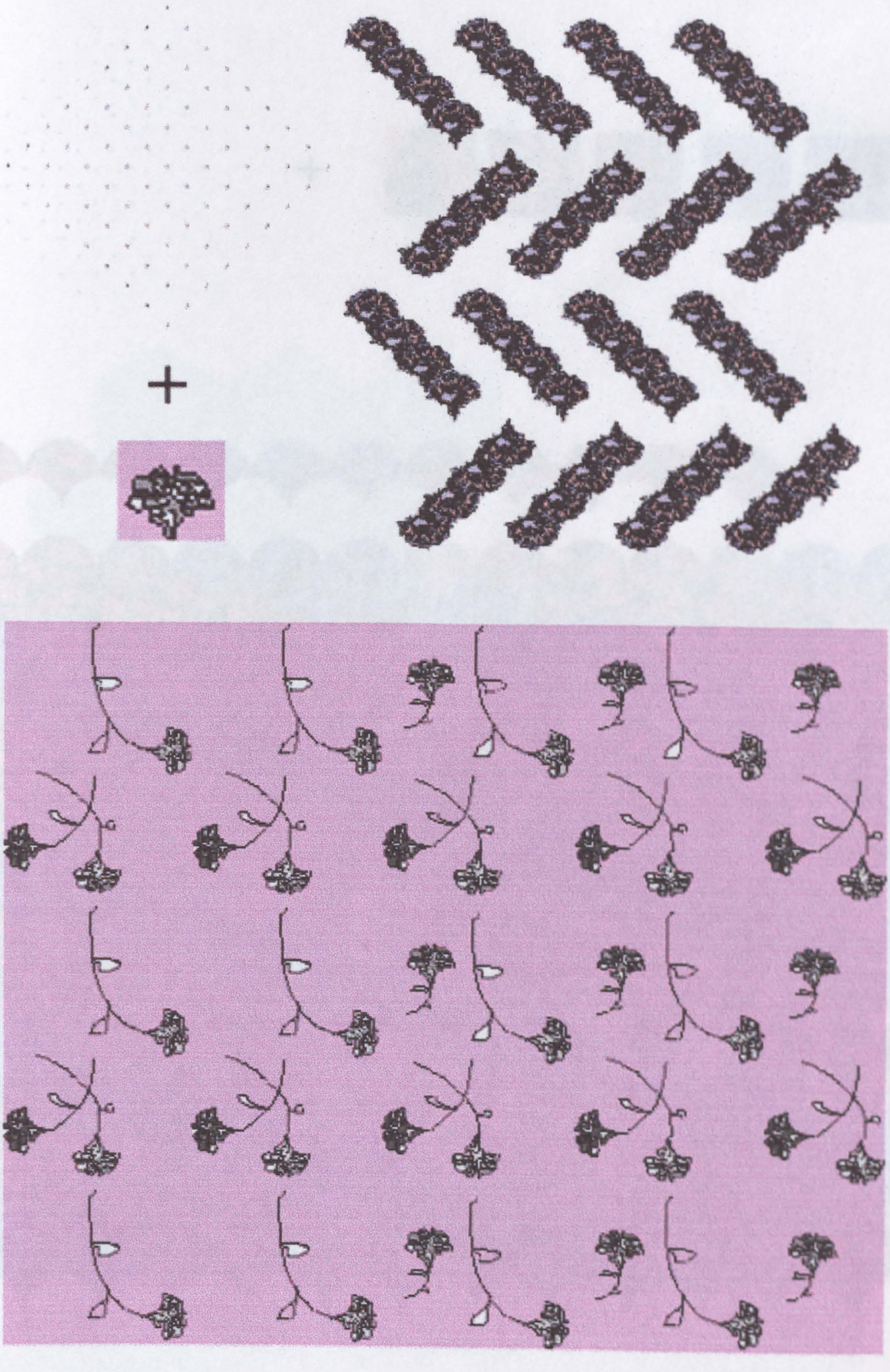


Figure A3.5 A square-based pattern generated from a series of five bacteria patterns, each of which is cropped in a scale shape

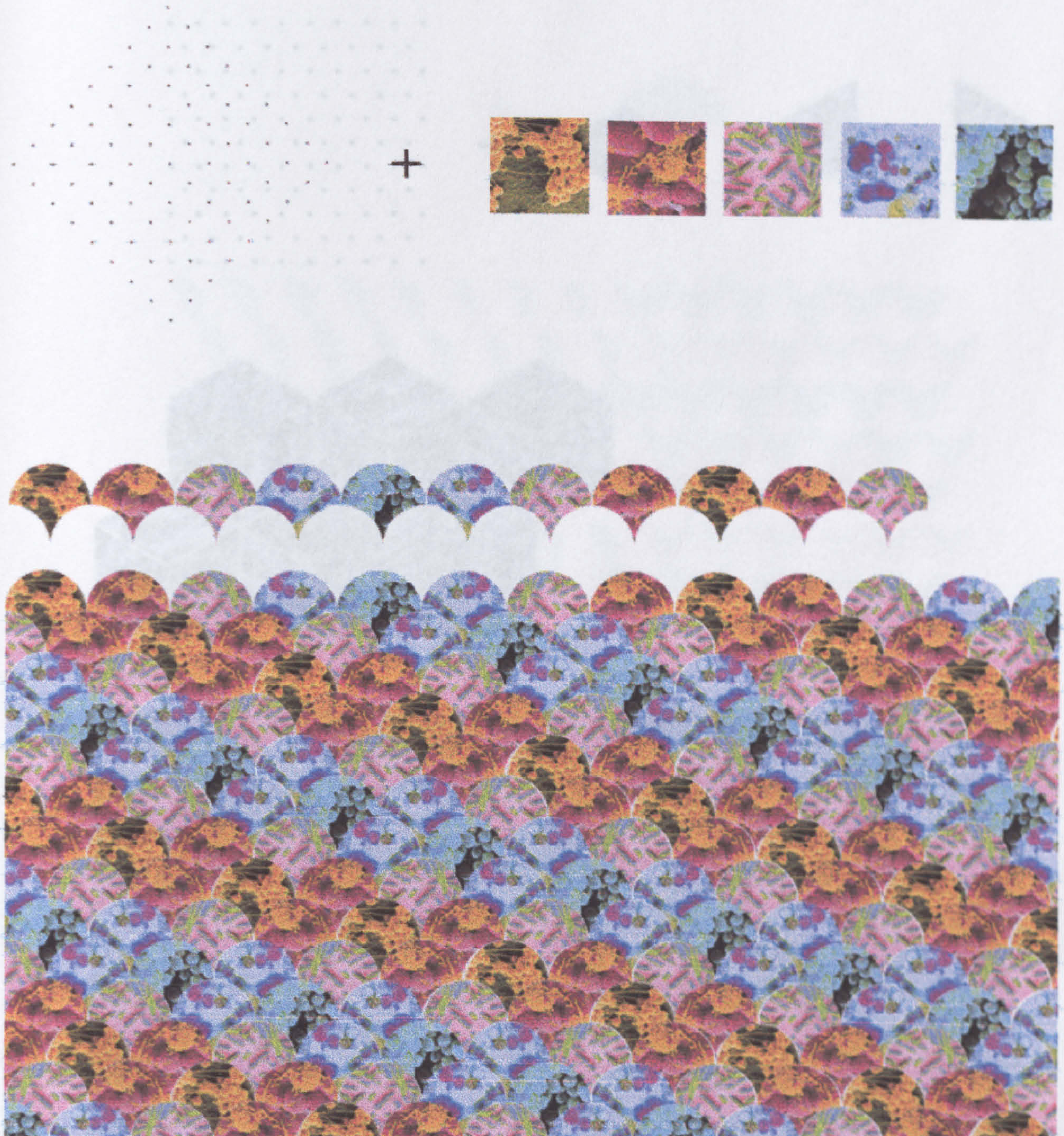


Figure A3.6 An isometric-based pattern generated from a series of three motifs cropped in rhombic shapes

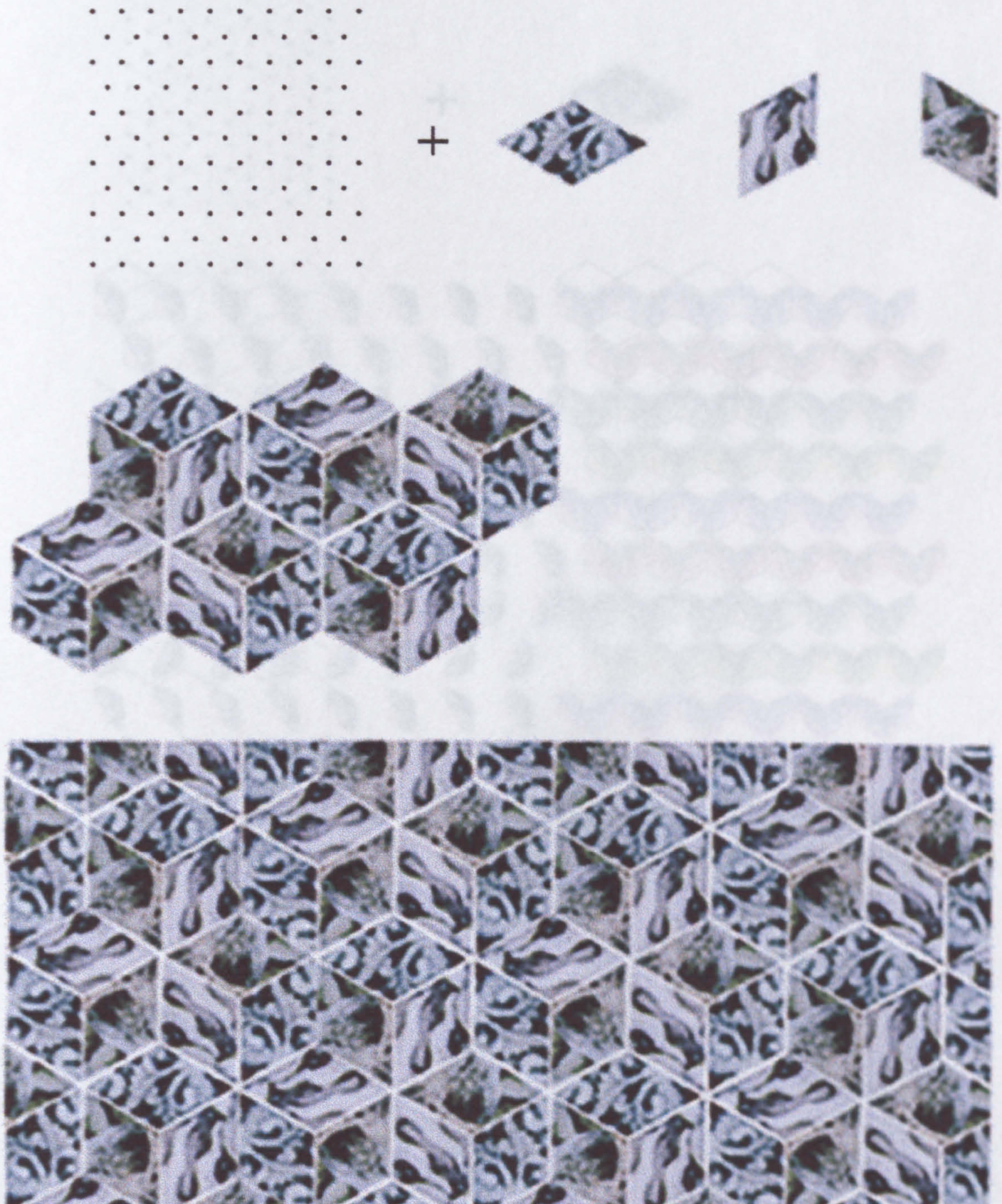


Figure A3.7 Three varieties of isometric-based patterns generated from eye motifs cropped in rhombic shapes